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Multiobjective Optimization Using Adaptive Range Genetic Algorithms with Data Envelopment Analysis

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Abstract

The present paper describe an implementation of the adaptive range genetic algorithms (ARange GAs) in multi-objective optimization by using the data envelopment analysis (DEA). ARange GAs is a new genetic search algorithms which adapt the searching range according to the optimization situation and make it possible to obtain highly accurate results effectively. DEA is to measure the efficiency of decision making units, and it is used mainly in the field of economy. When we combine both methods, we can obtain a great number of Pareto solutions, that might give an important aspect of the design, within a single GAs process effectively. The purpose of this study is to verify the characteristics and effectiveness of the proposed method through demonstrative examples.

Introductions

Recently, requirements of design become more and more complicated and sophisticated and the customers try to decide what they really need to buy from many aspects. Thus, we need to satisfy multiple requirements to meet these purposes. In such cases, it is rational and natural to formulate the problem in the multi-objective optimization (MO). However, in MO, there usually exist conflict among objective functions, so that the solution cannot be determined uniquely. In general, we try to fine a set of non-inferior solutions call Pareto solutions with results of a number of scalar optimizations and try to give the implicit desired preference with local information and approximation of a given Pareto solutions. However, these processes are not that easy decision makings and we have to do many try and errors before we finally satisfy with the results, and which cause the cost of MO very high(1).

When we try to think of the design as treasure hunting, these decision makings with local information seems to hunt treasure without a map. The map in MO might be a set of Pareto solutions, so that we would like to obtain nearly entire set of Pareto solutions with almost the same computational cost with that we need in a single scalar optimization. One possibility to meet this complicated requirement is the use of genetic algorithms (GAs), because it can be considered as multi-point search. So that GAs seems to be preferable in MO. A number of studies have been done in using GAs in MO. Hajela(2) used weighted method to deal MO for structural system with mix of continuous, integer and discrete design variables. There are several studies which tried to keep Pareto solution as rank 1 and try to keep rank 1 individuals in the next generation, (3-6). The other is to divide population in a number of small groups(7) and try to maintain special characteristics in each small groups. Tamaki(8) combined these two approaches and obtained relatively good results. We have newly introduced a strategy for survival among phenotype expression as something of game between individuals and developed a new methodology(9), and we also have extended and revised the method to consider environment and use adaptive range GAs(10) to give evolution of the species(11).

In this study, we also try to keep Pareto solutions and try to give higher fitness function for the frontier of Pareto solutions. We do not use ranking method like Goldberg or Fonseca, but to use DEA(12,13,14). DEA is an approach comparing the efficiency of decision making units (DMU) by measuring their efficiency by ratio of weighted sum of outputs and weighted sum of inputs. By using this efficiency measure, we can calculate DMU efficiency with in the range of [0,1] in continuous number. So that it might be suitable to use as fitness function in GAs.

In this article, we demonstrate the proposed method by using simple numerical examples, and try to figure out the characteristics and effectiveness of the proposed method.

Adaptive Range Genetic Algorithms

The ARange GAs is developed by one of the author in order to treat continuous number effectively by using the same frame work of simple GAs. Details can be seen in the Refs. (10,15&16).

Expression of continuous variables

From the second generation, we can calculate mean (μ_i) and standard deviation (σ_i) of each design variable of the individuals who are remained after GAs processes. By using these values, we can determine some sort of distribution like normal distribution normalized to have maximum value 1 as

$$N(x_i) = \exp\left(-(x_i - \mu_i)^2 / 2 / \sigma_i^2\right) \qquad (1).$$

These distributions show situation of each generation and they adapt automatically to the best fitted searching range in some generation. By using these distributions, continuous variables are given as

$$R(p_i) = \begin{cases} \mu_i - \sqrt{-2\sigma_i^2 \ln\left(LB + \dfrac{(UB - LB)C(p_i)}{2^{m-1} - 1}\right)} & \text{for } C(p_i) < 2^{m-1} \\ \mu_i + \sqrt{-2\sigma_i^2 \ln\left(UB - \dfrac{(UB - LB)(C(p_i) - 2^{m-1})}{2^{m-1} - 1}\right)} & \text{for } C(p_i) \geq 2^{m-1} \end{cases} \qquad (2).$$

Where p_i is the chromosome for design variable x_i and $C(p_i)$ is the integer decoded by using gray coding, $R(p_i)$ is the real number decoded from p_i, m is the number of bits, and UB and LB are system parameters. (See Fig. 1)

In this method, searching range will move according to the value μ_i (mean value of the previous generation), thus we do not have to care on giving priori set boundaries. Moreover, if it comes close to convergence, distribution becomes narrow and it will speed up convergence.

As the searching range will move according to the mean values of the previous generation, there is a possibility to miss the maximum variables, which have obtained during initial generation to previous generation, within the searching range. To avoid these situations, we make some efforts in the value σ_i as following and keep them in the searching range.

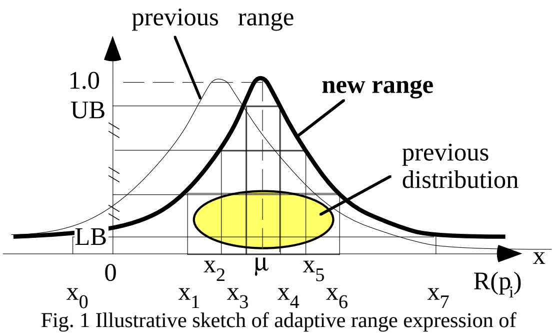


Fig. 1 Illustrative sketch of adaptive range expression of continuous variables

$$\sigma_{i,new} = \sqrt{-\frac{(\max_i - \mu_i)^2}{2\log(LB)}} \qquad (8)$$

If there are any explicit side constraints for each design variable, there are possibilities that the searching ranges will break these constraints in the ARRange GAs. As we do not want to pass these problems in the fitness function as penalty, we operate both LB and s as following and keep side constraints.

for upper bound

$$LB_{i,new} = \exp\left(\frac{(upper_i - \mu_i)^2}{2\sigma_i^2}\right)$$

if $LB_{i,new} > UB - margin$ *then*

$LB_{i,new} = UB - margin$ *and*

$$\sigma_{i,new} = \sqrt{-\frac{(upper_i - \mu_i)^2}{2\log(LB_{i,new})}} \qquad (3),$$

for lower bound

$$LB_{i,new} = \exp\left(\frac{(\mu_i - lower_i)^2}{2\sigma_i^2}\right)$$

if $LB_{i,new} > UB - margin$ *then*

$LB_{i,new} = UB - margin$ *and*

$$\sigma_{i,new} = \sqrt{-\frac{(\mu_i - lower_i)^2}{2\log(LB_{i,new})}} \qquad (4).$$

System parameters for ARange GAs

There are five system parameters for ARange GAs; *UB, LB,* σ_{min}, σ_{max} *and margin*. And we give default values by assuming that the searching range will have the width of 10 as,{*UB, LB,* σ_{min}, σ_{max},*margin*}={0.99, 0.044, 2.0, 0.1, 0.2}. However, these values must have different values according to the precision for each design variables. σ_{min} *and* σ_{max} will especially play important roles in improving accuracy. So that we give these values as,

$$\sigma_{min} = \sigma_{min} * w / 10.0 \qquad (5),$$

$$\sigma_{max} = \sigma_{max} * w / 10.0 \qquad (6),$$

where w represent the width of searching range and it will be given by upper and lower bound for each design variables. If there are no side constraints, it can be determined by the initial given boundary.

Expression of discrete variables

In the conventional method, integer variable are determined by

$$DI(p_i) = x_{i,min} + C(p_i) \qquad (7),$$

where $x_{i,min}$ is a priori set lower bound, as for discrete variable, they are

$$DC(p_i) = Database[[C(p_i)]] \qquad (8),$$

where Database[[k]] means number k-th discrete variable in the given set of discrete variables. In ARange GAs,

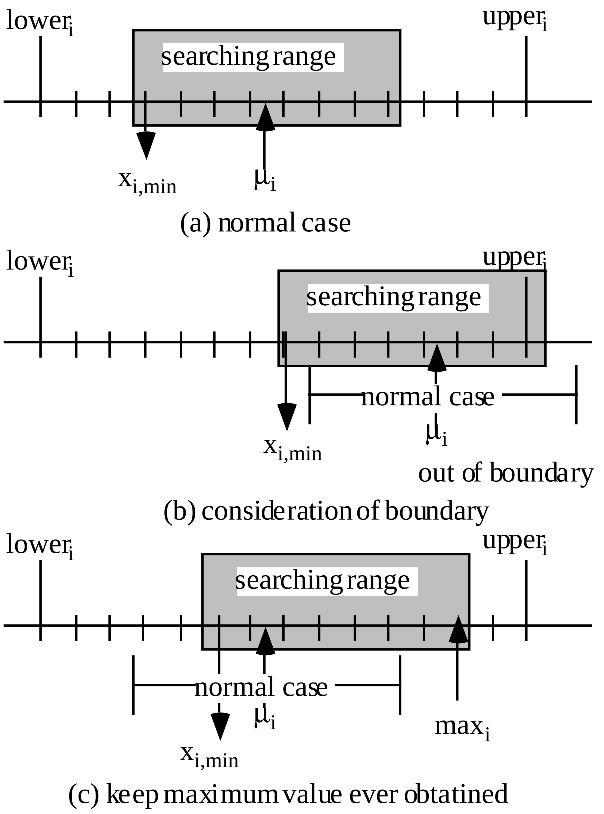


Fig.2 ARange GAs for integr and discrete variables

$x_{i,min}$ is determined by the situation of the optimization using μ (Fig. 2),

$$x_{i,min} = Int(\mu_i + 0.5) - 2^{m-1} \quad (9),$$

if $x_{i,min} < lower_i$ then
$x_{i,min} = lower_i$,
else if $x_{i,min} + 2^m - 1 > upper_i$ then
$x_{i,min} = upper_i - 2^m + 1$.

Where Int(•) transforms real number to integer. To keep maximum value within the searching range, $x_{i,min}$ will be revised as,

if $x_{i,min} > max_i$ then
$x_{i,min} = max_i$ (10),
else if $x_{i,min} + 2^m + 1 < max_i$
$x_{i,min} = max_i - 2^m + 1$.

Demonstrative Example

In order to show the effectiveness of the ARange GAs, we applied the problem to the Golinski's speed reducer which was applied in Azarm[17]. Here only the results is shown in Fig. 3 and Table 1. Formulation can also be seen in Web Page (http://fmad-www.larc.nasa.gov/mdob/ mdo.test/class2prob4/descr.html). As you can see in Fig. 3 and Table 1, we have very good convergence and we can obtain the results which have high accuracy. After 750 generation, all 5 trials obtained the same results, which will proof the stability of the proposed method.

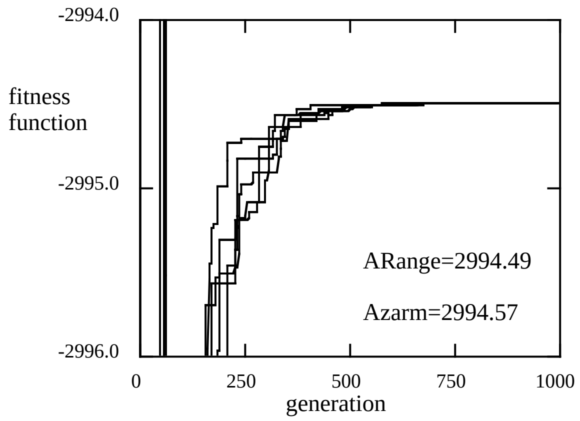


Fig.3 Convergence of Golinski's speed reducer (5 trials)

Table 1 Comparison of the results in fitness function

it	trial 1	trial 2	trial 3	trial 4	trial 5
150	-2996.84	-3003.43	-2997.03	-2998.37	-2998.07
200	-2995.30	-2996.74	-2994.99	-2995.50	-2995.56
250	-2995.19	-2995.18	-2994.71	-2994.98	-2994.82
300	-2994.75	-2994.95	-2994.71	-2994.90	-2994.82
350	-2994.56	-2994.64	-2994.56	-2994.72	-2994.65
400	-2994.53	-2994.57	-2994.55	-2994.59	-2994.60
450	-2994.51	-2994.54	-2994.53	-2994.59	-2994.54
500	-2994.51	-2994.52	-2994.51	-2994.53	-2994.52
550	-2994.50	-2994.51	-2994.50	-2994.52	-2994.50
600	-2994.49	-2994.50	-2994.49	-2994.50	-2994.50
650	-2994.49	-2994.50	-2994.49	-2994.50	-2994.50
700	-2994.49	-2994.49	-2994.49	-2994.49	-2994.49
750	-2994.49	-2994.49	-2994.49	-2994.49	-2994.49
800	-2994.49	-2994.49	-2994.49	-2994.49	-2994.49
850	-2994.49	-2994.49	-2994.49	-2994.49	-2994.49
900	-2994.49	-2994.49	-2994.49	-2994.49	-2994.49
950	-2994.49	-2994.49	-2994.49	-2994.49	-2994.49
1000	-2994.49	-2994.49	-2994.49	-2994.49	-2994.49

Data Envelopment Analysis

General Formulation of DEA

Data envelopment analysis (DEA) is first formulated by Charnes, Cooper and Rhodes [12]. It provides a new definition of scalar efficiency of participating units, along with methods for objectively determining the weights by reference to the observational data for the multiple outputs and inputs that characterize such programs.

In order to calculate efficiency of the units, we need inputs and outputs data of all the units which we would like to compare. The definition of efficiency is,

$$\theta = \frac{\sum_{i=1}^{s} u_i y_i}{\sum_{j=1}^{m} v_j x_j} \quad (11),$$

where

x_i= input data

y_i= output data

v_i=weight for input data x_i

u_i=weight for output data y_i

m=number of input data

s= number of output data

θ= efficiency (called D eff. from now).

When there are n decision making units (DMU), D eff of unit “o” can be calculated by;

$$\text{for unit "}o\text{", find } u_{i\,o}, v_{jo} \text{ such that}$$

$$\max\ \theta_o = \frac{\sum_{i=1}^{s} u_{io} y_{io}}{\sum_{j=1}^{m} v_{jo} x_{jo}}$$

subject to

$$\frac{\sum_{i=1}^{s} u_{io} y_{ik}}{\sum_{j=1}^{n} v_{jo} x_{jk}} \le 1 \ (k = 1,\ldots,n) \qquad (12),$$

$$u_{i\,o}\quad 0 \ \ (i=1,\ldots,s)$$

$$v_{j\,o}\quad 0 \ \ (j=1,\ldots,m)$$

where subscript “o” is efficiency and weights for unit “o” and “k” is data for unit “k”. Eq. (12) can be converted into linear programming and by using dual method, it can be rewritten as,

find θ_o and λ_o such that

$$\max\ \theta_o$$

subject to

$$\theta_o \boldsymbol{x}_o - \boldsymbol{X}\lambda_o \ge \boldsymbol{0}$$

$$\boldsymbol{y}_o - \boldsymbol{Y}\lambda_o \le \boldsymbol{0}$$

$$\lambda_o \ge \boldsymbol{0}$$

where (13)

λ_o: Lagrange multiplier

θ_o: efficiency

x_{jk} : input data sets

y_{ik} : output data sets

.

Each data has its specific meaning, like the determined weights mean that the weight which give highest efficiency, Lagrange multipliers means to determine superior sets and the direction of improvement and so on. But in this study, we only need efficiency θ_o, thus we do not go into the detail any more, Illustrative explanation is given in Fig. 4, and when θ_o=1.0, it means the unit “o” is located at the frontier of the efficiency.

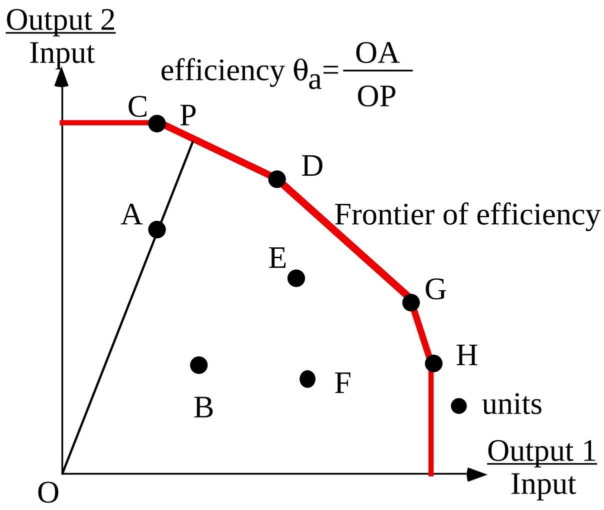


Fig. 4 Illustrative explanation of DEA with 1 input and 2 outputs

Remarks

Many advantages are reported in the refs (11,12) by using the results of DEA. However, we only need to calculate efficiency in this study. So that we are loosing many other advantages in DEA. Even though, we can benefit some of the advantages;

1.We do not have to care the order of given data.

2. We can obtain efficiency in scalar value. And it shows how far the reference data will be from the frontier.

3. When optimization process goes by, frontier of the efficiency will become a set of Pareto solution.

Only the lack in DEA is that it assume the convex nature of the frontier. Which means we cannot measure efficiency correctly when there are concave nature in the Pareto optimum solution sets.

Multi-objective Optimization in GAs

Here we only give a formulation of multi-objective optimization and present the conventional way to estimate fitness function by using ranking strategy.

Formulation

Find $\boldsymbol{x}$ such that

$$\text{minimize } F(\boldsymbol{x})=\{f_1(\boldsymbol{x}),\ldots, f_L(\boldsymbol{x})\}^T \qquad (14)$$

subject to

$$g_j(\boldsymbol{x}) \ge 1.0$$

$$x_i^L \le x_i \le x_i^U,$$

where

$\boldsymbol{x}$= design variable (=$\{x_1,\ldots, x_N\}^T$)

$F(\boldsymbol{x})$= objective functions($=\{f_1(\boldsymbol{x}),..., f_L(\boldsymbol{x})\}^T$)
$g_j(\boldsymbol{x})$= constraints (j=1,...,M)
x_i^L, x_i^U= side constraints

Penalty function

In GAs, we cannot treat constraints. So that we have to include them into fitness function using penalty functions.

$$pen_i(\boldsymbol{x}) = f_i(\boldsymbol{x}) + \sum_{j=1}^{M} p_j \times P[1.0 - g_j(\boldsymbol{x})]^c \qquad (15),$$

where

pj= penalty coefficient

a= penalty exponent

$$P[y] = \begin{cases} y & for\ y > 0 \\ 0 & otherwise \end{cases} \qquad (16).$$

By using Eq. (15), we can convert them into fitness function (fit_i) for each objective function. (Usually fitness function will be maximize in GAs.)

Ranking method

By using the penalty function for each objective function, we would like to give higher fitness value to the frontier to apply GAs. One of the method is ranking method[3]. We will illustrate how we rank each individuals in Fig. 5. In this method, count the number of individuals which has higher fitness value for every fitness function and add 1 to its number. For example, individual D has no individual which has higher objective function of both fit1 and fit2, thus its ranking is 1. Individual 4 has 2 individuals (G and H), thus its ranking is 3. In this method, even if individual E has ranking 1, it is almost the same distance with F from the frontier which is determined by DEA. It seems very strange and it will cause zigzag Pareto solutions for the result of GAs.

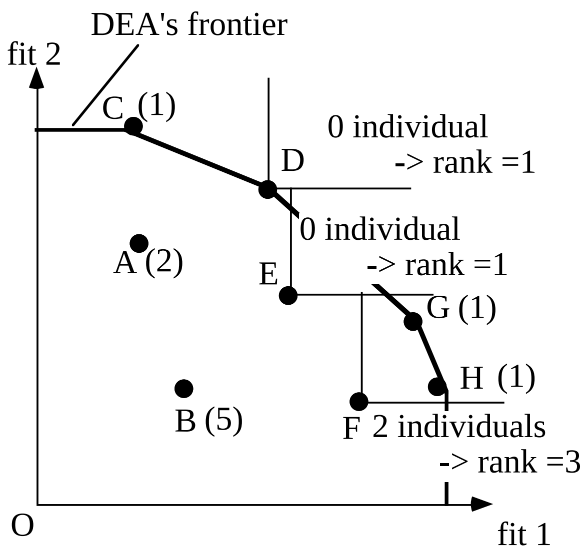


Fig. 5 Illustrative explanation of DEA with 1 input and 2 outputs

The proposed method

In the proposed method, we try to estimate fitness value by using DEA. We have to prepare data for DEA. Input in DEA will be the objective function which we would like to minimize and output will be the objective function which we would like to maximize. So it might be straight forward after we calculate objective functions and constraints and convert them in to fitness function using Eq. (15). However, there are some conditions in DEA that we have to convert data for its purpose.

Preparation of data

1. In DEA, we need at least two input and one output (or one input and two output) data. If we do not have enough data, add unit data set for output data.
2. In DEA, all data need to be plus, thus, when there are minus value in the fitness function, convert fitness function value as,

$$fit_i = fit_i - \min(fiti) + \varepsilon \qquad (17),$$

where ε is a small number. (ε=0.1 in the following examples)

Flow of the proposed method

Flow of the proposed method is shown in Fig. 6.

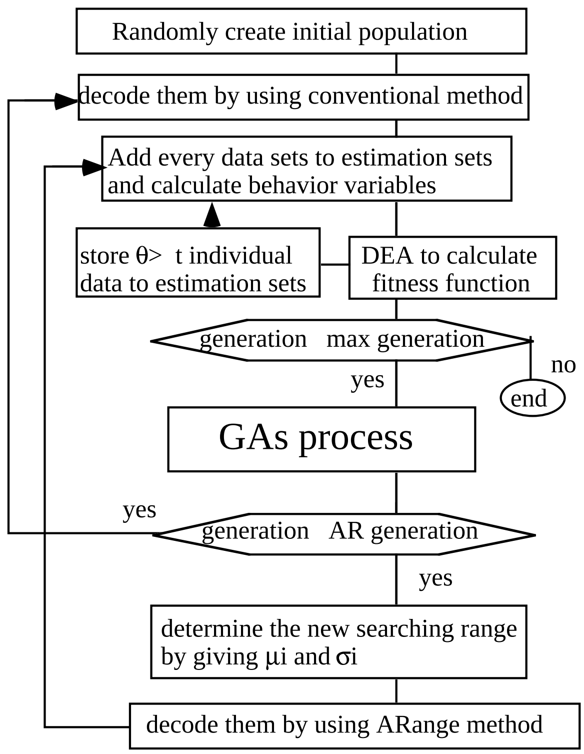


Fig. 6 Flowchart of the proposed method

Initially, populations are given randomly and decode by using conventional GAs. We prepare data for DEA and calculate fitness function. If fitness function is nearly equal to 1 (> t), we store them into DEA calculation data. Simple GAs will be ran for several times with conventional decoding method for a while (10 generation in the following examples). In this process we would like to find global information of the frontier. After that we will use ARange decoding. Unfortunately, mean value does not have any importance like it has in the single objective optimization case. So we determine its value by following. We repeat the process until generation become maximum generation.

Determination of a new range

In ARange decoding, a new searching range will be determined by μ_i and σ_i. However, because there are many different objectives, mean value does not have any important meaning. What we would like to have in MO is a precise set of Pareto solutions. Thus, we would like to give searching range near the Pareto solutions.

First, we will find two individuals (a and b), which is in the neighbor and has maximum distance (Fig. 7), Then, μ_i is determined as,

$$\mu_i = w\, x_{i,a} + (1 - w)\, x_{i,b} \tag{18}$$

where w is a parameter randomly given by [-0.25, 1.25] in the following. Then, σ_i is determined by the conventional ways.

By using these ranges, we can fill the Pareto solutions which are given by using conventional decoding method.

Counter-plan for concave characteristics

In DEA, we only can solve convex characteristics. When we have any pre-knowledge or we found that there are any concave characteristics in the frontier, we can convert them to convex character by using following equation.

$$fit_i = Exp(fit_i) \tag{19}$$

Although it is only a counter-plan for concave characteristics, if we can give some good weight for them, we can solve concave characteristics problem like we can see in Athan and Papalambros[18].

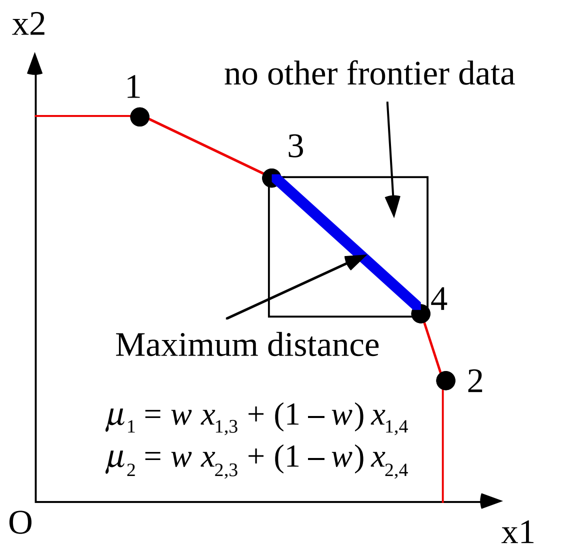


Fig. 7 Determinatio of the searching range

Demonstrative Examples

Tamaki's simple problem

In order to show the effectiveness of the proposed method, we carry out a simple numerical example shown in Tamaki[7].

$$\begin{aligned}
&\textit{Minimize } f_1(x_1,x_2) = 2\,x_1^2 - x_2 \\
&\textit{and } f_2(x_1,x_2) = -x_1 \\
&\textit{subject to} \\
&\quad (x_1 - 1)^3 + x_2 \le 0 \\
&\quad (x_1,x_2) = ([0,),[0,))
\end{aligned} \tag{20}$$

To show the effectiveness of the proposed method, we compare the results with cases.

Case 1: Fonseca's method with conventional decoding
Case 2: Fonseca's method with old ARange decoding
Case 3: DEA with conventional decoding
Case 4: DEA with new ARange decoding
Case 5: DEA with Eq. (19) with conventional decoding
Case 6: DEA with Eq. (19) with new ARange decoding

In the conventional decoding, we use 6 bits for each design variables, and in ARange decoding we use 4 bits for each design variables. And AR generation equals to 10. Results of each case in the design variable is shown in Fig. 8 to 12. Comparison of the results in objective function space of case 4 and 6 is shown in Fig. 13. In case 1, we can obtain over all Pareto solution sets after 100 generations. But the results include zigzag relations, which can be seen explicitly in the results after 30 generation. In that sense, we have to examine the results, which are good Pareto solution and which are not.

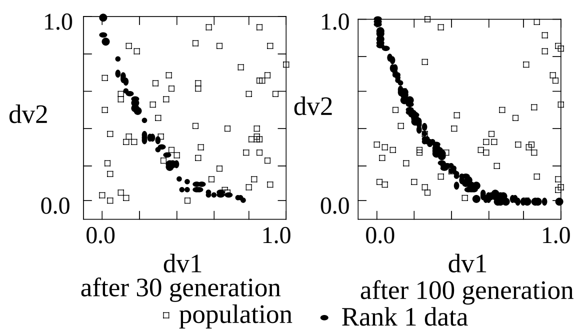


Fig. 8 Results of Case 1

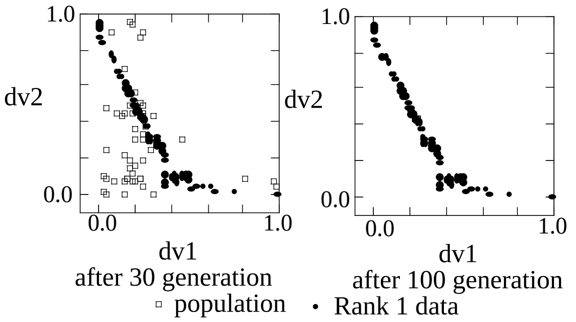


Fig. 9 Results of Case 2

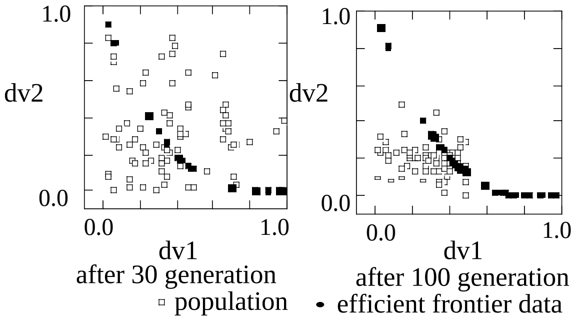


Fig. 10 Results of Case 3 (t=0.995)

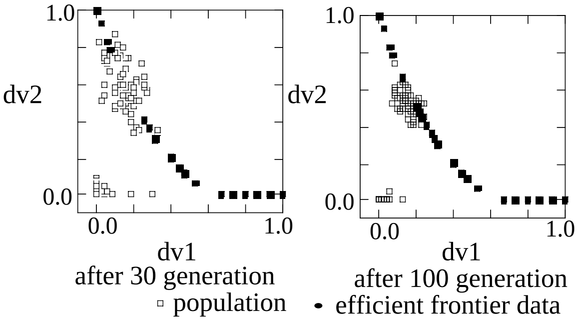


Fig. 11 Results of Case 4 (t=0.995)

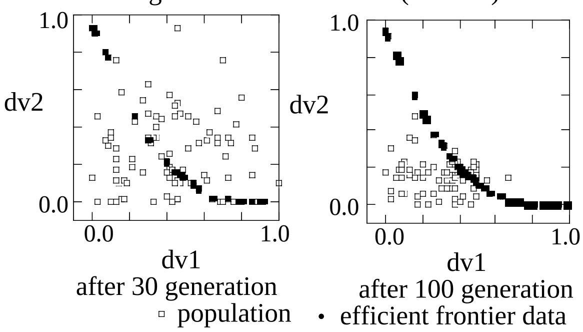


Fig. 12 Results of Case 5 (t=0.999)

In case 2, we use old ARange decoding that is to use mean value. We can see that zigzag nature is exaggerated and it would not disappeared after 100 generation, because the searching range became so narrow to search the other possibility. In that sense, usage of old ARange in Fonseca's method was failed. In case 3, even the results after 30 generation, we have obtained data in need to predict the actual Pareto solution sets. After 100 generation, we have

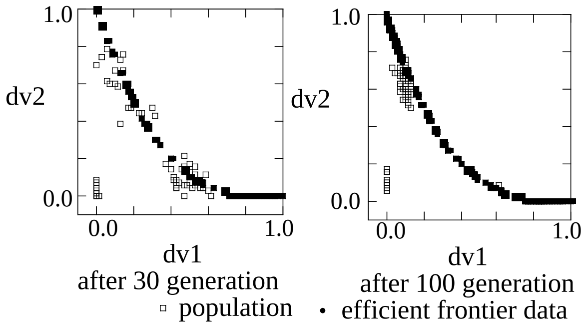


Fig. 13 Results of Case 6 (t=0.999)

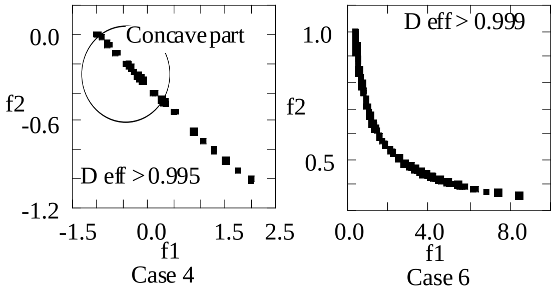


Fig. 14 Comparison of the results in objective function space

a good set of Pareto solutions without zigzag nature. Which mean that we only have the data in need. However, the data between x1=[0.1,0.2] is missing. In Fig. 11, we can see that the proposed ARange tried to find the results of missing part in case 3. As ARange can have more precision that it can obtained some missing part of case 3 and obtained over all Pareto optimum sets. When we use convex conversion of fitness function, we can obtain the missing part of case 3 even with the conventional decoding (Fig. 12). In Fig. 13, we can obtain more Pareto solution than in case 5. Even in the result after 30 generation, we can obtain almost the same number of Pareto solutions which is obtained after 100 generation in case 5. Compared with the results with case 4, we can obtain more precise Pareto sets. This results are quite natural, because DEA can estimate its efficiency more accurate in the convex case. As we can see in Fig. 14, the problem has some concave character in its Pareto solution sets. Thus, there are some missing part in the original method. However, after converted to the convex problem, we can obtained all over the Pareto solution precisely.

A static three-bar truss problem

The problem is first solved by Koski[19] and it is also used to explain the efficiency by Athan and Papalambros [18].

The total volume of the truss and a linear combination of the two nodal displacements are to be minimized. The design variables are the three cross sectional area of

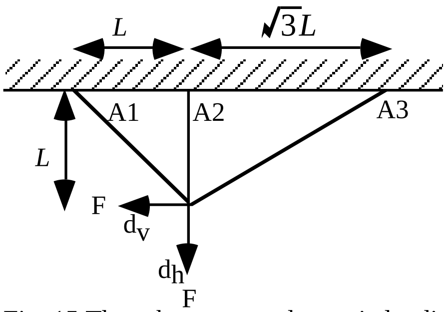
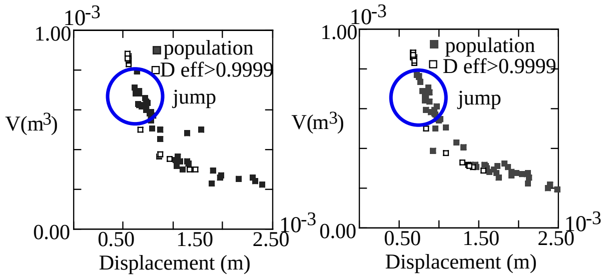
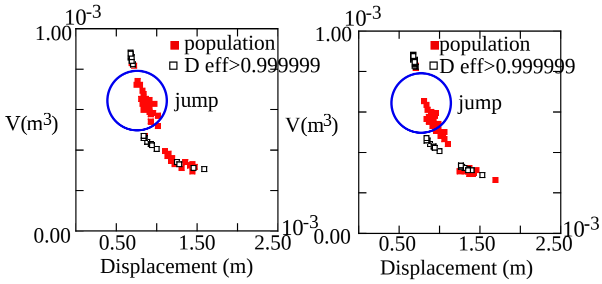


Fig. 15 Three-bar truss under static loading



After 30th generation After 100 generation

Fig. 16 Results in the case we do not use Eq. (19)



After 30th generation After 100th generation

Fig. 17 Results in the case we use Eq. (19)

the members. Stress and side constraints are imposed. The three bar truss is shown in Fig. 15. Problem formulations are as follow;

Find {A1,A2,A3} such that

$$\text{minimize } V = \sqrt{2}LA_1 + LA_2 + 2LA_3 \qquad (21)$$

and d=0.25 d_v + 0.75 d_h

subject to

$\sigma_c \le \sigma \le \sigma_t$ (i=1,2,3)

$A_L \le A \le A_U$ (i=1,2,3),

where

$$d_v = \frac{8FL}{E}\,\frac{b-d}{b^2-ad}$$

$$d_h = \frac{8FL}{E}\,\frac{b-a}{b^2-ad}$$

$$\sigma_1 = \frac{E(d_v - d_h)}{2L}$$

$$\sigma_1 = \frac{Ed_v}{L}$$

$$\sigma_1 = \frac{E(d_v + \sqrt{3}d_h)}{4L}$$

$$a = 8A_2 + \sqrt{8}A_1 + A_3$$

$$b = \sqrt{3}A_3 - \sqrt{8}A_1$$

$$d = 3A_3 + \sqrt{8}A_1$$

F=20KN, L=1.00m,E=200GPa,

σ_t=200MPa, σ_c=-200Mpa

A_L=1.0 e-5 m^2, A_U=2.0 e-4 m^2

We applied the proposed method in both case with convex conversion and without convex conversion. Results are shown in Figs. 16 & 17. In the both cases, we have obtained sufficient number of Pareto solutions, in order to predict over all Pareto solution sets. Although the problem seems to have a convex character, there are some jump in the Pareto sets, because even if we convert the problem to convex character they do not vanish. In such a case, Eq. (18) try to search the range which are some missing range on design variable space, so that the searching range will become those range where there seems no Pareto solutions like we can see in the circle of Fig. 17. This might be one of the shortcoming of using Eq. (18). However, even in the results after 30 th generations, we can predict the over all Pareto solution sets sufficiently, and it shows effectiveness in the searching of the solutions of the proposed method. Comparing the both results, convex conversion results seems to obtain Pareto solutions more effectively. Even we give higher "D eff" value, we obtained more Pareto solution than the other. This is quite natural results, because of the nature of DEA that need convex characteristics in the objective functions.

Conclusions

1. In this article, we proposed a new multi-objective optimization method, using ARange GAs with DEA.
2. When we use ARange decoding, we do not have to care about the initial searching range and the number of bits to determine precision of the results. As ARange decoding try to adapt the searching range adaptive to the range where we would like to obtain solutions and when the convergence goes by, it will narrow the range to obtain more precise results.
3. DEA gives some mathematical meaning for Pareto optimality. In these sense, we can give rational fitness function to each individuals, so that we can avoid to obtain some zigzag nature which will cause by estimating Pareto optimality with conventional ranking methods. Moreover, efficiency in DEA will be much more suitable for GAs.
4. Through some simple numerical examples, we showed

the effectiveness of the proposed method. One of the shortcoming in using DEA is that they cannot treat the problem with concave case, We proposed one counterplan to convert the problem into convex characteristics and obtained the results much better than the original ones.

5. In the proposed method, we can obtain enough number of Pareto solutions to predict over all Pareto solutions sets within relatively small number of generations. Which will show the effectiveness of the proposed method in the stand point of convergence. We still need some effort to give searching range to obtain the over all Pareto solutions. We would like to investigate that point in the future.

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