VISUALIZATION TECHNIQUE FOR ANALYZING NON-DOMINATED SET COMPARISON

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ABSTRACT

Multi-objective evolutionary algorithms (MOEAs) have been proved successfully in many different applications. Over 900 publications [4] have since proposed various MOEA implementations and applications. However, there is still a significant lack of studies on metrics for MOEA comparison. In this paper, we propose a new visualization technique that will provide better analysis of the non-dominated solutions for any number of objectives.

1. INTRODUCTION

Real world problems often require the simultaneous optimization of multiple, possibly conflicting objectives. Typically, such a problem does not admit a single optimal solution, but rather a set of solutions known as Pareto-optimal, or non-dominated. This non-dominated set describes the trade-offs in the problem, and may help the designer understand the options available and select final solution for implementation.

Evolutionary algorithms are stochastic methods for search and optimization inspired by the process of natural selection. They offer greater flexibility in the handling of multiple objectives than conventional optimization methods. The growing interest devoted by the scientific community to evolutionary algorithms for multi-objective optimization has led to an increasing number of different approaches being proposed in the literature. Such techniques have been demonstrated on several practical engineering applications, and their power and usefulness is recognized. Detailed summaries of the state-of-the-art in MOEA were discussed in [5,7,11,15,17]. Unfortunately, there is a serious lack of quantitative methods to describe their performance.

The aim of this paper is to propose a new visualization technique to analyze the final non-dominated set of solutions, which in turn can assist in the comparison process. Section 2 give a brief background information on why such a visualization technique is useful and essential. The proposed technique will be described in Section 3. Sections 4 and 5 discuss experimental results and observations, followed by our conclusions in Section 6.

2. BACKGROUND

The conventional way of comparing non-dominated set of solutions is through visual comparison in the objective space. This method is simple and straightforward. The criterion is to have solutions close to the true Pareto front and must be well distributed over the Pareto frontier. However, this method is limited to maximum three objectives. Following, various performance metrics were proposed and quite a few of them were designed upon the basic criterion of a good MOEA.

Some of the infamous metrics proposed were diversity [6], attainment surface [9], attainment surface sampling [12], generational distance [18], spacing [18], error ratio [18], maximum Pareto front error [18], overall nondominated vector generation and ratio [18], size of the dominated space [19], coverage of two sets [19], coverage difference of two sets [19] etc.. Detailed summaries of the metrics were discussed in [2,7,13,14].

Currently, all the proposed metrics have their limitations. The main problem is the lack of decision maker preferences in the comparison, hence causing difficulty in some cases of comparison. Hansen & Jaszkiewicz [10] have proposed a formal framework for evaluating the quality of the non-dominated set. However, the proposed metrics only emphasized on the distance between competing non-dominated sets or distance between competing non-dominated sets and a reference set. In addition, there were quite a few settings that users need to determine for e.g. the choice of the set of utility function, the choice of probability distribution of the utility functions and utility functions' scaling.

The common way of proving the creditability of a new proposed performance metric is always limited to two or three objectives problem. A common practice of proofing is to display those competing non-dominated solutions in objective space and show the metric report results. However, it is difficult to provide any evidence that a particular metric works for a two objectives problem will also work for problems with more than three objectives.

In the following section, a new visualization technique based on distance and distribution will be proposed to assist in the comparison process. This technique is not meant to replace any existing metrics but may be used to visualize the comparison process for problems with any number of objectives.

3. PROPOSAL

Instead of plotting the non-dominated solutions in the objective space (which is only limited to three objectives), we proposed to plot the non-dominated solutions against their distance to the approximate Pareto front [1] and their distance between each other. Here we called it the Distance & Distribution (DD) chart. The DD chart consists of three elements namely the approximate Pareto front, distance measure and distribution measure.

Approximate Pareto front can be easily generated by two methods. First method is to have an archive to store all the best-found non-dominated solutions. Second method is to use all the non-dominated solutions found by the competing algorithms and use it as an approximate true Pareto front.

Distance measure is simply the normalized Euclidean distance of each solution to the nearest approximate Pareto front solution. This measure is similar to generational distance metric except that it is measuring the individual distance rather than the overall average distance. A zero value indicate that the solution is Pareto-optimal and any values above zero indicates that the solution deviates from the approximate Pareto front.

Distribution measure is simply the normalized Euclidean distance between each solution and taking into consideration the distance between the boundary solutions and the approximate Pareto front. This measure is similar to diversity metric except that it is measuring the individual gap distance rather than the overall average gap distance. Thus, a low performance measure characterizes an algorithm with a good distribution capacity.

The computation for distance measure is straightforward. As for the distribution measure, it will get complicated when the number of objectives is more than two. In this case, Deb [7] proposed to use the non-dominated solutions to construct a higher-dimensional surface by employing the so-called triangularization method. As several distance measures can be associated with such a triangularized surface, the average distance of all edges can be used as the gap distance. Note that this method is extremely computationally expensive. Hence, we propose another method to compute distribution measure that is applicable to any number of objectives. This method is not accurate but it can still serve as a rough estimation for the distribution measure. First, the non-dominated solutions found must be sorted. It is recommended to sort based on the first objective, for e.g. if the first objective is to minimize then the solutions should be sorted in ascending order based on the first objective value. Now regardless of how many objectives, the two-boundary gap distance calculation is simply the normalized Euclidean distance between the first and last non-dominated solution and the first and last solution of the approximate Pareto front respectively. For example, the two-boundary gap distances (g1 and g4) can be calculated based on the distance between the first solution found and the first solution of the approximate Pareto front as shown in Fig. 1. In Fig. 1, circle represents the non-dominated solutions found, square represents the approximate Pareto front, d1 to d3 represents the distance measure and g1 to g4 represents the distribution measure.

The number of non-dominated solutions required for the DD chart is about 10 to 100. Although, the amount of the competing non-dominated solutions does not need to be the same, but they should not be different by more than 50%.

Our proposal is to view the distance and distribution measure of each non-dominated solutions found by an algorithm. Using one simple line chart to plot the non-dominated solutions against its distance and another line chart to plot the non-dominated solutions against its distribution measures. The distance chart will not only provide information on the overall distance of the solutions to the approximate front but will also reveal the maximum Pareto front error. As for the distribution chart, it can reveal the coverage of the non-dominated solutions in the objective space.

4. SIMULATIONS

Here we will illustrate the visualization technique for distance and distribution measures. In all the past publications, the visual comparison technique has been applied to view the objective space of the problem and that severely limit the technique to only a maximum of three objectives problem.

The experiment setup is to use from our previous studies but now with a different view. The two algorithms used are the multi-objective evolutionary algorithm toolbox (MOEA_NUS) [16] and (1+1)-Pareto Archived Evolution Strategy (PAES) [12]. Though, MOEA_NUS and PAES are of different nature as MOEA_NUS is population-based and PAES is not. This might not be a fair comparison, but the focus of this experiment is to demonstrate the new visualization technique rather than debating on the validity of the comparison.

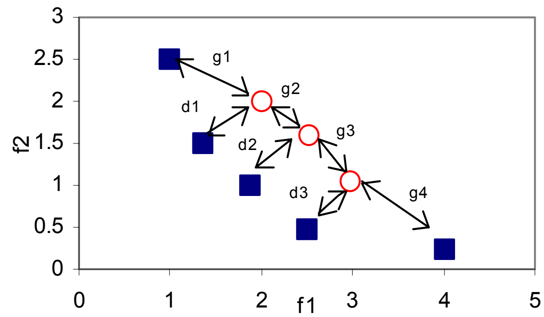

Fig. 1. An Example Plot

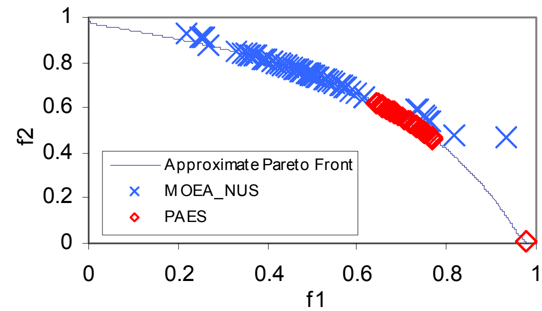

Fig. 2. Result of a single run

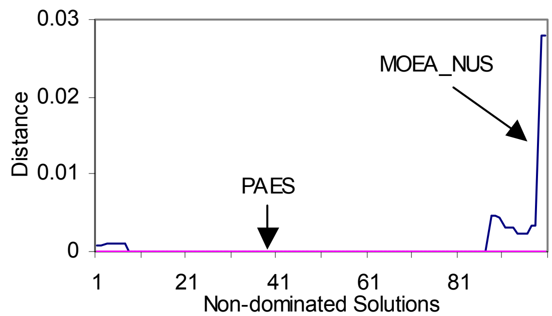

Fig. 3. Distance chart of a single run

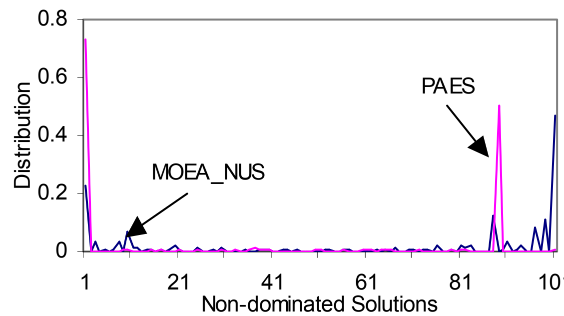

Fig. 4. Distribution chart of a single run

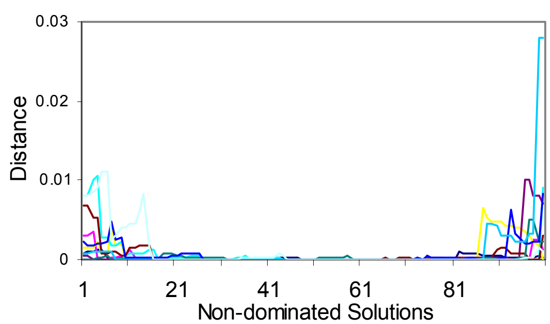

Fig. 5. Distance chart of MOEA_NUS (10 runs)

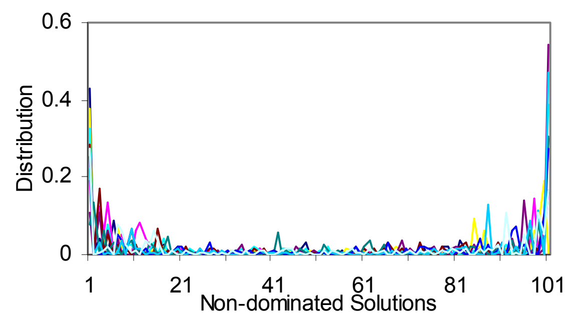

Fig. 6. Distribution chart of MOEA_NUS (10 runs)

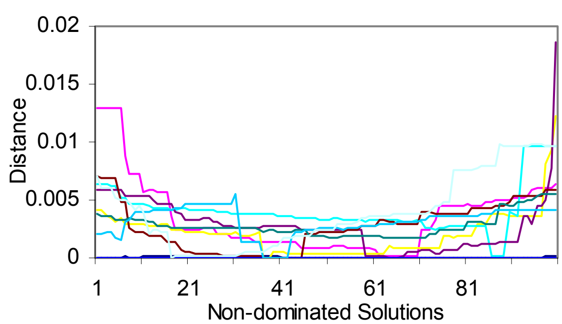

Fig. 7. Distance chart of PAES (10 runs)

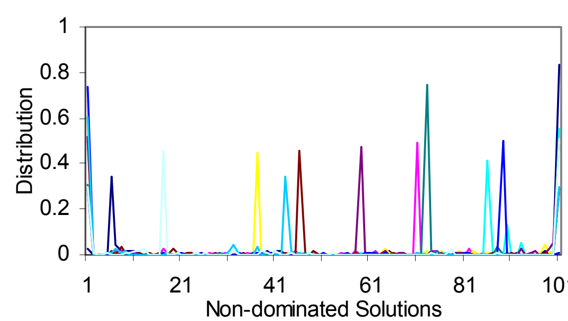

Fig. 8. Distribution chart of PAES (10 runs)

For MOEA_NUS, the population size is set to 100 and generation size to 250. It is using tournament selection, one-point crossover with a rate of 0.9 and using classical mutation with a rate of $1/l$ (where l is the string length for binary-coded chromosome). As for PAES, the iteration is set to 25000; depth of 4 and archive size of 100. We used 30 bits to represent each decision variable.

The two-objectives minimization test problem [8] illustrated here is:

$$f_1(x) = 1 - \exp(-\textstyle\sum_{i=1}^{n}(x_i - 1/\sqrt{n})^2\ ), \tag{1a}$$

$$f_2(x) = 1 - \exp(-\textstyle\sum_{i=1}^{n}(x_i + 1/\sqrt{n})^2\ ), \tag{1b}$$

where, $-4.0 \le x_i \le 4.0$, $n = 1,2,3$

4.1. Single Run

This is to illustrate the result of a single run though it does not have any statistical significant. The result plotted in the objective space is shown in Fig. 2. Fig. 3 and 4 shows the distance and distribution charts respectively.

Based on Fig. 2, it is clear that solutions found by PAES are Pareto-optimal but not well distributed. As for the solutions found by MOEA_NUS, it is better distributed but not all the solutions are Pareto-optimal as compared with PAES. In Fig. 3, we can confirm that all PAES solutions are indeed Pareto-optimal. As for MOEA_NUS, the center portion of the solutions are Pareto-optimal and the boundary solutions deviate from the approximate Pareto front. In Fig. 4, there are two very high 'spike' in the distribution measure for PAES solutions. The first 'spike' on the left shows that the solutions are 'far away' from the left boundary of the approximate Pareto front. The second 'spike' simply indicates that there is a break in the continuity of the solutions. Note that the second 'spike' is common if the true Pareto front is disconnected.

4.2. Multiple Runs

In order to have any statistical significant, multiple runs of each algorithm is required. Here each algorithm execute 10 times on the test problem. Fig. 5 and 6 shows the distance and distribution charts of MOEA_NUS and Fig. 7 and 8 shows the distance and distribution charts of PAES.

Fig. 5 again shows that the center portion of the solutions found by MOEA_NUS is close to Pareto-optimal. Fig. 6 shows that the extent of the solutions does not span widely enough. In general, MOEA_NUS performance is considered quite consistent based on the results on single and multiple runs. Thus, DD chart can also be used to detect any performance inconsistency.

Fig. 7 shows that on average, solutions found by MOEA_NUS are closer to the approximate Pareto front than PAES. Fig. 8 indicates that there is a break in continuity among the solutions. However, this might be true for the case of a disconnected true Pareto front but based on Fig. 6 it can be shown that the true Pareto front is not disconnected.

5. OBSERVATIONS

In the previous section, we have done a simple single run and a multiple runs simulation in order to illustrate the capability of DD chart. The distance chart is straightforward and easy to analyze but the distribution chart is not. Care must be under taken when analyzing the distribution chart as the true Pareto front might be disconnected and not well distributed. One of the possible solutions is to plot the DD chart for the approximate Pareto front. Then user may have an idea what the best result should look like. However, the approximate Pareto front solutions must be reduced to almost the same number as the competing non-dominated solutions.

Another noticeable problem is the result range of distance and distribution measures [3]. Without any knowledge of the range, it is almost impossible to determine if the results can be considered similar. For example, if two result values are 20 and 80, so if the range is from 0 to 100 then we can determine that the differences between these two results are significant. However, if the range is from zero to infinity, then the differences between these two results can be considered as unnoticeable.

This visualization technique is not limited to only distance and distribution measures and in fact it can be extended to any performance metrics that is suitable. More works are still needed for handling disconnected Pareto front and more accurate distribution measure calculation for three or more objectives.

6. CONCLUSIONS

In this paper, a novel and simple visualization technique has been proposed. This technique can reveal the coverage of the non-dominated solutions in the objective space with no limitations on the number of objectives. It can also be used to validate the analysis by any performance metrics.

7. ACKNOWLEDGEMENT

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