A Multiobjective Evolutionary Algorithm
The Study Cases

To Thanh Binh
Institute of Automation and Communication
Steinfeldstr. 3, 39179 Barleben, Germany

Abstract

In the few last years, among other tools a multiobjective evolutionary algorithm (MOBEA) for successfully solving many multicriteria optimization problems (MOPs) was proposed. However, there is a lack of the systematically testing our approach with other benchmark MOPs that may cause the algorithm very difficult to achieve its performance: robustness of the convergence to the true pareto-optimal surface; uniform distribution of the population on it. In this work after briefly discussing a concept for our approach we illustrate its effectiveness for solving some difficult MOPs and propose some basic way to improve it in the future.

1 INTRODUCTION

The MOBEA system is implemented for solving the following MOPs with linear and non-linear constraints:

$$\min_{\vec{x}} \ \vec{f}(\vec{x}) = \min_{\vec{x}} \ (f_1(\vec{x}), f_2(\vec{x}), \cdots, f_{\mathrm{N}}(\vec{x}))$$

where $\vec{x} = (x_1, x_2, \cdots, x_{\mathrm{n}})^T \in \mathcal{F} \subseteq \mathcal{S} \subseteq \mathcal{R}^{\mathrm{n}}$; N and n are the number of objective functions and the number of variables, respectively. The *search space* $\mathcal{S}$ is a region of the n-dimensional space $\mathcal{R}^n$, for example, a n-dimensional rectangle defined by lower and upper bounds of variables:

$$x_{\mathrm{i}}^{(\mathrm{lower})} \leq x_{\mathrm{i}} \leq x_{\mathrm{i}}^{(\mathrm{upper})}, \ \forall i = \overline{1,n}$$

whereas the *feasible region* is defined by a set of m additional linear and nonlinear constraints ($m \geq 0$)[1]:

$$g_{\mathrm{j}}(\vec{x}) \quad \leq \quad 0, \ \forall j = \overline{1,m}$$

[1]Here only constraints in terms of inequations are considered, because constraints in terms of equalities can easily be converted into that of inequations

The solution of the multicriteria optimization problem is to find all feasible trade-offs among the multiple, often conflicting objectives, known as a set $\mathcal{P}_{\mathrm{x}}$ of *global feasible pareto-optimal solutions* ($\mathcal{P}_{\mathrm{x}} \subseteq \mathcal{F}$) in the variable space (Goldberg, 1989; To and Korn, 1997a; Deb, 1998). Because the set of pareto-optimal solutions can be easily recognized in the objective function space using the definition of the pareto-optimality (Goldberg, 1989) it is reasonable to represent it in this space.

According to the discussion in (Deb, 1998) there are some challenging problems that multiobjective evolutionary algorithms have to be faced with:

- Some MOPs may have both *local* and *global* feasible pareto-optimal solutions. The MOBEA should have a possibility to overcome the *local* solutions and robustly to find the *global* ones.
- Different to single-objective optimization, which has one single optimal solution, a multi-objective optimization problem usually contains a set of pareto-optimal or non-dominated solutions. The size and the shape of the pareto-optimal fronts often depend on the number of objective functions, properties of objective functions and interactions among the individual objective functions. A pareto-optimal front may be a point (when all objectives are co-operating) or a multidimensional hyper-surface (when objectives are conflicting to each other), while as the result the MOBEA provides a finite number of solutions (the population size). For this reason, in the solving of MOPs the MOBEA should not only make the current population to evolve towards the global feasible pareto-optimal region, but also maintain the population diversity in the current non-dominated front.
- A pareto-optimal front may be convex or non-convex, continuous or discontinuous. In the cases

of discontinuous pareto-optimal set, there is a need to explore all of its true subregions.

- The presence of the constraints may cause for the MOBEA further difficulties in converging to the true feasible pareto-optimal region and in maintaining a diverse pareto-optimal solutions.

The main aim of this paper is to study which difficulties the MOBEA system has by achieving its performance: robustness of the convergence to the true pareto-optimal surface, uniform distribution of the population on it (Zitzler and Thiele, 1998; Rudolph, 1998).

2 STRUCTURE OF THE MOBEA

In order to make our MOBEA possible to solve challenging MOPs mentioned above, the following algorithms are implemented in this system. We propose a so-called $\mathcal{C}$-fitness (degree of (in)feasibility) for effectively evolving the population towards the feasible region:

$$\mathcal{C}(\vec{x}) = (\sum_{i=1}^{m}[c_i(\vec{x})]^p)^{\frac{1}{p}}, \quad (c_i(\vec{x}) = \max\{g_i(\vec{x}), 0\}; p > 0).$$

Then, some operators on the $\mathcal{C}$-fitness and the fitness $\vec{f}(\vec{x})$ are designed both to make the diversity of the (in)feasible population and to maintain a mixed population (containing infeasible and feasible individuals) in every generation (To and Korn, 1997b). It is advisable especially for the optimization cases, where the feasible region is concave so that it is very difficult to generate a feasible individual from the feasible parent using well-known evolutionary mechanisms. For maintaining of the population diversity on the current non-dominated front a special preselection method is implemented. This method seems to be a combination of the pareto-based ranging and a dynamical window-based selection.

3 EXPERIMENTS

Keeping the challenging problems of MOPs in mind, and based on the idea proposed recently in (Deb, 1998) we try to construct the test problems with linear/nonlinear constraints and then to test our MOBEA system. The use of constrained MOPs allows us to study how our methodology for handling the constraints works correctly. The results of this study are devoted to analyse the performances and to improve our system. Our experiments show that MOBEA can adopt to a different kind of the pareto-optimal front while evolving from local (concave front) to global (convex front) pareto-optimal front. It is also able to explore many subregions of a discontinuous pareto-optimal front even by a small population (e.g. 30 individuals). In some test cases with the high search dimension (e.g. $n > 10$) the population very slowly evolves towards the true pareto-optimal front. Using other methods (e.g. simulated annealing) to additively change the evolution strategy parameters of the whole population, the convergence of the system can be evidently improved for these test cases. But uptodate there is no systematic way to effectively control evolution strategy parameters for all individuals.

4 CONCLUSIONS

In this paper, we attempt to give a brief survey about our MOBEA system and to construct some difficult MOPs for testing our system. For all tests, MOBEA seems to be very robust to achieve good performances.

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