Cryptographic Algorithms Implemented on FPGAs

Francisco Rodríguez Henríquez

Why Secure Hardware?

- …bedded systems now common in the industry
  - …Hardware tokens, smartcards, crypto accelerators, internet …pliances
- … analysis & reverse engineering …s available to all
- …fficulty of attack
- … exist

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Attacker resources and methods vary greatly

	Teenager	Academic	Org. Crime	Gov't
	Limited	Moderate	Large	Large
	<$1000	$10K-$100K	$100K+	Unknown
	…aries	High	Varies	Varies
	…gh	High	Low	Low
	…allenge	Publicity	Money	Varies
	…y	Moderate	Few	Unknown
		No	Yes	Yes
		Yes	Varies	No

Source: Cryptography Research, Inc. 1999, "Crypto Due Diligence"

Minimal key lengths for symmetric ciphers

Source: Blaze/Diffie/Rivest/Schneier/Shimoura/Thompson/Wiener: www.bsa.org/policy/encryption

…e …er	Budget	Tool	Time and cost per key recovered		Length needed for protection in late 1995
			40 bits	56 bits	
	tiny	scavenged computer time	1 week	infeasible	45
	$400	FPGA	5 hours ($0.08)	38 years ($5,000)	50
	…,000	FPGA	12 min ($0.08)	556 days ($5,000)	55
	…K	FPGA	24 sec ($0.08)	19 days ($5,000)	60
		ASIC	18 sec ($0.001)	3 hours ($38)	
		FPGA	7 sec ($0.08)	13 hours ($5,000)	70
		ASIC	0.005 sec ($0.001)	6 min ($38)	
		ASIC	0.0002 sec ($0.001)	12 sec ($38)	75

Reconfigurable Hardware

nfigurable Hardware (RCHW) means in
mmercial applications mostly:

Programmable Gate Arrays (FPGAs)

le Programmable Logic Devices (EPLD).

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Field Programmable Gate Arrays

realize a variety of circuits:

be reprogrammed in-system,

st of boolean and storage elements,

alize fairly large circuits > 100; 000 gates.

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Reconfigurable Computing - Characteristics

is the middle ground between ASICs and processors. ASICs are the ultimate in speed but xibility while processors have the ultimate in y but lack speed.

ture is the ability to perform computations in increase performance, while retaining much ility of a software solution.

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Choosing a Platform

of implementation is driven by:

- rithm performance
- Per-unit cost, Development cost]
- onsumption (wireless devices!)
- y
- er change
- ity
- n agility
- urity

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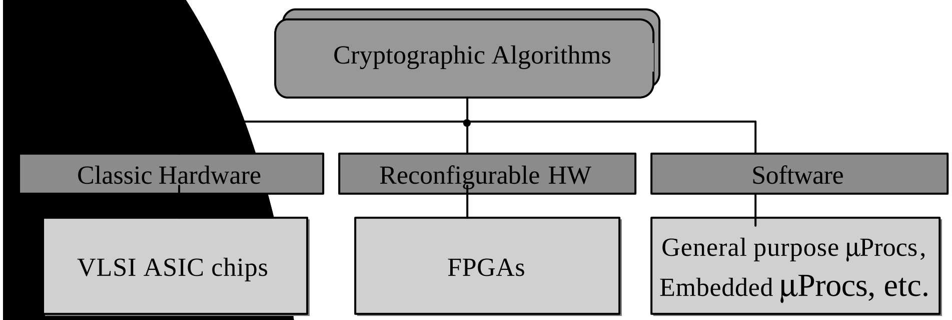
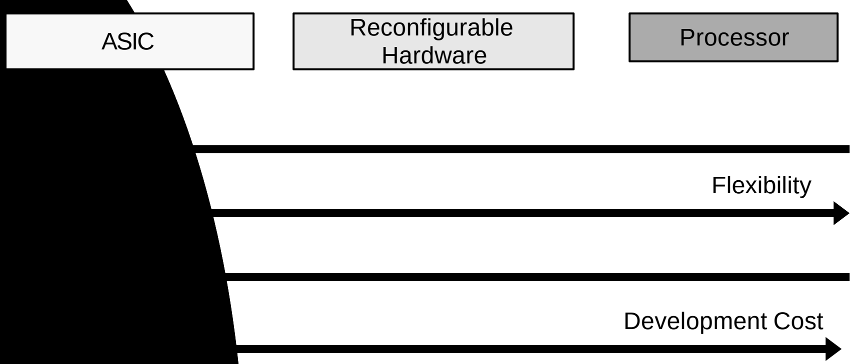
Platform Implementation for Cryptographic Algorithms



Reconfigurable Computing - defined



Why Crypto-algorithms in Hardware

[redacted] main reasons:

[redacted] tware implementations are too slow for some
[redacted] cations (symmetric alg: encryption rates
[redacted] bit/sec public-key alg: > 10 msec)

[redacted] are implementations are intrinsically more
[redacted] lly secure: Key access and algorithm
[redacted] on is considerably harder.

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But why reconfigurable hardware?

[redacted] tial advantages of crypto algorithms implemented on
[redacted] onfigurable platforms:

[redacted] rithm Agility
[redacted] thm Upgrade
[redacted] cture Efficiency
[redacted] Efficient
[redacted] n Modification
[redacted] put relative to software)
[redacted] ciency relative to ASICs)

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Crypto and FPGAs: Algorithm Agility

rvation: Modern security protocols are defined to
rithm independent:

ryption algorithm is negotiated on a per-session

variety of ciphers can be required. Ex: IPsec-
algorithms: DES, 3DES, Blow-Fish, CAST,
C4 and RC6, & future extensions!

olds for public-key algorithms, e.g., Diffie-
nd ECDH.

SIC solutions can provide algorithm agility
sts.

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Crypto and FPGAs: Algorithm Upgrade

ons may need upgrade to a new algorithm because:

nt algorithms was broken (DES)

rd expired (again DES)

ndard was created (AES)

list of algorithm independent protocol was

-implemented algorithm is practically
devices are affected or in applications
ommunications.

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Crypto and FPGAs: Architecture Efficiency

In certain cases a hardware architecture can be much
ore efficient if it is designed for a specific set of
rameters. Parameters for cryptographic algorithms
be for example the key, the underlying finite
the coefficient used (e.g., the specific curve of
C system), and so on. Generally speaking, the
pecific an algorithm is implemented the more
t it can become.

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Crypto and FPGAs: Resource Efficiency

vation: The majority of security protocols uses
key as well as public-key algorithms during
ssion, but not simultaneous.

A device can be used for both through run
uration.

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Crypto and FPGAs: Algorithm Modification

- ne applications require Public algorithms (such as candidates) with proprietary modules, e.g., tary S-boxes or permutations.
- of modes of operations (feedback modes, ode, etc.)
- alytical implementation, such as key-search may use slightly altered version of the

e changes can readily be implemented.

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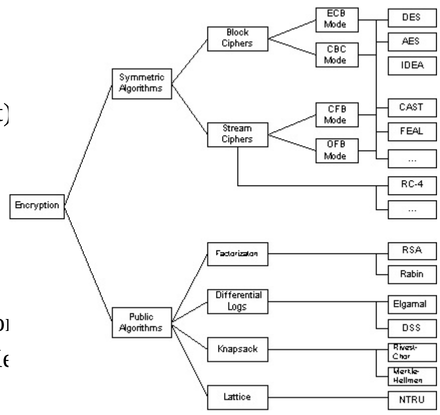
Cryptography

- dentiality is provided by on primitives
- the process of an readable (plaintext) unintelligible rtext)
- e inverse
- ion primitives
- phers (Conventio
- iphers (Public Ke



Cryptographic Primitives for Encryption

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Case of Study 1: GF(2^m) Squaring

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GF(2^m) Squarer

- …n most algorithms the modular product is computed in two steps: …ynomial multiplication followed by modular reduction. Let *A(x)* Î …m) be an arbitrary element in the field and *P(x)* be the irreducible …nerator polynomial.
- …o compute the modular square of the element *A*(*x*) we first … polynomial product *C(x)*, of degree at most *2m-2*, as

…omial product …oordinates

$$C(x) = A(x)A(x) = \left(\sum_{i=0}^{m-1} a_i \alpha^i\right)\left(\sum_{i=0}^{m-1} a_i \alpha^i\right)$$

- …ond step, a reduction operation is performed in order …-1 degree polynomial *C'(x)* defined as

…ction step …rdinates

$$C'(x) = C(x) \bmod P(x)$$

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Squaring: Example

[redacted] be an element of the finite field $F=GF(2^5)$. Then, the square of A [redacted] as,

$$a_4a_3a_2a_1a_0 \;*\; a_4a_3a_2a_1a_0$$

$a_4a_0\ \ a_3a_0\ \ a_2a_0\ \ a_1a_0\ \ a_0a_0$ $+$

$a_4a_1\ \ a_3a_1\ \ a_2a_1\ \ a_1a_1\ \ a_0a_1$

[redacted] $a_3a_2\ \ a_2a_2\ \ a_1a_2\ \ a_0a_2$

[redacted] $a_2a_3\ \ a_1a_3\ \ a_0a_3$

[redacted] a_0a_4 $=$

[redacted] $a_2\quad 0\quad a_1\quad 0\quad a_0$

[redacted] element A in the field $F=GF(2^5)$, we have,

[redacted] $= A^2(x) = \left(\sum_{i=0}^{m-1} a_i x^i\right)\left(\sum_{i=0}^{m-1} a_i x^i\right) = \sum_{i=0}^{m-1} a_i x^{2i}$

Squaring: Software Solution

```
[redacted]_low[256] = {
[redacted] 5, 16, 17, 20, 21, 64, 65, 68, 69, 80, 81, 84, 85,
[redacted] 272, 273, 276, 277, 320, 321, 324, 325, 336, 337, 340, 341,
[redacted] 1040, 1041, 1044, 1045, 1088, 1089, 1092, 1093, 1104, 1105, 1108, 1109,
[redacted] 1296, 1297, 1300, 1301, 1344, 1345, 1348, 1349, 1360, 1361, 1364, 1365,
[redacted] 4112, 4113, 4116, 4117, 4160, 4161, 4164, 4165, 4176, 4177, 4180, 4181,
[redacted] 4368, 4369, 4372, 4373, 4416, 4417, 4420, 4421, 4432, 4433, 4436, 4437,
[redacted] 5136, 5137, 5140, 5141, 5184, 5185, 5188, 5189, 5200, 5201, 5204, 5205,
[redacted] 5393, 5396, 5397, 5440, 5441, 5444, 5445, 5456, 5457, 5460, 5461,
[redacted] 16400, 16401, 16404, 16405, 16448, 16449, 16452, 16453, 16464, 16465, 16468
[redacted] 16656, 16657, 16660, 16661, 16704, 16705, 16708, 16709, 16720, 16721
[redacted] 17413, 17424, 17425, 17428, 17429, 17472, 17473, 17476, 17477, 17488
[redacted] 17668, 17669, 17680, 17681, 17684, 17685, 17728, 17729, 17732, 17733
[redacted] 20481, 20484, 20485, 20496, 20497, 20500, 20501, 20544, 20545, 20548
[redacted] 20736, 20737, 20740, 20741, 20752, 20753, 20756, 20757, 20800, 20801
[redacted] 20821, 21504, 21505, 21508, 21509, 21520, 21521, 21524, 21525, 21568
[redacted] 21588, 21589, 21760, 21761, 21764, 21765, 21776, 21777, 21780, 21781
[redacted] 21841, 21844, 21845
```

Squaring: Software Implementation

```
FieldSqr2k_Random(rct_word   *ax,  rct_word   *tx,  rce_context *cntxt,
            rct_octet   *offsetptr)

index i;
ord C, S;
ex wlen, blen_p;
d *tmp;

txt->ecp->wlen;
ntxt->ecp->blen_p;

vord*) offsetptr;

p[1]=0;

n; i++) {
  sqr_table_low[(ax[i]&0xff)];
  (sqr_table_low[(ax[i]>>8)&0xff]<<16);
  sqr_table_low[(ax[i]>>16)&0xff];
  (sqr_table_low[(ax[i]>>24)&0xff]<<16);

2] = S; tmp[i*2+1] = C;

UC2K(cntxt) (tmp, blen_p, cntxt->ecp->poly);

p, blen_p, cntxt->ecp->poly);

+) tx[i] = tmp[i];
```


Second step: reduction

blem: Given the polynomial product *C(x)* with at most, *2m-1*, obtain dular product *C'* with *m* coordinates, using the generating e polynomial *P(x)*.

$$C'(x) = C(x) \bmod P(x)$$

we are interested in the polynomial reminder of the e can safely add any multiple of $P(x)$ to $C(x)$ without d result. This simple observation suggest the following reduce k bits of the polynomial product *C* at once.

Second step: reduction

...s assume that the *m+1* and *2m-1* coordinates of $P(x)$ and $C(x)$, ...ely, are distributed as follows:

$$C = [c_{2m-2} \quad c_{2m-3} \quad \dots \quad c_{2m-1-k} \quad c_{2m-2-k} \quad \dots \quad c_1 \quad c_0]$$

$$= [p_m \quad p_{m-1} \quad \cdots \quad p_1 \quad p_0]$$

...ays exists a *k*-bit constant scalar *S*, such that

$$[p_m \quad p_{m-1} \quad \dots \quad p_{m-k+1} \quad p_{m-k} \quad \dots \quad p_1 \quad p_0]$$

$$\dots_{2m-2} \quad c_{2m-3} \quad \dots \quad c_{2m-k-1} \quad p'_{m-k} \quad \dots \quad p'_1 \quad p'_0]$$

...otice that all the *k* MSB of *SP* become identical to the ... of the number *C*. By left shifting the number *SP* ...-*k-1* positions, we effectively reduce the number in

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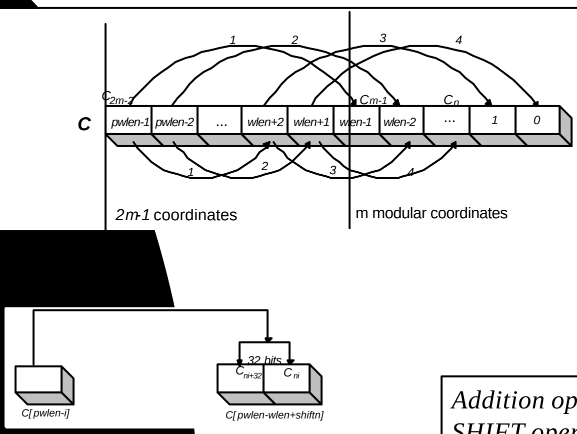
Software reduction implementation



$$wlen := \frac{m}{w}$$

Addition operations < 4wlen;
SHIFT operations < 4wlen;
Comparisons = 2wlen.

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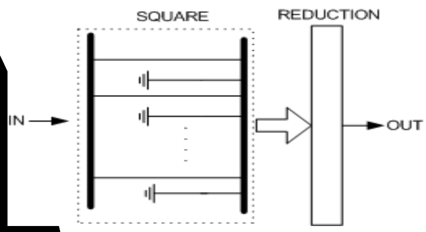
Squaring: Polynomial Multiplication Step FPGA Implementation [by Nazar Saqib]



$$A = a_3x^3 + a_2x^2 + a_1x + a_0$$
$$A^2 = a_6x^6 + a_4x^4 + a_2x^2 + a_0$$

A = 1111
A² = 1010101

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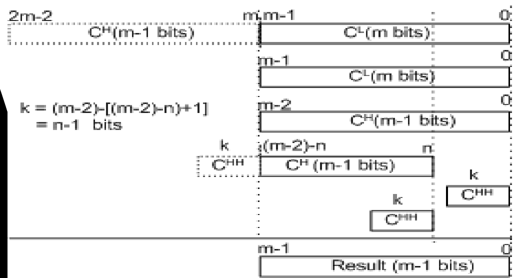
Squaring: Reduction Step FPGA Implementation [by Nazar Saqib]



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Case of Study: Modular Exponentiation

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But why are we interested in modular exponentiation in the first place?

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RSA cryptosystem by layers

Protocols and Applications: SSL, TLS, WTLS, WAP, etc.

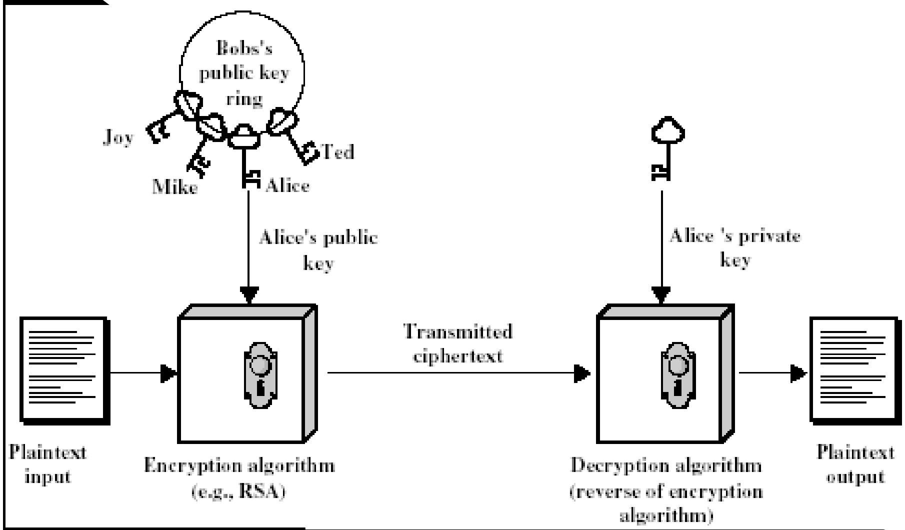
PKCS User Functions:PKCS1_OAEP_Encrypt, PKCS1_OAEP_Decrypt, PKCS1_v15_Sign,

PKCS Primitives: PKCS1_OAEP_Encode, PKCS1_OAEP_Decode, etc

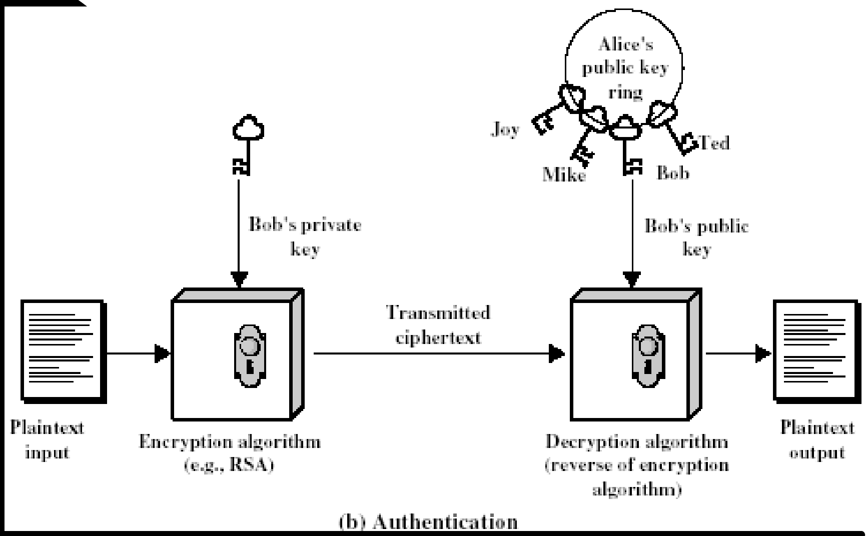
RSA primitive Operations: Encryption: $C = M^e \bmod n$, Decryption $M = C^d \bmod n$.

F_P finite field operations : Addition, Squaring, multiplication, inversion and exponentiation

Public-Key Cryptography



Public-Key Cryptography



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RSA: Key Generation

Key Generation

Select p, q	p and q both prime
Calculate $n = p \times q$	
Calculate $\phi(n) = (p-1)(q-1)$	
Select integer e	$\gcd(\phi(n), e) = 1;\ 1 < e < \phi(n)$
Calculate d	$d = e^{-1} \bmod \phi(n)$
Public key	$KU = \{e, n\}$
Private key	$KR = \{d, n\}$

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RSA: Encryption, Decryption

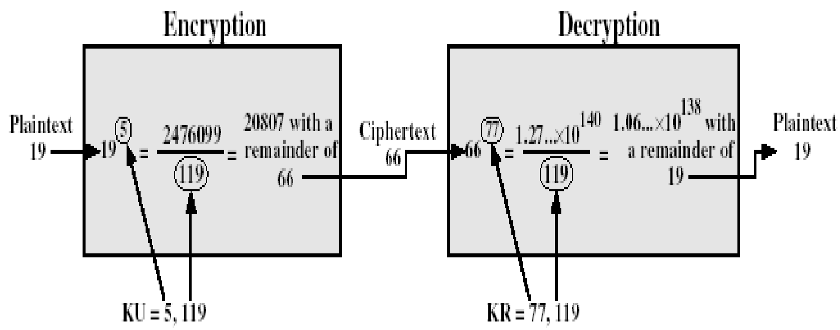
Encryption

Plaintext:	$M < n$
Ciphertext:	$C = M^e \pmod{n}$

Decryption

Ciphertext:	C
Plaintext:	$M = C^d \pmod{n}$

RSA: An Example



Example of RSA Algorithm

Modern Cryptosystems: A Top-Down Model

Applications: e-commerce, smart cards, digital money, secure communications, etc.

Crypto-protocols: Diffie-Hellman, authentication protocols, etc.

Top level Crypto-primitives: Key-pair generation, Signing and Verification

Low-level crypto-primitives: addition, doubling, scalar multiplication

F_{2^m} finite field operations : Addition, Squaring, multiplication and inversion

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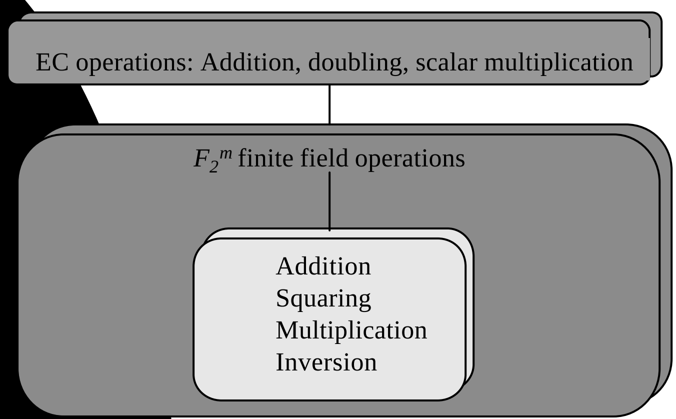
Elliptic curves over finite fields



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Arithmetic on Elliptic Curves

…dition and Doubling

- $P = (x_1, y_1)$ and $Q = (x_2, y_2)$, then $P + Q = (x_3, y_3)$
- $x_3 = \lambda^2 - x_1 - x_2$
- $y_3 = \lambda(x_1 - x_3) - y_1$

$(y_2 - y_1) / (x_2 - x_1)$ for $P \neq Q$

$(3x_1^2 + a) / 2y_1$ for $P = Q$ (doubling)

…lication

$P+P+\dots+P$ ——— k times

…ormed in the finite field $F=GF(2^m)$ over $K=GF(2)$.

…lliptic curves requires addition, squaring, …nd inversion in finite fields.

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Case of Study: Modular Exponentiation

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Modular Exponentiation

Ve do ***NOT*** compute $C := M^e \bmod n$

first computing M^e

then computing $C := (M^e) \bmod n$

rary results must be reduced modulo

each step of the exponentiation.

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Modular Exponentiation

M^{15}

ow many multiplications are needed??

wer (requires 14 multiplications):

$M^3 \rightarrow M^4 \rightarrow M^5 \rightarrow \ldots \rightarrow M^{15}$

od (requires 6 multiplications):

$^3 \rightarrow M^6 \rightarrow M^7 \rightarrow M^{14} \rightarrow M^{15}$

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Modular Exponentiation: Binary Method

e ***binary method*** requires:

quarings: *k*-1

ltiplications: The number of 1s in the binary nsion of *e*, excluding the MSB.

umber of multiplications:

(*k*-1) + (*k*-1) = 2(*k*-1)

(*k*-1) + 0 = *k*-1

k-1) + 1/2 (*k*-1) = 1.5(*k*-1)

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Modular Exponentiation

scanning the bits of *e*

time: **quaternary method**

me: **octal method**

: ***m*-ary method**.

e quaternary method: 250 = <u>11</u> <u>11</u> <u>10</u> <u>10</u>

ocessing required.

2 squaring performed.

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Modular Exponentiation: Quaternary Method

[redacted]ample:

[redacted]	bits	j	M^j
[redacted]	00	0	1
[redacted]	01	1	M
[redacted]	10	2	$M \times M = M^2$
[redacted]	11	3	$M^2 \times M = M^3$

Modular Exponentiation: Quaternary Method

[redacted]ample: e = 250 = 11 11 10 10

[redacted]	bits	*Step 2a*	*Step 2b*
[redacted]	11	M^3	M^3
[redacted]	[redacted]	$(M^3)^4 = M^{12}$	$M^{12} \times M^3 = M^{15}$
[redacted]	[redacted]	$(M^{15})^4 = M^{60}$	$M^{60} \times M^2 = M^{62}$
[redacted]	[redacted]	$(M^{62})^4 = M^{248}$	$M^{248} \times M^2 = M^{250}$

[redacted] of multiplications: 2+6+3 = 11

Modular Exponentiation: Average Number of Multiplications

[redacted]	BM	MM	d	Savings %
[redacted]	11	10	2	9.1
[redacted]	23	21	2	8.6
[redacted]	47	43	2, 3	8.5
[redacted]	95	85	3	10.5
[redacted]	191	167	3, 4	12.6
[redacted]	383	325	4	15.1
[redacted]	767	635	5	17.2
[redacted]	[redacted]535	1246	5	18.8
[redacted]	[redacted]071	2439	6	20.6

Addition Chains

[redacted] a sequence of integers $a_0, a_1, a_2, \ldots, a_r$ [redacted] and $a_r = e$. The sequence is constructed in such a way [redacted] k there exist indices $i, j = k$ such that, $a_k = a_i + a_j$.

[redacted] chain is r. A short chain for a given e implies an [redacted] rithm for computing M^e.

[redacted] BM: 1 2 3 6 12 13 26 27 54 55

[redacted] QM: 1 2 3 6 12 13 26 52 55

[redacted] M: 1 2 4 5 10 20 40 50 55

[redacted] M: 1 2 3 5 10 11 22 44 55

Addition Chains

ding the shortest addition chain is NP-complete.

bound is given by binary method:

$$\lfloor \log_2 e \rfloor + H(e) - 1$$

he *Hamming* weight of *e*.

given by Schönhage:

$$\lfloor \log_2 e \rfloor + H(e) - 2.13$$

ary, m-ary, adaptive m-ary, sliding windows,

or.

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Modular Exponentiation: Binary Method Variations

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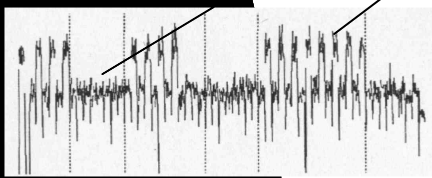
Side Channel Attacks

[redacted]inary exponentiation
[redacted] exponent d = (d_k,d_{k-1},…,d_0)
[redacted]s the most significant bit)
[redacted]G
[redacted]n to 0;
[redacted]= c*a;

The time or the power to execute c^2 and c*a are different (side channel information).



Algorithm Coron's exponentiation
Input: a in G, exponent d = (d_k,d_{k-1},…,d_{l0})
Output: c = a^d in G
1. c[0] = 1;
2. For i = k-1 down to 0;
3. c[0] = c[0]^2;
4. c[1] = c[0]*a;
5. c[0] = c[d_i];
6. Return c[0];

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Mod. Exponentiation: LSB-First Binary

[redacted] be the number of bits of *e*, i.e.,

$$k = 1 + \lfloor \log_2 e \rfloor$$

[redacted]*e*, *n*.

$$e = (e_{k-1}e_{k-2}\ldots e_1 e_0) = \sum_{i=0}^{k-1} e_i 2^i$$

[redacted]= M^e mod *n*

for $e_i \in \{0,1\}$

[redacted]:= *M;*
[redacted]to *n*-1
[redacted]1 then *R* := *R*·*C* mod *n*
[redacted]mod *n*

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Modular Exponentiation: LSB First Binary

[redacted] = 250 = (11111010), thus k = 8

[redacted]	e_i	Step 3 (R)	Step 4 (C)
[redacted]	0	1	M^2
[redacted]	1	$1*(M)^2 = M^2$	$(M^2)^2 = M^4$
[redacted]	0	M^2	$(M^4)^2 = M^8$
[redacted]	1	$M^2 * M^8 = M^{10}$	$(M^8)^2 = M^{16}$
[redacted]	1	$M^{10} * M^{16} = M^{26}$	$(M^{16})^2 = M^{32}$
[redacted]	1	$M^{26} * M^{32} = M^{58}$	$(M^{32})^2 = M6^4$
[redacted]	1	$M^{58} * M^{64} = M^{122}$	$(M^{64})^2 = M^{128}$
[redacted]	1	$M^{122} * M^{128} = M^{250}$	$(M^{128})^2 = M^{256}$

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Modular Exponentiation: LSB First Binary

[redacted]he LSB-First ***binary method*** requires:

[redacted]quarings: k-1

[redacted]ltiplications: The number of 1s in the binary [redacted]nsion of e, excluding the MSB.

[redacted] number of multiplications:

[redacted]: (k-1) + (k-1) = 2(k-1)

[redacted] (k-1) + 0 = k-1

[redacted] (k-1) + 1/2 (k-1) = 1.5(k-1)

[redacted]efore, but here we can compute the [redacted]ation operation in parallel with the [redacted]!!

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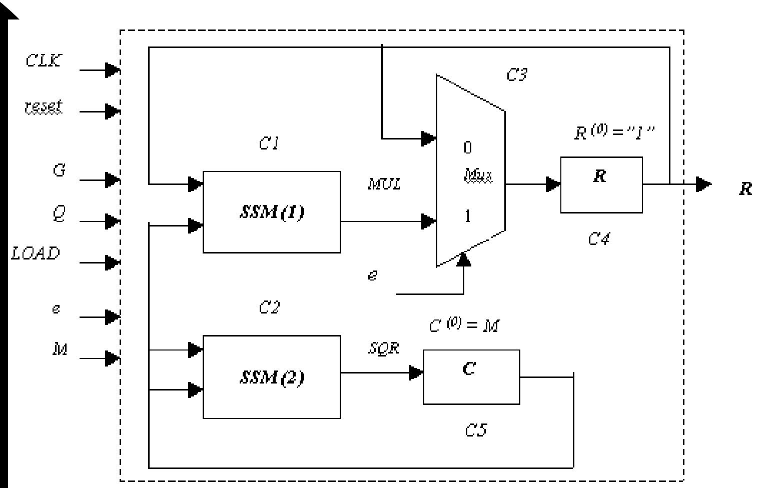
Arquitectura del Multiplicador [Mario García et al ENC03]



Desarrollo (Método q-ario)

Input: $\mathbf{x} \in GF(2^m)$: Element in the Galois field
e: Exponent
$r > 1$: Integer

Output: $\mathbf{z} = \mathbf{x}^e \in GF(2^m)$

{ $N := e$; $\mathbf{x_2} := \mathbf{x} \cdot \mathbf{x}$; $\mathbf{w}[1] := \mathbf{x}$;
  for $i = 2$ **to** $\lfloor \frac{r}{2} \rfloor$ **do** $\mathbf{w}[i] := \mathbf{w}[i-1] \cdot \mathbf{x_2}$;
  $\mathbf{z} := 1$;
  while $N > 0$ **do**
  { $d :=$ Mod$[N, r]$;
    $N := \lfloor \frac{N}{r} \rfloor$;
    if $d \neq 0$ **then**
    { express $d = 2^p q$ with p greatest possible integer, q odd ;
      $i := (q+1)/2$;
      $\mathbf{z_1} := \mathbf{w}[i]$;
      $\mathbf{z} := \mathbf{z} \cdot \mathbf{z}_1^{2^p}$
    } ;
    for $i = 1$ **to** $\lfloor \frac{r}{2} \rfloor$ **do** $\mathbf{w}[i] := \mathbf{w}[i]^r$
  } ;
  output z
}

Desarrollo (Método q-ario)

- [redacted]ecálculo de W.
- [redacted]ño de q.
- [redacted] de d = 2^p * q

Desarrollo (Análisis)

- [redacted]maño de memoria y tiempo de [redacted]ución del precómputo W.
- [redacted]o de multiplicaciones y [redacted]ones al cuadrado para método q-[redacted]

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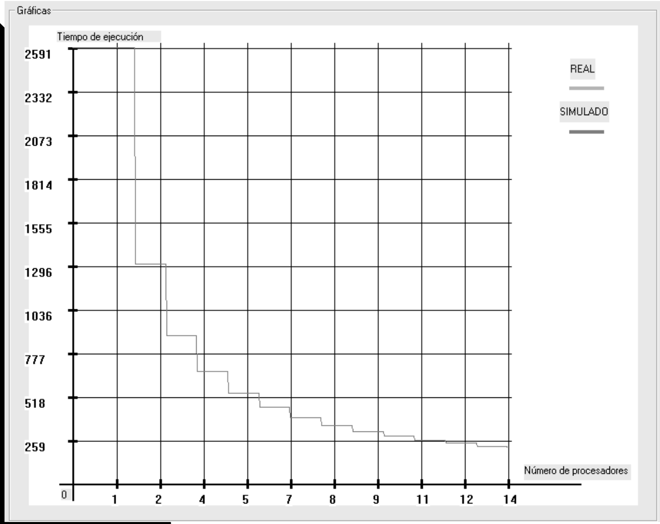
Tiempo de Ejecución Vs. Número de Procs.



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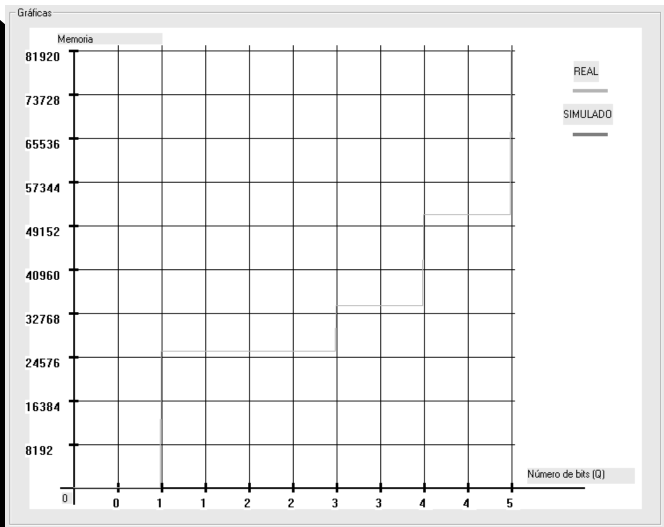
Tamaño de Memoria



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Elliptic Curve Point Multiplication Revisited

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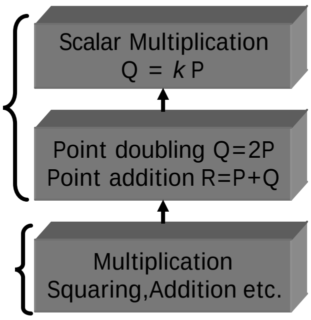
Elliptic Curve Cryptography



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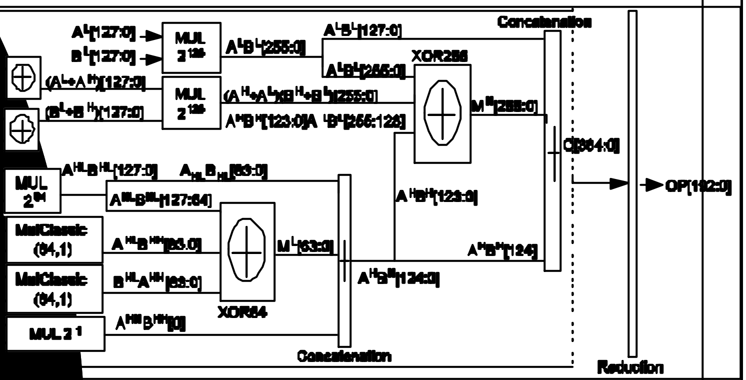
First Layer: Field Multiplication

[illegible]reliminary results yield a time delay of 50-70 ηSec [illegible] and ≈9K Slices of hardware resources utilization.



EC Point Addition and Doubling: A model

[illegible]tic Curve [illegible]rm	Point Addition	Point Doubling	*# of Multipliers*
[illegible]e [illegible]s	9M	4M	1
[illegible]	12M	6M	1
[illegible]	6M	3M	2
[illegible]	4M	2M	3
[illegible]	4M	2M	1
[illegible]	2M	1M	2

EC Point Multiplication: A model

…orithm	Number of Point Addition	Number of Point Doubling	PA and PD Boxes
[illegible]	1/2m	m	Sequential
[illegible]	1/2m	1/2m	Parallel
[illegible]	1/3m	m	Sequential
[illegible]	1/3m	2/3m	Parallel
[illegible]	m	m	Sequential
[illegible]	m	0	Parallel

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EC Point Multiplication: A model

Method	Total Number of Field Multiplications	Number of Multipliers
…roj	9/2mM+4mM = 8.5mM	1
…n2	3mM+3mM=6mM	2
…1	6mM+3mM=9mM	2
…2	2mM+3mM=5mM	2
…ar	2mM+1mM=3mM	2

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