Reduction of the Torque Ripple in Permanent Magnet Actuators by a Multi-Objective Minimization Technique

C.A. Borghi, D. Casadei, M. Fabbri, and G. Serra
Dipartimento di Ingegneria Elettrica, Università degli Studi di Bologna
Viale Risorgimento 2, I-40136 Bologna, Italy

Abstract — **In this paper a minimization technique is used to reduce the torque ripple of permanent magnet actuators. In order to do this a global stochastic (1+1) evolution strategy algorithm coupled to a filled function acceleration technique is utilized. The cost function is defined as a combination of the e.m.f. harmonics induced in the stator windings and the harmonic components of the magnet permeance. The relative importance of the different contributions are taken into account by penalty coefficients. The method described is used for a 3-phase, 6-pole Permanent Magnets (PM) synchronous motor.**

Index terms — **Optimization methods, Permanent magnet machines, Design methodology, Brushless rotating machines.**

I. INTRODUCTION

In an actuator a constant torque as function of time and angular position, is required. The permanent magnet actuators are widely used in industrial applications owing to their high efficiency and power density. However, a torque ripple may be present in these actuators, causing negative effects such as vibrations, noises, positioning errors and non-uniform movement at low speed.

When sinusoidal currents are assumed to flow in the armature windings, there are mainly two contributions to the torque ripple. The first one is the cogging torque which is generated by the tendency of the rotor to align with the stator at positions where the permeance of the magnetic circuit is maximized. The second one is the torque ripple caused by the presence of harmonics in the air-gap flux density distribution of the permanent magnets.

Many studies for the reduction of the cogging torque have been carried out [1]-[5]. The proposed solutions are the step skew of the rotor, the adjustment of the angular magnet width [3] and the shifting of pole pairs [4].

In order to reduce the field harmonics responsible for the second contribution of the torque ripple, a solution has been proposed in [5], which uses several magnet pieces per pole with different thickness. This solution can be only used for large machine and is of complex realization. The influence of field harmonics may be also reduced acting on stator winding distribution. Furthermore, it should be noted that the solutions employed to reduce the cogging torque are also effective for the reduction of the torque ripple due to the field harmonics of the magnets. As a consequence the problem of the torque ripple reduction should be considered on the whole.

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C.A. Borghi et Al., borghi@eletb2.ing.unibo.it, phone +39-51-644 3566, fax +39-51-644 3588, http://die.ing.unibo.it.

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In this paper the reduction of the total torque ripple will be studied without separating the different causes. Reference is made to a synchronous machine with permanent magnets placed on the rotor surface. The problem of the torque ripple reduction is formulated as an optimization design problem. In order to do this a global minimization algorithm based on the Evolution Strategy (ES) method coupled to the Filled Function (FF) technique is utilized [6]-[8]. The minimization considers the two contributions on the torque ripple and the enhancement of the fundamental harmonic component. In order to do this a multi-objective problem is stated and solved by means of a penalty technique. The example presented considers a 6-pole PM synchronous machine. The design optimization unknowns are the magnet width, the magnet position within the pole pitch, the rotor skew angle and the stator winding distribution.

II. TORQUE RIPPLE CONTRIBUTIONS

The electromagnetic torque can be expressed in terms of coenergy variation as follows

$$T = \left.\frac{\partial W_{co}}{\partial \vartheta}\right|_{i = \text{cost.}} \tag{1}$$

With reference to a synchronous machine with surface mounted permanent magnets, it can be noted that the self- and mutual inductance coefficients of the armature windings are independent from the rotor angular position ϑ. Thus, the electromagnetic torque can be expressed as

$$T = \sum_{i=1}^{z} \mathcal{F}_m i_i \frac{dM_{im}(\vartheta)}{d\vartheta} + \frac{1}{2}\mathcal{F}_m^2 \frac{d\mathcal{P}_m}{d\vartheta} \tag{2}$$

where z is the number of phases, $\mathcal{F}_m$ and $\mathcal{P}_m$ represent the equivalent m.m.f. and the magnetic circuit permeance of the magnets, respectively. i_i is the stator winding current and M_{im} the mutual inductance between a stator winding and the one-turn equivalent circuit of the magnets. The e.m.f. of a stator winding is related to the magnet flux by

$$e_i = -\frac{d\phi_{im}}{dt} = -\omega_m \mathcal{F}_m \frac{dM_{im}(\vartheta)}{d\vartheta} \tag{3}$$

where ω_m is the rotor angular speed. From (3) and (2) follows:

$$T = -\sum_{i=1}^{z} \frac{e_i i_i}{\omega_m} + \frac{1}{2}\mathcal{F}_m^2 \frac{d\mathcal{P}_m}{d\vartheta} \tag{4}$$

In Eq.(4) the first term takes into account the main contribution to the electromagnetic torque. In this term, the torque ripple due to the field harmonics of the magnets is also included. The second term in (4) represents the cogging torque. Assuming sinusoidal winding currents, a constant torque is obtained when the machine is designed to compensate the harmonic components of e_i and to reduce the magnet permeance variations. On the basis of these considerations the cost function of the minimization algorithm has been defined.

III. FIELD ANALYSIS MODEL

The flux density distribution generated by the magnets has a rectangular shape, modulated by a series of harmonic component due to the presence of slots in the stator. As a first step, we consider the flux density distribution of the magnets having a rectangular shape. The flux density distribution B along the rotor surface for one rotor segment is shown in Fig. 1. Here six permanent magnets of width l_m are placed at six different positions y_i along the rotor surface. The total length of the rotor is $2p\tau$, where p is the number of pole pairs. When a step skew is used, the field can be considered as the superposition of the value of B in each rotor segment. Defining with Δ the length corresponding to the skew angle and considering a single step rotation, the total value of B in the position y is given by

$$B_{tot}(y) = B(y) + B(y+\Delta) \tag{5}$$

where $B(y)$ is the flux density distribution due to the first rotor segment and $B(y+\Delta)$ is the flux density distribution due to the second rotor segment.

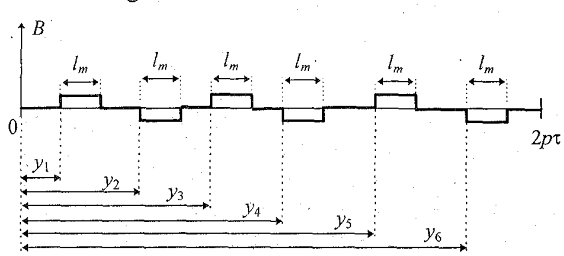

Fig. 1. Flux density distribution along the rotor surface for one rotor segment.

Under these assumptions and for a constant rotor speed, the Fourier series expression of the e.m.f. induced in a stator winding is

$$e_i(t) = -\frac{2qnlp\tau\omega}{\pi} \sum_{k=1,\mathrm{odd}}^{\infty} (-1)^{\frac{k-1}{2}} K_{dk} K_{pk} \cdot \left[a_{e,k}\cos k\omega t + b_{e,k}\sin k\omega t\right] \tag{6}$$

where q is the number of slots per pole and per phase, n is the number of wires per slot, l is the axial length of the machine and ω is given by ω_m/p. K_{dk} is the k-th harmonic distribution factor, K_{pk} the k-th harmonic pitch factor. The amplitudes of the harmonic components $a_{e,k}$ and $b_{e,k}$ are functions of q, τ, K_{dk}, K_{pk}, the slot pitch τ_c and the skew length Δ.

In determining (6) the harmonics of the flux density distribution of the magnets generated by the stator slots have been neglected. It has been verified that this assumption can be usually made owing to the low amplitude of the corresponding e.m.f. harmonic components.

The effects of the stator slots have been considered in modeling the magnet permeance and the cogging torque. For this purpose the following relationship is used to represent the magnetic circuit permeance $\mathcal{P}_{m1}$ of a rotor segment

$$\mathcal{P}_{m1}(\vartheta) = \mu_0 l_a \sum_{i=1}^{6} \int_{y_i}^{y_i+l_m} \frac{dy}{\delta(y+R\vartheta)} \tag{7}$$

where l_a is the axial length of the magnets, δ is the non-uniform iron-to-iron air gap and R is given by $p\tau/\pi$. When the inverse of $\delta(y+R\vartheta)$ is expressed by the Fourier series, the cogging torque produced in the machine becomes

$$T_c(\vartheta) = T_0 \sum_{k=1}^{\infty} \left[a_{T,k}\cos 2k\pi \frac{R\vartheta}{\tau_c} + b_{T,k}\sin 2k\pi \frac{R\vartheta}{\tau_c} \right] \tag{8}$$

In (8) the constant T_0 is expressed in torque unit and is given by

$$T_0 = -4R\mathcal{F}_m^2 \frac{\mu_0 l_a}{\delta_0} \tag{9}$$

where δ_0 is the air gap between the rotor core and the stator teeth. In (8) k=1 represents the fundamental harmonic component of the cogging torque having a number of oscillations equal to the number of stator slots.

IV. OPTIMIZATION PROCEDURE

The determination of the main design parameters of the machine is obtained as the solution of an optimisation problem of the type

$$\min \quad (f) \qquad \text{subject to} \begin{cases} q_1 = 0 \\ q_2 < 0 \end{cases} \tag{10}$$

where the cost function f is given by

$$f = \sum_{k=3,\mathrm{odd}}^{\infty} c_{E,k} \frac{E_k^2}{E_1^2} - c_{E,1}\frac{E_1^2}{E_{1,Max}^2} + \sum_{k=1}^{\infty} c_{T,k}\frac{T_{c,k}^2}{T_0^2} \tag{11}$$

and q_1, q_2 are constraints defined in the following.

E_k^2 is the squared amplitude of the k-th e.m.f. harmonic component, E_1^2 and $E_{1,Max}^2$ are respectively the squared amplitude of the first harmonic component and its maximum value. $T_{c,k}^2$ is the squared amplitude of the k-th cogging torque harmonic component. In the cost function each term is weighted by the penalty coefficients $c_{E,k}$, $c_{E,1}$, $c_{T,k}$. The first term on the right hand side of (11) causes the minimization of e.m.f. harmonic content. The second term is for the maximization of the e.m.f. fundamental component. The third term is for the minimization of the cogging torque.

The constraint $q_1 = 0$ is taken to guarantee the geometrical consistency of the rotor. In the constraints $q_2 < 0$, an upper limit to the length of the magnets and an upper limit to their eccentricity ε (distance of the centre of mass from the rotor axis), are taken. The number of poles, the number of phases, the number of slots per pole and per phase and the number of wires per slot are given. The unknowns of the optimisation are the length of the magnets and their positions, the skew length Δ and the number of tooth intervals r by which the pitch of the primary winding is shortened.

The minimization procedure is based on the (1+1) ES algorithm combined with the FF acceleration technique [6]-[8]. This method is stochastic and it does not utilise the gradient of the cost function. Moreover, the sensibility to numerical errors is lower than that of deterministic minimisation algorithms. Therefore, this method is indicated for the minimisation of functions with singularities and stiffnesses. The evolution strategy method has also an high probability to converge to the global minimum.

The (1+1) ES technique is used to find local minima of problem (10). Starting from a local minimum of f, the FF method is used to find a point in the basin of attraction of a lower minimum of f. Inside this region the ES technique is adopted to find the lower minimum. Successive utilisations of the ES method and of the FF method allows one to reach the lowest minimum. Convergence to the global minimum is proved [9].

V. Numerical Results

The minimization technique has been applied to determine the main design parameters of a 3-phase star-connected, 6-pole, PM synchronous machine. It is assumed q=2 and a slot opening equal to $0.3\tau_c$.

In order to validate the optimization procedure, a first example has been carried out considering the second term only in (11). This leads to the maximization of the fundamental harmonic component of the e.m.f.. As it could be expected the optimization process gives the values of r and Δ equal to zero. Furthermore, the length of the permanent magnets results 0.999τ, which is the maximum allowed by the geometrical constraints. The amplitudes of the lower order e.m.f. harmonic components, in p.u. of the fundamental component, are given in Table I. As it can be seen, the amplitudes of the harmonic components of order 5, 7, 11 and 13 are not negligible. It should be noted that the harmonic component of order $3h$ (h=1,2,...) are not relevant in star-connected 3-phase machines.

In Table I a performance index related to the cogging torque is given. It is defined as

$$K_{T_c} = \sqrt{\sum_{k=1}^{\infty} \frac{T_{c,k}^2}{T_0^2}} \qquad (12)$$

According to the assumptions made on the flux density distribution, the value of this index is very small as the ratio between the magnet length and the slot pitch is an integer number.

In Table I the eccentricity ε of the permanent magnets, in p.u. of the rotor radius, is also given. In this case the symmetry of the magnet geometry leads to a zero value of the eccentricity.

TABLE I

$E_1 / E_{1,Max}$	0.999
Amplitude of the 3rd e.m.f. harmonic	243×10^{-3}
Amplitude of the 5th e.m.f. harmonic	53.8×10^{-3}
Amplitude of the 7th e.m.f. harmonic	37.4×10^{-3}
Amplitude of the 9th e.m.f. harmonic	81.2×10^{-3}
Amplitude of the 11th e.m.f. harmonic	90.5×10^{-3}
Amplitude of the 13th e.m.f. harmonic	76.8×10^{-3}
Amplitude of the 15th e.m.f. harmonic	47.9×10^{-3}
Amplitude of the 17th e.m.f. harmonic	15.5×10^{-3}
K_{Tc}	2.9×10^{-5}
ε	0.000

In a second validation example the design optimisation has been carried out in order to maximise the e.m.f. fundamental component and to minimise the cogging torque. In this case the optimisation procedure leads to a symmetric magnet arrangement with r=0, Δ=0. The magnet length equals $5/6\tau$, which corresponds exactly to $5\tau_c$. The fundamental harmonic component of the e.m.f. is reduced to $0.966E_{1,Max}$ and the harmonic content is slightly lower than that of Table I.

The solutions obtained are in agreement with the basic principles of PM synchronous machine design. The optimisation procedure is now applied in order to obtain a machine design with some particular features.

A) Case 1

In this case a machine design with minimum e.m.f. harmonic content and maximum fundamental component is required. The results obtained are shown in Table II.

TABLE II
MAIN DESIGN PARAMETERS

	Case 1	Case 2
r	0.0	0.0
l_m(*)	0.81568	0.83332
Δ (*)	0.07541	0.08445
y_1 (*)	0.05135	0.00000
y_2 (*)	1.21184	1.08482
y_3 (*)	2.11284	2.17141
y_4 (*)	3.09151	3.00473
y_5 (*)	4.15314	4.08358
y_6 (*)	4.99249	5.16667

* referred to the pole pitch

In order to visualise the arrangement with six magnets, a schematic plot of one rotor segment is shown in Fig. 2. As it is shown in the figure, the magnets are shifted with respect to their symmetric position. Furthermore the magnet length is not an integer multiple of the slot pitch.

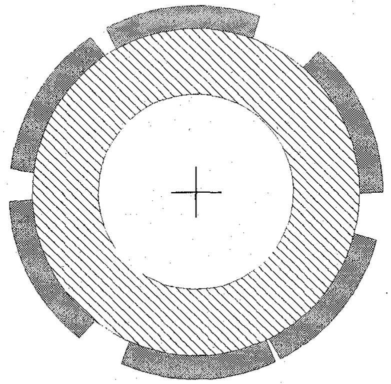

Fig. 2. Plot of the rotor geometry for Case 1.

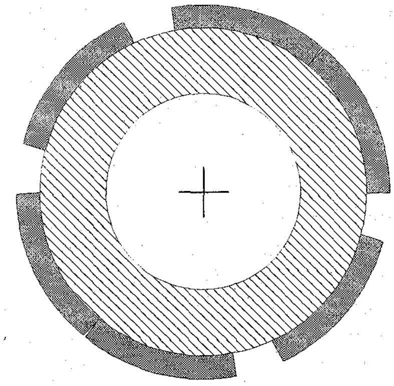

Fig. 3. Plot of the rotor geometry for Case 2.

The stator winding is of full pitch type and the skew Δ corresponds to about one half of the slot pitch.

In Table III the harmonic components of the e.m.f are given in p.u. of the fundamental component. By comparing Table III with Table I it is possible to note that in spite of a small decrease of the e.m.f. fundamental component a large reduction of the e.m.f. harmonic content is obtained. The performance index related to the cogging torque and the eccentricity are both increased.

TABLE III
MAIN RESULTS

	Case 1	Case 2
$E_1 / E_{1,Max}$	0.928	0.935
Amplitude of the 3rd e.m.f. harmonic	126×10^{-3}	134×10^{-3}
Amplitude of the 5th e.m.f. harmonic	3.00×10^{-3}	5.90×10^{-3}
Amplitude of the 7th e.m.f. harmonic	2.70×10^{-3}	1.02×10^{-3}
Amplitude of the 9th e.m.f. harmonic	1.05×10^{-3}	3.11×10^{-3}
Amplitude of the 11th e.m.f. harmonic	0.44×10^{-3}	3.23×10^{-3}
Amplitude of the 13th e.m.f. harmonic	0.13×10^{-3}	3.74×10^{-3}
Amplitude of the 15th e.m.f. harmonic	0.75×10^{-3}	2.07×10^{-3}
Amplitude of the 17th e.m.f. harmonic	0.42×10^{-3}	0.14×10^{-3}
K_{Tc}	0.20	6.8×10^{-4}
ε	2.9×10^{-2}	1.4×10^{-3}

B) Case 2

In this case the whole cost function has been minimised in order to obtain a machine design with low e.m.f. harmonic content, high fundamental component, low cogging torque, under the constraint of an upper limit for the magnet eccentricity. The results obtained are shown in Table II. In order to visualise the arrangement of the magnets a schematic plot of one rotor segment is shown in Fig. 3.

The geometry of the rotor has been modified so that the eccentricity of the magnets is strongly reduced. The magnet length is very close to 5/6 of the pole pitch. The harmonic content of the e.m.f. is given in Table III. As it can be noted the fundamental component is almost the same as in Case 1, while the harmonic content shows an increase. The cogging torque index is reduced with respect to Case 1.

VI. CONCLUSIONS

In this paper it has been verified the possibility to reduce the torque ripple in PM synchronous motors using a global stochastic ES algorithm coupled to a FF acceleration technique. The cost function has been defined taking all the contributions to the torque ripple into account.

The minimisation technique has been firstly validated, then it has been employed to obtain the main design parameters of PM synchronous motors having some particular features.

Although some simplifying assumptions about the flux density distribution have been made, the results obtained by the minimisation technique are very interesting showing particular solutions for the arrangement of the permanent magnets. The accuracy of the design could be improved by a more detailed representation of the magnetic field density distribution.

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