SENSITIVITY ANALYSIS IN PARETO OPTIMAL DESIGN

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1. ABSTRACT

In real world engineering design problems we have to search for solutions that simultaneously optimize a wide range of different criteria. Furthermore, the optimal solutions also have to be robust. Therefore, this paper presents a method where a multi-objective genetic algorithm is combined with response surface methods in order to assess the robustness of the identified Pareto optimal solutions.

The objects of study are two different concepts of hydraulic actuation systems, which have been modeled in a simulation environment to which the optimization strategy has been coupled. The outcome from the optimization is a set of Pareto optimal solutions that elucidate the tradeoff between the energy consumption and the control error for each actuation system.

With the help of response surface methods sensitivity analysis have been performed at different regions on the Pareto front. Thus it could be determined how different design parameters affect the system at different points on the Pareto front.

2. INTRODUCTION

Many real-world engineering design problems involve simultaneous optimization of several conflicting objectives. In many cases, the multiple objectives are aggregated into one single overall objective function. Optimization is then conducted with one optimal design as the result. Another approach is to search the solution space for a set of Pareto optimal solutions, from which the decision-maker may choose the final design. Pareto-optimality is defined as a set where every element is a problem solution for which no other solutions can be better in all design objectives. A solution in the Pareto optimal set cannot be deemed superior to the others in the set without including preference information to rank competing objectives. For the two-dimensional case, the Pareto front is a curve that clearly elucidates the tradeoff between the objectives.

However, there might be other aspects that are not reflected in the objective functions that have to be considered as well. One such aspect that is addressed in this paper is system robustness. In real world applications, we can not rely upon the normative values of the design parameters due to effects of for example manufacturing tolerances, wear, and environmental changes. Therefore this paper presents a method where response surface methods are used together with a genetic algorithm for Pareto optimization. The optimization results in a set of optimal solutions that the designer has to consider. The sensitivity analysis then gives insight into how robust different solutions are and how different parameters affect the optimal performance. To study the sensitivity of solutions has hitherto been a neglected topic in evolutionary computation.

The paper starts with presenting a nomenclature for the multi-objective design problem together with a background on genetic algorithms and the optimization method used. Thereafter we discuss response surface methods and how they could be applied together with Pareto optimization. Then a design problem consisting of two different hydraulic actuation concepts is studied with the help of simulation models and the proposed optimization strategy. Sensitivity analyses are then performed and finally, different ways of presenting the result of the sensitivity analysis is introduced.

3. OPTIMIZATION

A general multi-objective design problem could be expressed by equations (1).

$$\left.\begin{aligned} &\min \mathbf{F}(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})\right)^T \\ &s.t.\ \ \mathbf{x} \in S \\ &\mathbf{x} = \left(x_1, x_2, \ldots, x_n\right)^T \end{aligned}\right\} \quad (1)$$

where $f_1(x), f_2(x), \ldots, f_k(x)$ are the k objectives functions, $(x_1, x_2, \ldots, x_n)$ are the n optimization parameters, and $S \in R^n$ is the solution or parameter space.

The Pareto set is defined by equation (2). Considering a minimization problem and two solution vectors $\mathbf{x}$, $\mathbf{y} \in S$. $\mathbf{x}$ is said to dominate $\mathbf{y}$, denoted $\mathbf{x} \succ \mathbf{y}$, if:

$$\begin{aligned}&\forall i \in \{1,2,...,k\} : f_i(\mathbf{x}) \le f_i(\mathbf{y}) \quad and \\ &\exists j \in \{1,2,...,k\} : f_j(\mathbf{x}) < f_j(\mathbf{y})\end{aligned} \tag{2}$$

The space in R^k formed by the objective vectors of Pareto optimal solutions is known as the Pareto optimal front, $\mathcal{P}$. It is clear that any final design solution should preferably be a member of the Pareto optimal set. Pareto optimal solutions are also known as non-dominated or efficient solutions.

3.1. Genetic algorithms

Genetic algorithms (GA:s) are modeled after mechanisms of natural selection. Each optimization parameter (x_n) is encoded by a gene using an appropriate representation, such as a real number or a string of bits. The corresponding genes for all parameters $x_1,..x_n$ form a chromosome capable of describing an individual design solution. A set of chromosomes representing several individual design solutions comprise a population where the most fit are selected to reproduce. Mating is performed using crossover to combine genes from different parents to produce children. The children are inserted into the population and the procedure starts over again, thus creating an artificial Darwinian environment. For a general introduction to genetic algorithms, see Goldberg [6].

Additionally, there are many different types of multi-objective genetic algorithms. For a review of genetic algorithms applied to multi-objective optimization, readers are referred to work by Deb [4].

The optimization method used in this paper borrows some major ideas from the multi-objective GA (MOGA) presented by Foseca and Fleming [5]. In MOGA each individual is ranked according to their degree of dominance. The more population members that dominate an individual, the higher ranking the individual is given. An individual's ranking equals the number of individuals that it is dominated by plus one. Thus individuals on the Pareto front have a rank of 1 as they are non-dominated.

3.2. The proposed optimization method

In this paper the multi-objective struggle genetic algorithm (MOSGA), see Andersson et al. [1] and [2], was used for the Pareto optimization. MOSGA combines the struggle crowding genetic algorithm presented by Grueninger and Wallace [7] with Pareto-based ranking. In the struggle algorithm the child replaces the most similar individual in the entire population, but only if it has a better fitness. This replacement strategy counteracts genetic drift that can spoil population diversity. The principle of the MOSGA algorithm is outlined below.

Step 1: Initialize the population.

Step 2: Select individuals uniformly from population.

Step 3: Perform crossover and mutation to create a child.

Step 4: Calculate the rank of the new child, and a new ranking of the population that considers the presence of the child.

Step 5: Find the most similar individual, and replace it with the new child if the child's ranking is better.

Step 6: Update the ranking of the population if the child has been inserted.

Step 7: Perform steps 2-6 until the mating pool is filled.

Step 8: If the stop criterion is not met go to step 2 and start a new generation.

The likeness of two individuals is measured using a distance function. The method has been tested with distance functions based upon the Euclidean distance in both the attribute as well as parameter space. A mixed distance function combining both the attribute and parameter distance has been evaluated as well. The result presented in his paper was obtained using an attribute based distance function.

4. RESPONSE SURFACE METHODS

The approach presented in this paper is a statistically based method witch combines design of experiments (DoE), [3] with response surface methodology (RSM), see Myers and Montgomery [9]. RSM is a method for constructing approximations of the behavior of a system based on results at various points in the design space. The resulting surfaces, usually linear or quadratic, are fitted to these points. Often statistical methods such as design of experiments are used to determine where in the design space these points should be located in order to obtain best possible fit. In this paper we use quadratic polynomials to create the response surface, see equation (3). Equation (3) is also called the Response surface Equation (RSE).

$$y = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} b_{ii} x_{ii}^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} b_{ij} x_i x_j \tag{3}$$

y is the response, i.e. the function value we want to approximate in this case the objective functions. However, any other system characteristics could be estimated as well. b_0 is a constant term and b_i are the coefficients of the linear terms, better known as the main effect. b_{ii} are the coefficients of the pure quadratic terms and are known as quadratic effects, whereas b_{ij} are the coefficients of the cross products, which are also called second order interactions.

By examining the coefficient of the RSE it could be seen how the different parameters affect each objective function and knowledge about the underlying causes of the trade-off between the objectives could be gained. A more formal method of gaining such knowledge is to study the sensitivities of each response with respect to the different parameters. Here the gradients of the estimated surfaces are used as sensitivity measure, see equation (4).

$$\frac{\partial \hat{y}}{\partial x_j} = b_j + \frac{\partial}{\partial x_j}\left(\sum_{j=1}^{n}\sum_{k=j}^{n} b_{jk} x_j x_k\right) \tag{4}$$

5. THE DESIGN PROBLEM

The objects of study for the design problem are two different concepts of hydraulic actuation systems. Both systems consist of a hydraulic cylinder that is connected to a mass of 1000 kilograms. The objective is to follow a pulse in the position command with a small control error and simultaneously obtain low energy consumption. Naturally, these two objectives are in conflict with each other. A low control error implies high acceleration and retardation which consumes more energy. The problem is thus to minimize both the control error and the energy consumption from a Pareto optimal perspective.

Two different ways of controlling the cylinder are studied. In the first more conventional system, the cylinder is controlled by a servo valve, which is powered from a constant pressure system. In the second concept, the cylinder is controlled by a servo pump. Thus, the systems have different properties. The valve concept has all that is required for a low control error, as the valve has a very high bandwidth. On the other hand, the valve system associated with higher losses, as the valve constantly throttles fluid to the tank.

The different concepts have been modeled in the simulation package Hopsan [8]. The models of each component consist of a set of algebraic and differential equations taking aspects such as friction, leakage and non-linearities into account. The system models are depicted in Figure 1 and Figure 2 respectively.

The servo valve system consists of the mass and the hydraulic cylinder, the servo valve and a proportional controller that is controlling the motion. The servo valve is powered by a constant pressure pump and an accumulator, which keeps the system pressure at a constant level. The optimization parameters are the sizes of the cylinder, valve and the pump, the pressure lever, the feedback gain and a leakage parameter that is necessary to dampen the system. Thus, this problem consists of six optimization parameters and two objectives.

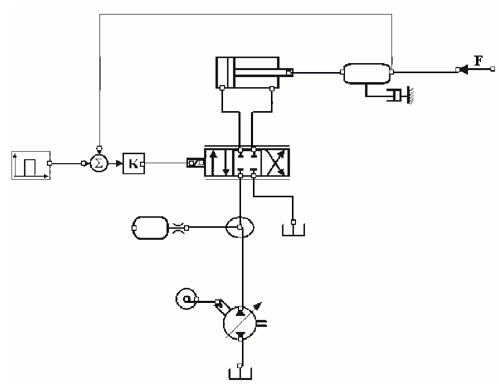


Figure 1: The servo valve concept for hydraulic actuation.

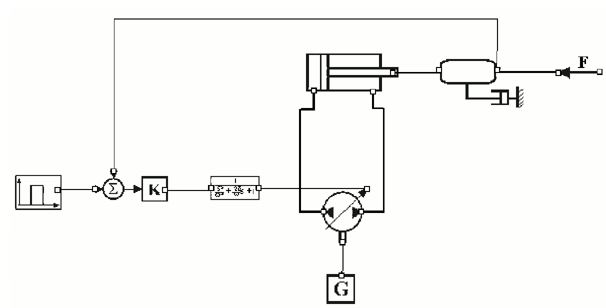


Figure 2: The servo pump concept of hydraulic actuation.

The servo pump concept contains fewer components, the cylinder and the mass, the controller and the pump. A second order low-pass filter is added in order to model the dynamics of the pump. The servo pump system consists of only four optimization parameters.

5.1. Optimization results

Both systems where optimized in order to simultaneously minimize the control error f_1 and the energy consumption f_2. The control error is obtained by integrating the absolute value of the control error and adding a penalty for overshoots, see equation (5). The energy consumption is calculated by integrating the hydraulic power, expressed as the pressure multiplied with the flow, see equation (6).

$$f_1 = \int_{t_{start}}^{t_{finish}} \left| x_{ref} - x \right| dt + \alpha \left(\int_{t_1}^{t_2} \left(x - x_{ref} \right) dt + \int_{t_3}^{t_4} \left(x_{ref} - x \right) dt \right) \tag{5}$$

$$f_2 = \int_{t_{start}}^{t_{finish}} \left(q_{pump} \cdot p_{pump} \right) dt \tag{6}$$

The optimization is conducted with a population size of 30 individuals over 200 generations. The parameters are real encoded and BLX crossover is used to produce new offspring.

As a Pareto optimization searches for all non-dominated individuals, the final population will contain individuals with a very high control error, as they have low energy consumption. It is possible to obtain an energy consumption close to zero, if the cylinder does not move at all. However, these solutions are not of interest, as we want the system to follow the pulse. Therefore, a goal level on the control error is introduced. The optimization strategy is modified so that solutions, which are below the goal level on the control error are always preferred to solutions that are above it regardless of their energy consumption. In this manner, the population is focused on the relevant part of the Pareto front. In order to elucidate the properties of the different systems, the obtained Pareto optimal fronts are depicted in the same graph, see Figure 3.

In order to achieve fast systems, and thereby low control errors, large pumps and valves are chosen by the optimization strategy. A large pump delivers more fluid, which enables a higher speed of the cylinder. However, bigger components consume more energy, which explains the shape of the Pareto frontiers.

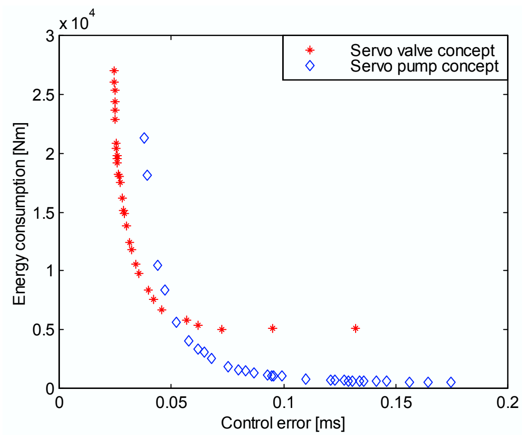


Figure 3: The Pareto frontiers for both concepts.

It is evident that the final design should preferably be on the overall Pareto front, which elucidates when to change between concepts. The servo pump system consumes less energy, and is preferred if a control error larger then 0.05ms is acceptable. The servo valve system is fast but consumes more energy. If lower control error then 0.05ms is desired, the final design should preferably be a servo valve system. In order to choose the final design, the decision-maker has to study the tradeoff between the control error and the energy consumption and select a solution point that matches his or her preferences.

However, first a sensitivity analysis should be conducted in order to gain further insight into the properties and robustness of individual solutions.

5.2. Sensitivity analysis

For both systems, five points evenly spread on the Pareto front were used as center points for the designed experiments. For each of these points the MODE software [10] was used to create a design setup using the D-optimality criterion, see Myers and Montgomery [9]. Based on these design points a second order response surface was created, which emulates the performance of each system at the selected center points.

The results are analyzed by looking at the values for the normalized coefficients and see how they vary as we move along the Pareto front. In Figure 4 the normalized coefficients for the servo Pump system are plotted for the five points on the Pareto front. Point one is a point with a low control error, i.e. to the left on the Pareto curve, whereas point five has a large control error and lies far out to the right on the Pareto curve, see Figure 3.

For each location on the Pareto front, all coefficients of the response surface equation are plotted as a point in the graph, which are then connected with straight lines. Coefficient values close to zero evidently indicate that the corresponding parameter has little influence of the response. Whereas points which have a high magnitude indicate coefficients that are important.

The abbreviations for the parameters are: pump displacement, (Dp), cylinder area (A1), control gain (Ga), and leakage coefficient (Kc). For the servo valve system ther is also the valve spool diameter (Sd).

There is a lot of insight that could be gained by studying such a graph. First we can conclude that in point 1 where the control error is small, the feedback gain is the most important parameter, whereas in point 5 the pump size is the most important parameter, this is true for both control error and energy consumption. Thus it could be seen how the relative importance of system parameters varies as we move along the Pareto front. If we study the control error graph we can also see that the second order terms are largest in point 1, which indicating that the smaller we make the control error the more sensitive the solution gets. By comparing the coefficients for the different responses we can also se the underlying causes to the trade-off between the objectives. In Figure 4, this could be exemplified by that a larger pump (larger Dp-value) gives a smaller control error but a lager energy consumption. A more thorough investigation of the impact of the parameters on the objectives could be gained by calculating the sensitivities by deriving the response surface equation according to equation (4). In Figure 5 this is done for the servo pump system.

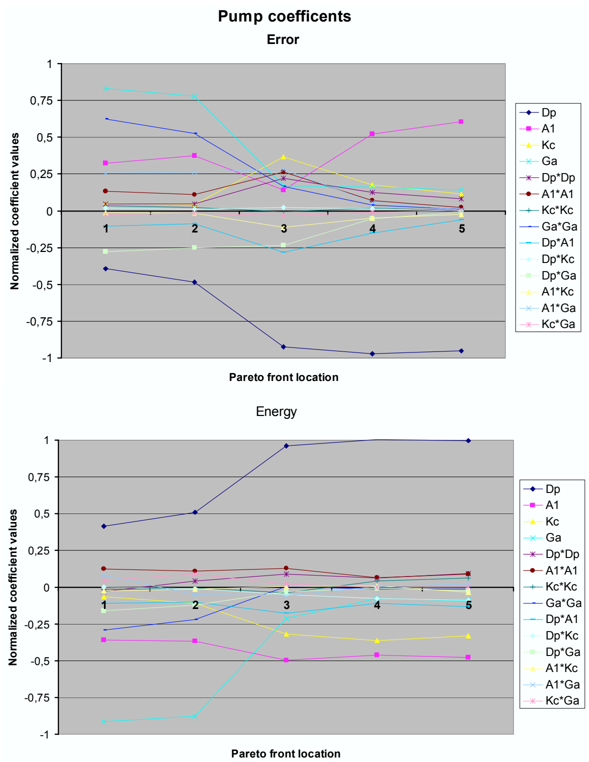


Figure 4. RSE coefficients for the servo pump system at five different points on the Pareto front.

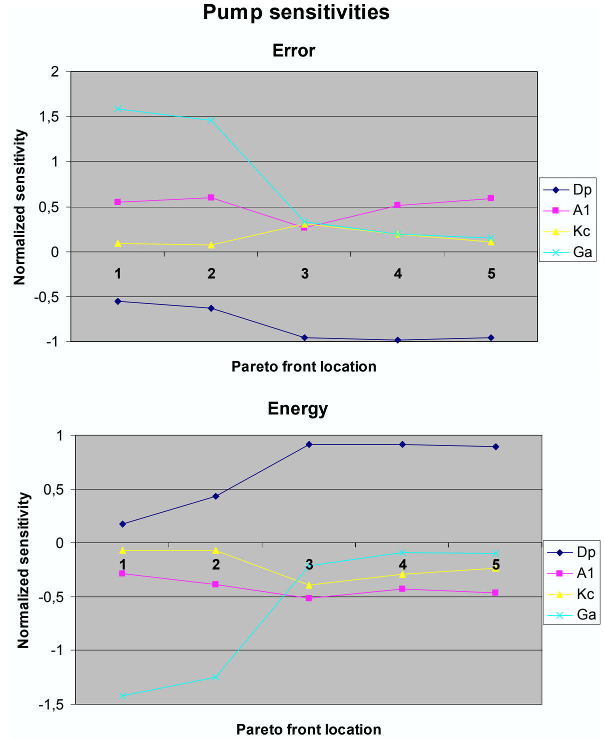


Figure 5. Sensitivities for the servo pump system at five different points on the Pareto front.

The sensitivity graph in Figure 5 shows what impact a small change in parameter value has on the objectives as we move along the Pareto front. It could be seen that for systems with low control error the feedback gain (Ga) is the most important parameter, for both energy consumption and control error. However, as we move to the right on the Pareto front the gain loses in importance and the size of the pump gets more important. We can also see how an increased pump size leads to lower control error but larger energy consumption. A perhaps more illustrative way of showing how the different parameters influence the objectives are shown in Table 1.

Table 1. Sensitivity table for the servo pump system

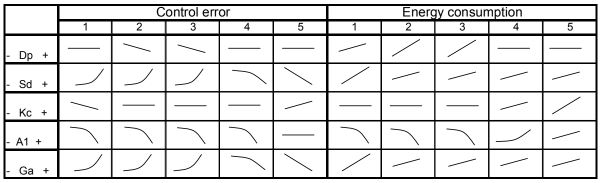
	Control error					Energy consumption				
	1	2	3	4	5	1	2	3	4	5
- Dp +										
- A1 +										
- Kc +										
- Ga +										

The columns in the table indicate the different points on the Pareto front. Each row shows how an increase in the corresponding parameter affects the control error and the energy consumption. A straight line indicates no effect, whereas the lines with a gradient indicate in what direction and how much the objectives changes as the corresponding parameter is increased. The curved lines indicates points where we have significant second order effects and where the system is more sensitive.

The sensitivity table contains all the information from the graphs presented earlier condensed to one table. It can thus be seen how the importance of the parameters varies along the Pareto front and where the second order effects are largest.

To summarize the information from the servo pump system it has been shown that the displacement and the control gain are the most influential parameters and that the faster the system the more sensitive it is to parameter changes. The same sensitivity analysis has been performed for the servo valve system. The results could be seen in Table 2.

Table 2. Sensitivity table for the servo pump system



The servo valve system has a more complex behavior and the trade-off between the parameters is not as clear as in the servo pump system. Furthermore, the second order effects are much greater then for the servo pump system, particularly for the control error. Thus it could be argued that this system is not as robust.

For this system the cylinder area, the spool diameter, and the gain are the most influential parameters. However the way they influence the system changes as we move along the Pareto front. In the first three points the trade-off is due to the pump size, (Dp), larger pump gives a small control error but large energy consumption. However as we move along the Pareto front the system gets slower and less fluid are taken from the pump and more from the accumulator. Thus pump size has no longer any influence. Then the Trade-off is shifted towards spool diameter and gain. It can bee seen how an increased spool diameter and gain reduces the control error but gives an increased energy consumption.

6. DISCUSSION AND CONCLUSION

In this paper a multi-objective genetic algorithm is used to optimize two different hydraulic actuation systems. The outcome of the optimization is a set of Pareto optimal designs, where the tradeoff of the conflicting objectives is clearly visualized. The resulting Pareto optimal frontiers elucidate the advantages of the different concepts and, advice the decision-maker which concept to choose depending on his or her preferences. If a very fast system is desired, a servo valve system should be chosen. However, if a slower system is acceptable a servo pump system is more favorable as it consumes more energy. Furthermore, the algorithm suggests when to switch between the concepts.

Then sensitivity analysis is performed in order to gain more information about the properties and the robustness of each concept. The sensitivity analysis tells us what effect a small change in a parameter value has on the objectives depending on the location on the Pareto front. For the servo pump example it has been shown that for a fast system the control gain is the most important parameter, but for a slower system the pump size is the most important one. This type of information could be very useful as it tells the designer where to focus his efforts. When designing large systems, sensitivity analysis could guide the designer towards parts or sub-systems that have the greatest influence on the performance of the system.

The method presented in this paper combines modern optimization techniques with response surface methods and supports the engineer when a design is based on simulation models. It visualizes the tradeoff between the objectives and points out which parameters that has the greatest influence on the results. However, the main benefit is not in finding an optimal and robust solution, but in learning more about the properties of the system being designing and about the behavior of the system model. By conducting optimization together with a thorough sensitivity analysis much more knowledge could be gained out of our simulation models. Furthermore, the ability to assess the sensitivity of the optimal solutions makes the method more suitable for real world engineering design applications where robustness is always a critical issue. This is an area towards which more research needs to be directed in the future.

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