

# EVOLUTIONARY ALGORITHMS FOR NAVIGATION OF UNDERWATER VEHICLE

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**Abstract--** Path planning for an underwater vehicle can be formulated as a multiobjective optimization problem, which can be solved by modern heuristic techniques. For assessment of a trajectory, three crucial criteria are used: a total length of a path, a smoothness of a trajectory, and a measure of safety. A multiobjective evolutionary algorithm for finding Pareto-optimal solutions is proposed. Then, the underwater vehicle and navigation in three dimensions is considered. Some results of numerical simulations are presented.

**Index terms—**underwater vehicle, multi-criterion optimization, evolutionary algorithms

## I. INTRODUCTION

Path planning for an underwater vehicle can be formulated as a multiobjective optimization problem, which can be solved by modern heuristic techniques. We focus on the Super Achilles M4 that is a remotely operated vehicle designed for underwater observation in hostile environment [16]. The main characteristic of the vehicle is its power capability and its compactness.

The vehicle consists of two parts. The upper part ensures the vehicle positive buoyancy and houses the sonar head. The lower part consists of a watertight frame made of welded pressure-resistant tubular stainless steel. The underwater vehicle is equipped with four three-phase asynchronous thruster motors with propellers. A vehicle weights 120 kg. Its length is 720 mm, the width 600 mm, and the height 520 mm. There is a surface control unit with its power cable. It is possible to extend unit's capabilities by finding trajectory of the vehicle.

Evolutionary strategies and genetic algorithms are the alternative approaches compare to the other heuristic multiobjective optimization methods such, as simulated annealing, tabu search, Hopfield models of neural networks, and Lagrangean relaxation [10].

Both, evolutionary strategies and genetic algorithms can combine with recurrent neural networks for solving optimization problems [2]. This approach is very efficient because of the massively parallel processing. Recently, interest has risen in the application of evolutionary algorithms to solving combinatorial optimization problems. Evolutionary algorithms develop genetic algorithms for solving optimization problems by another chromosome representation, more complex operators, and a specific knowledge related with the optimization problem [8]. In the paper, an evolutionary strategy and genetic algorithm are developed as a novel approach to a multicriteria path planning of the underwater vehicles.

For evaluation of a path (trajectory) three main criteria are used: a total length of a path, a measure of safety, and a smoothness of a trajectory. A multiobjective evolutionary algorithm for finding Pareto-optimal solutions is proposed. A navigation of the underwater vehicle in three dimensions is considered. Some results of numerical simulations are presented.

## II. EVOLUTION STRATEGY

For solving optimization problems an evolution strategy was proposed and developed by Rechenberg [9] and Schwefel [13]. An extension of evolution strategy on multi-objective optimization was introduced by Kursawe [7].

Chromosome in evolution strategies consists of two main parts, as follows:

$$\bar{X} = (x, \sigma), \quad (1)$$

where

$x$  - decision variable vector,

$\sigma$  - deviation standard vector for  $x$ .

Figure 1 shows the diagram of evolutionary strategy EA in a version  $(\mu+\lambda)$  [8]. A strategic mutation of chromosome

changes a value of decision variable  $x_m$  by randomly chosen number  $\Delta x_m$  representing value of random variable with a normal distribution  $N(0, \sigma_m)$ .

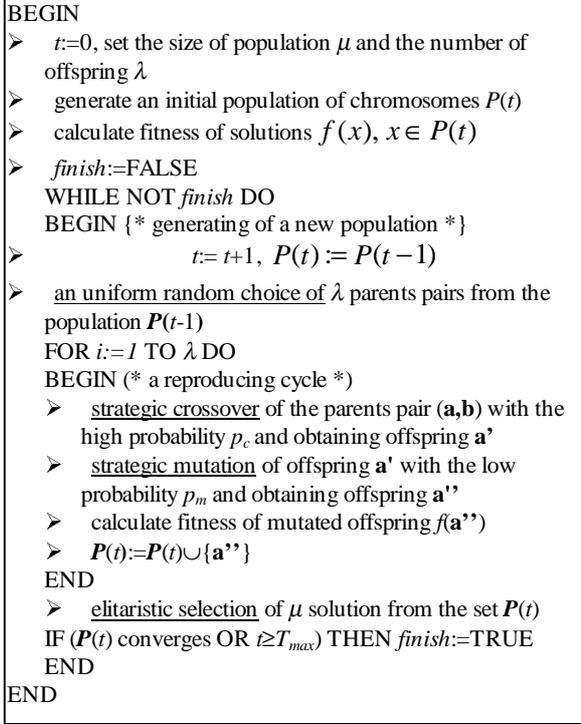


Fig. 1. Diagram of evolution strategy

### III. QUALITY CRITERIA FOR ANTI-COLLISION CONTROL

An underwater vehicle should plan a path between two specified locations in a three dimensional space, which is collision-free and satisfies optimization criteria. If  $(x_1, y_1, z_1)$  is the starting point and  $(x_M, y_M, z_M)$  is the destination point, then the path can be represented as follows:

$$x = (x_1, y_1, z_1, \dots, x_m, y_m, z_m, \dots, x_M, y_M, z_M). \quad (2)$$

The point  $(x_m, y_m, z_m)$  is feasible, if an segment from  $(x_{m-1}, y_{m-1}, z_{m-1})$  to  $(x_m, y_m, z_m)$  and an segment from  $(x_m, y_m, z_m)$  to  $(x_{m+1}, y_{m+1}, z_{m+1})$  do not cut forbidden areas for the vehicle. A path  $x$  can be either feasible (collision-free) or infeasible. A path with at least one unfeasible point is non-feasible, too. We assume that at the time  $t$  the forbidden areas are given, and it is possible to determine if any trajectory is feasible or not. The number  $M$  of points defined a trajectory  $x$  can be changed. There is a maximum number of points  $M_{max}$  and a minimum number of points  $M_{min}$ . Moreover, some formal constraints for including trajectories in the given water area are formulated, as follows:

$$\begin{aligned}
 M_{min} &\leq M \leq M_{max}, \\
 X^{min} &\leq x_m \leq X^{max}, \quad m = \overline{1, M}, \\
 Y^{min} &\leq y_m \leq Y^{max}, \quad m = \overline{1, M}, \\
 Z_m^{min} &\leq z_m \leq 0, \quad m = \overline{1, M},
 \end{aligned} \quad (3)$$

where

$X^{min}, X^{max}$  – area constraints for the coordinate  $x_m$ ,

$Y^{min}, Y^{max}$  – area constraints for the coordinate  $y_m$ ,

$Z_m^{min}$  – a deep of water on the position  $(x_m, y_m)$ .

For evaluation the quality of the planning underwater vehicle path several criteria can be used. Usually, the total length of the path  $x$  is discussed because of the time and economy of motion [14, 15]. Let  $p_m = (x_m, y_m, z_m)$  denotes a point of trajectory direction changing. The total length of the path  $x$  can be expressed, as follows:

$$F_1(x) = \sum_{m=1}^{M-1} d(p_m, p_{m+1}), \quad (4)$$

where  $d(p_m, p_{m+1})$  denotes a distance between two adjacent path points  $p_m = (x_m, y_m, z_m)$  and  $p_{m+1} = (x_{m+1}, y_{m+1}, z_{m+1})$ .

The distance  $d(p_m, p_{m+1})$  between two adjacent path points can be calculated, as below:

$$d(p_m, p_{m+1}) = \sqrt{(x_m - x_{m+1})^2 + (y_m - y_{m+1})^2 + (z_m - z_{m+1})^2}. \quad (5)$$

A. The length of a trajectory is the same from formula (3), (4) and (5), if it goes through a forbidden area or through a clear field. So, a safe aspect of navigation is considered.

### IV. SAFE MEASURE OF TRAJECTORY

The second criterion  $F_2$  for evaluation the quality of a trajectory is a safe measure, which can be defined according to the following formula [14]:

$$F_2(x) = \max_{m=1, M-1} b(p_m, p_{m+1}), \quad (6)$$

where  $b(p_m, p_{m+1})$  denotes the penalty value for the line segment from the point  $p_m$  to the point  $p_{m+1}$ , if the segment cuts any forbidden area.

The penalty value  $b(p_m)$  is defined as follows:

$$b(p_m, p_{m+1}) = \begin{cases} d_{min} - r(p_m, p_{m+1}), & \text{if } r(p_m, p_{m+1}) \geq d_{min} \\ e^{\beta(d_{min} - r(p_m, p_{m+1}))} - 1, & \text{otherwise,} \end{cases} \quad (7)$$

where

$r(p_m, p_{m+1})$  – the smallest distance from the line segment connecting path points  $(p_m, p_{m+1})$  to an object from all detected objects created forbidden areas,

$d_{\min}$  – a parameter defining a minimal safe distance from the underwater vehicle to another object,

$\beta$  – a positive penalty coefficient.

If the smallest distance from the line segment connecting path points  $(p_m, p_{m+1})$  to an object from all detected objects is non-smaller than the safe distance  $d_{\min}$ , then the penalty  $b(p_m, p_{m+1})$  is negative. When the distance between a path segment and the closest obstacle is smaller than  $d_{\min}$ , then the penalty  $b(p_m, p_{m+1})$  is positive and it grows exponentially. The function  $F_2$  is defined as a maximum of  $b(p_m, p_{m+1})$  for all segments to make sure that if a certain segment of a path is dangerously close to an obstacle, i.e. within distance  $d_{\min}$ , then the path is penalized strongly even if all other path segments are safe. The safe criterion  $F_2$  should be minimized to obtain a trajectory as safe as possible.

## V. SMOOTHNESS OF TRAJECTORY

The third criterion  $F_3$  should maintain a smooth trajectory to avoid large changes of direction according to the following formula [15]:

$$F_3(x) = \max_{m=2, M-1} s(p_m), \quad (8)$$

where  $s(p_m)$  denotes the measure of a trajectory “curvature” at the point  $p_m$ .

The trajectory curvature at the point  $p_m$  can be defined as follows:

$$s(p_m) = \frac{\alpha_m}{\min\{d(p_{m-1}, p_m), d(p_m, p_{m+1})\}}, \quad (9)$$

where  $\alpha_m$  is the angle between the extension of the line segment  $(p_{m-1}, p_m)$  and the line segment  $(p_m, p_{m+1})$  on a plane determined by both above segments.

We assume, that  $\alpha_m \in [0, \pi]$ . For the same distances the trajectory is smoother, if the maximal angle in it is smaller. If the minimal length from distances  $d(p_{m-1}, p_m)$  and  $d(p_m, p_{m+1})$  is longer, then there are less points  $p_m$ , where the direction of trajectory is changed. The criterion  $F_3$  should be minimized.

Another approach for improving the smoothness of trajectory is related with the consideration of the sum of all trajectory curvatures at points  $p_m, m = \overline{1, M}$ , as follows:

$$\overline{F}_3(x) = \sum_{m=2}^{M-1} s(p_m). \quad (10)$$

Moreover, the minimization of sum-squared function can be carried out, as below;

$$\tilde{F}_3(x) = \sqrt{\sum_{m=2}^{M-1} s(p_m)}. \quad (11)$$

## VI. MULTICRITERIA PROBLEM FORMULATION

The standard problem of the mobile robot path planning in two-dimensional plane was formulated as one criterion minimization problem by Yap [15]. A transformation of above three criteria in one global criterion for a two-dimension plane trajectory was carried out in [14]. Some evolutionary algorithms for navigation of mobile robots are presented in [8]. So, the formulation of the trajectory finding problem in a three dimension space as a multiobjective optimization problem is the next step for finding a short, safe and smooth trajectory of mobile robot or another object such, as an underwater vehicle.

There are several classes of multiobjective optimal solutions related with the preferences for criteria. If criteria are ordered from the most important criterion to the least important criterion, then a hierarchical solution can be found. In a multicriteria navigation of the underwater vehicle the safe measure  $F_2$  seems to be the most important.

If all criteria have the same priority, then Pareto-optimal solutions can be considered [1]. Because of the great number of Pareto-optimal solutions some reducing techniques can be used. For instance, the compromise solutions with the “democracy” parameter  $p$  equal 1, 2 or  $\infty$  may be extracted from Pareto set. Moreover, an additional criterion can be used. If the anti-collision situation permits on a dialog with the navigator, then some dialog techniques can be introduced, where the navigator choose the best trajectory from the proposed set of trajectories during several iterations.

Let us consider the multicriteria optimization problem for finding optimal trajectory for the underwater vehicle as the Pareto solution in the following form:

$$(X, F, R), \quad (12)$$

where

$X$  – the set of admissible trajectories,

$F$  – the vector criterion,

$R$  – the relation for finding Pareto-optimal trajectories.

Because of the variable number of points in trajectory, a set of all trajectories (admissible or non-admissible) consists of

vectors with no more than  $3M_{\max}$  coordinates. It can be denoted as  $\mathcal{X} = 2^{\mathcal{T}}$ , where  $\mathcal{T} = \mathcal{R}^{3M_{\max}}$  and  $\mathcal{R}$  is a set of real numbers.

The set of feasible trajectories is defined, as follows:

$$\begin{aligned} X = \{x \in \mathcal{X} \mid x = (x_1, y_1, z_1, \dots, x_m, y_m, z_m, \dots, x_M, y_M, z_M), \\ M_{\min} \leq m \leq M_{\max} \\ X^{\min} \leq x_m \leq X^{\max}, m = \overline{1, M}, \\ Y^{\min} \leq y_m \leq Y^{\max}, m = \overline{1, M}, \\ Z_m^{\min} \leq z_m \leq 0, m = \overline{1, M}\} \end{aligned} \quad (13)$$

The vector criterion  $F : \mathcal{X} \rightarrow \mathcal{R}^3$  has three scalar criteria, as follows:

$$F(x) = [F_1(x), F_2(x), F_3(x)], x \in X, \quad (14)$$

where  $F_1(x), F_2(x), F_3(x)$  are calculated according to (3), (6) and (8).

The relation  $R$  for finding Pareto-optimal trajectories is a subset of  $Y \times Y$ , where  $Y = F(X)$ . If  $a \in Y, b \in Y$ , and  $a_n \leq b_n, n = \overline{1, N}$ , then the pair of evaluations  $(a, b) \in R$ . Above definition of the Pareto relationship respects the minimization of all criteria. For Pareto-optimal trajectory  $x^* \in X$  there is no trajectory  $a \in X$  such, that  $(F(a), F(x^*)) \in R$ .

## VII. MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM

Evolutionary algorithms based on genetic algorithms can be an alternative approach for evolution strategies. For solving multiobjective optimization problem (12) with the considered three criteria an evolutionary algorithm can be used. Genetic algorithms are applied for solving several optimization problems. Holland [6] developed this approach and its theoretical foundation. Rosenberg noticed abilities of GA for development many criteria [11].

Schaffer [12] considered GA for solving multiobjective optimization problems by a vector evaluated genetic algorithm VEGA. VEGA is an extension of system GENESIS prepared by Grefenstete [5]. VEGA uses dividing of the population on  $N$  subpopulations, where  $N$  is the number of criteria. For each  $n$ th subpopulation the criterion  $F_n$  is a fitness function. But, selection, crossover, and mutation are carried out for whole population. This method for fitness evaluation has the disadvantage related with the discrimination of Pareto solutions situated in the interior of the Pareto frontier. Indeed, mainly lexicographic solutions are preferred. An overview of evolutionary

algorithms for multiobjective optimization problems is presented by Fonseca and Flaming [3].

To avoid the discrimination of the interior Pareto solutions Goldberg introduced the ranging system for non-dominated individuals, which is similar to the Baker's ranging system for one function [4]. In [1] the evolutionary algorithm for finding Pareto-optimal trajectories of the underwater vehicle is presented. At the beginning  $L$  randomly chosen trajectories from the given point  $A$  to  $B$  are generated. Each initial trajectory performs the coordinate constraint, according to the formula (3).

Above constraints are satisfied, when an evolutionary algorithm is in progress. According to the considered vector criterion all trajectories in the population are evaluated. If  $F_2(x) > 0$ , then the trajectory  $x$  is non-feasible, and it gets the fitness function value  $f(x) = 1$ . If the trajectory  $x$  is feasible, then the fitness function value is calculated, as below:

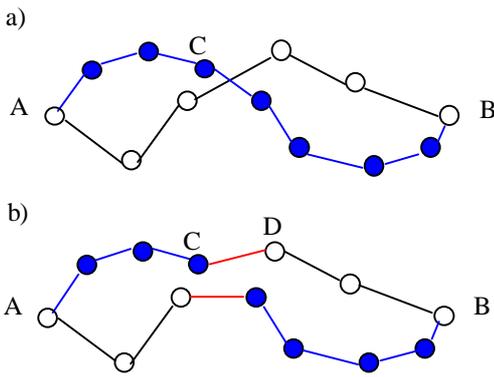
$$f(x) = -r(x) + L + 1. \quad (15)$$

where  $r(x)$  denotes the rank of a feasible solution.

(a) If there are some feasible solutions in a population, then the Pareto-optimal trajectories are sought, and they get the rank 0. Then they are temporary eliminated from the population. From reduced population the new Pareto-optimal trajectories are found and get the rank 1. This procedure with increasing of the rank is repeated until the set of feasible solutions will be exhausted. That is why, all non-dominated solutions have the same rank and the same fitness to reproduction.

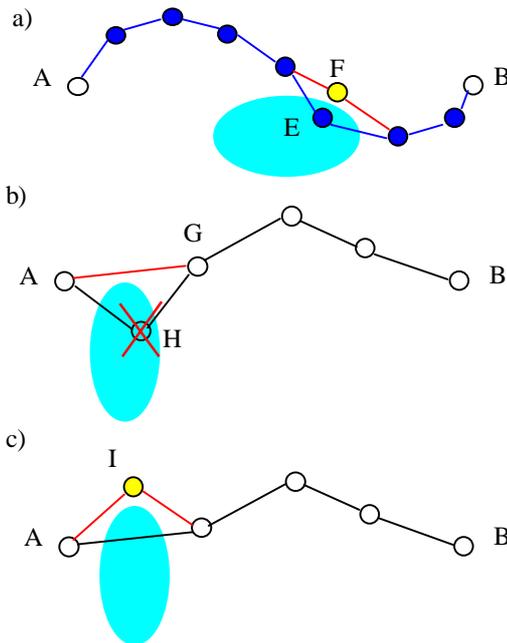
## VIII. CROSSOVER AND MUTATION

During selection a new multicriteria competition approach is introduced because of its advantage to the proportional selection or the other standard selections. Two trajectories are randomly chosen to the competition with respect to their fitness. If the trajectory  $A$  dominates the trajectory  $B$  in the Pareto relationship  $R$  sense, then  $A$  is a winner, and  $A$  is recommended to a reproduction. The dominated trajectory  $B$  is not recommended. If both trajectories are non-dominated each to other, then they are accepted, too. Figure 2a) shows two trajectories chosen from a population to crossover. At the beginning, the point  $C$  was chosen from the trajectory points represented by gray dots with the same probability  $1/M$  for each point in this trajectory. Then, the point  $D$  from the co-parent trajectory is matched.  $D$  is the closest point to  $C$  from the closer points to  $B$ . The first offspring is created from part  $(A, C)$  of the first parent trajectory, an additional edge  $(C, D)$ , and from the part  $(D, B)$  of the second parent trajectory.



**Fig. 2.** A crossover operator in the evolutionary algorithm for randomly chosen point C  
 a) two parent trajectories,  
 b) two offspring trajectories.

A crossover of two accepted trajectories is carried out with the low probability from 0.1 to 0.2. If the crossover point is  $p_m$ , then the first offspring is created from the first part trajectory of parent A and from the second part trajectory of parent B. The first part of trajectory starts from the point  $p_1$  and finishes in  $p_m$ . Similarly, the second part of trajectory starts from the point  $p_m$  and finishes in  $p_M$ .



**Fig. 3.** A mutation operator in the evolutionary algorithm:  
 a) a shift of a point on a trajectory,  
 b) removing a point from a trajectory,  
 c) insertion a point to a trajectory

A mutation is carried out with the high probability from 0.6 to 0.8. For randomly chosen trajectory several versions of mutation are taken.. The mutation changes elementary parameters of the trajectory such, as node coordinates or insert/delete node to satisfy constraints.

On Figure 3, there are some examples of a mutation. A shift of a point on a trajectory can reduce the trajectory length, avoid a collision situation, or improve the smoothness of a trajectory. On Figure 3a), the point E is shifted to the randomly chosen position F from the non-collision neighborhood of a point E. Then, two additional edges are inserted.

```

BEGIN
t:=0, set L the number of trajectories in population
generate the initial population of trajectories P(t)
calculate the evaluation for each trajectory in population
    F(x), x ∈ P(t)
calculate the rank for each trajectory in population
    r(x), x ∈ P(t)
calculate the fitness for each trajectory in population
    f(x), x ∈ P(t)

finish:=FALSE
WHILE NOT finish DO
    BEGIN /* generation of a new population */
    t:= t+1, P(t) := ∅
    calculate probability of trajectory selection
    p_s(x), x ∈ P(t-1)
    FOR L/2 DO
        BEGIN /* reproduction cycle */
            ♦ selection with multicriteria competition to obtain a
              potential parent pair (a,b) from population P(t-1)
            ♦ crossovering a pair of parents (a,b) with the
              crossover probability p_c
            ♦ mutation of a pair of offspring (a',b') with the
              mutation propability p_m
            ♦ calculate the fitness for each trajectory offspring
              F(a'), F(b')
            ♦ P(t):=P(t) ∪ {a',b'}
        END
    calculate the rank for each trajectory in population
        r(x), x ∈ P(t)
    calculate the fitness for each trajectory in population
        f(x), x ∈ P(t)

    IF (P(t) jest zbieżna OR t ≥ T_max) THEN finish:=TRUE
    END
END
  
```

**Fig.4.** An evolutionary algorithm for finding subset of Pareto-optimal trajectories

Removing the point from trajectory can produce similar results as a point shift. On the fig. 3b), the point H is removed, and a new trajectory without a conflict with an obstacle is obtained. In some collision situation the insertion of a new point to a trajectory can be necessary as shown on the fig. 3c). Mutation operators can fail in some admissible trajectories, but in a population of trajectories greater chances have individual with higher rank in

Pareto-optimality sense. That is why, the Pareto-suboptimal solutions are obtained after enough number of new generations.

Figure 4 shows an evolutionary algorithm for finding Pareto-optimal trajectories with using like-Pascal notation.

## IX. SIMULATION RESULTS FOR UNDERWATER VEHICLE

The multicriteria evolutionary algorithm was applied for finding trajectories of theoretical objects in different environments. Another approach based on models of neural networks was developed, too [1]. Let consider finding trajectory at the time  $t$  for the real object the underwater vehicle called Koral 100, which was designed by Technical University of Gdańsk. It can operate on the deep up 100 m with the maximal speed 1,5 meters per second. It uses five electrical engines.

Its weight is 90 kilos and the size is 0,7 meters in each dimension. It is equipped with a board computer and a vision system consisted of a TV-camera, a photo-camera, and reflectors. It can penetrate water space in three dimensions. The vehicle can be controlled from the ship or it can work alone with using the board computer and the navigation system. The multiobjective optimization problem (11) can be adapted for the motion of the underwater vehicle by using trajectory  $x$ .

The Pareto-optimal trajectory of the underwater vehicle model for a simulated water environment with some obstacles is presented in [1]. The evolutionary algorithm performed 250 new generations, and from the last population all trajectories were admissible. 4 trajectories were Pareto-optimal. Also, an evolutionary algorithm starts from different randomly chosen initial point, but after enough number of generations it reaches admissible trajectories, and the P-suboptimal trajectories can be found.

## X. CONCLUDING REMARKS

Techniques for solving related multiobjective optimization problems can use the proposed evolutionary strategy “with plus” or the evolutionary algorithm. The presented approach seems to be very elastic for adaptation in the other cases.

In the evolutionary algorithm some new mutation operators can be introduced to satisfy a local collision situations. Moreover, evolutionary strategies can be compared to evolutionary algorithms. A canonical genetic algorithm gave worse results, because it is more general and it do not use the knowledge related with the specific optimization problem.

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