Differential evolution based tuning of fuzzy automatic train operation for mass rapid transit system

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Abstract: Train performance of mass rapid transit systems can be improved with the use of fuzzy controllers in automatic train operation (ATO) systems. The tuning of these fuzzy controllers is presented using the algorithm of differential evolution (DE). The basic DE algorithm is modified to optimise a multiobjective function comprising punctuality, riding comfort and energy usage. Using this algorithm, the fuzzy ATO controller is tuned for each interstation train run. In operation, the controller adjusts each train's control according to the current operating conditions. A fuzzy ATO controller model was previously developed by the authors and is used to demonstrate the effectiveness of the tuning scheme.

List of major symbols

NP = population
F = amplification probability
CR = crossover probability
D = dimension of population in the algorithm of differential evolution
MG = maximum permitted generation number
P = punctuality
E = energy usage
j = traction fluctuation
TS = safety performance indices
P = punctuality performance indices
CT = riding comfort performance indices
AC = energy saving performance indices

1 Introduction

Automated transit systems are designed to provide the safest and most cost-effective railway service. Within these systems, automatic train operation (ATO) plays a very important role. Under normal situations, the ATO controller schedules train movement from departure to the next scheduled station-stop with interstation parameters collected at each station. At the same time, the conventional ATO controller also manoeuvres each train using data received from the automatic train supervision system (ATS), which monitors and co-ordinates the movement of all trains operating at any time. Usually the ATO controller is not equipped with on-schedule controller. This means that the ATS does not have the capability to make self-adjustment if there are deviations from the normal schedule. For example, during peak hours the dwell time will be longer because of increased passenger flows. Therefore, the conventional ATO controller alone might not guarantee punctual schedule and ATS might require human intervention. Trains might have to be under manual control.

In view of the above, many studies were carried out to find better control schemes [1–4]. Fuzzy ATO control is one of the more successful methods. Yasunobu and Miyamoto [1] made a noteworthy contribution. They designed a fuzzy ATO controller, which can automatically change the train operation status to offset these unforeseen deviations caused by various factors. It actually controls each train's departure, speed regulation, and station-stop time at target points and at each station. It was implemented in the city of Sendai in Japan in 1987. Chang and Sim [2] also applied fuzzy logic to the ATO controller to provide multiobjective control for satisfying various railway operational requirements. Both these two fuzzy ATO controllers can perform as skilfully as human experts do, and are superior to a conventional PID automatic train operation controller in terms of stopping precision, energy usage, riding comfort and running time as described in [1, 2] separately.

Membership functions play an important role in ensuring the control precision and robustness of the fuzzy ATO controller. The initial design of membership functions can be accomplished heuristically. Railway control systems are, however, very complicated and usually affected by many factors such as interstation distance, gradient profiles (tunnel, upward slope, and downhill slope) and different schedules. These have made it quite difficult to tune the fuzzy membership functions manually.

To solve this problem, a new method is developed to optimally tune the fuzzy membership functions as in [2]. A modified differential evolution (DE) algorithm is adopted to accomplish this task. The DE algorithm is conceptually simple and easy to use, and has good convergence [5, 6]. These features have made it an ideal algorithm for optimising functions with continuous variables. In addition, with the use of float-point strings, the dynamic range of the DE's search space is greatly increased as compared with the genetic algorithms. Higher resolution and wider range have been achieved. To obtain faster convergence and better results, the basic algorithm is modified to make it more robust and suitable for tuning the fuzzy ATO controller.

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2 Fuzzy ATO controller and tuning requirements

This Section reviews the fuzzy ATO controller [2], which prescribes braking, coasting, and powering of the train based on evaluation of safety, riding comfort, punctuality and energy usage. It decides, for each moving train at each time instant, the need and the effort of powering F or braking B, or whether coasting should be initiated to optimise the overall performance.

Fig. 1 shows the layout of the fuzzy ATO controller as proposed in Fig. 2. There are five basic inputs needed: train kinetics, distance of the designated station from the train, ATP codes, scheduled time and track gradient profile. Fuzzy sets and performance indices are based on multiple objectives as predefined in the form of rules to ensure correct decisions. For each choice of command status (motoring, coasting, brake-to-target-speed, brake-to-stop), a group of rules are fired for determining the strength for the selection of the status. The command that has the largest weighting after defuzzification will be chosen as the next train command status to be taken.

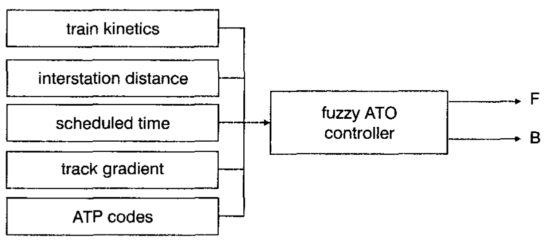

Fig. 1 *Layout of fuzzy ATO controller*

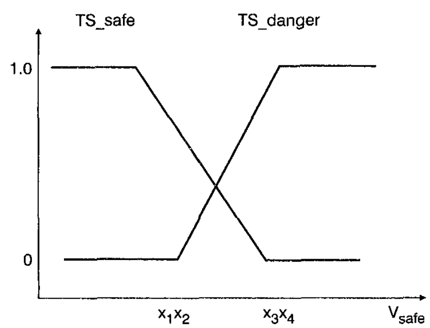

Fig. 2 *Membership function of safety performance indices*

The major fuzzy performance indices are classified as follows:

(i) safety performance indices: TS (TS_safe and TS_danger).

(ii) punctuality performance indices: P (P_early and P_late).

(iii) energy saving performance indices: AC (AC_short and ES_long).

(iv) riding comfort performance indices: CT (CT_short and RC_long).

2.1 Assumption

The following assumptions are made in the design of the proposed ATO controller:

(i) Factors which affect the kinematics of the train such as the mass, acceleration, velocity and distance, can be estimated or measured with onboard sensors.

(ii) The train voltage profile is read from a database, which is established through previous electrical network analysis [3]. The train voltage profile can be used for adjusting each train's acceleration and braking rates during optimisation.

(iii) Fixed block signalling schemes are adopted.

2.2 Fuzzy ATO controller design based on fixed block signalling scheme

The fixed block-signalling scheme has fixed-length ATP blocks connected along the entire track. When the train enters an ATP block, the ATP code is transmitted to the train, which decodes for the desired target speed and the maximum safe speed. Subsequent ATP blocks are set to prevent the trailing trains from encroaching upon the headway or from exceeding the civil speed limit. The following requirements are considered for designing the fuzzy ATO controller [2]:

(i) train and passenger safety

(ii) punctuality in meeting schedule

(iii) energy conservation

(iv) passenger comfort

The following fuzzy membership functions provide a means of measuring and maintaining performance of the fuzzy ATO controller. All the constants used here are chosen, based on extensive sensitivity simulations of fuzzy memberships.

2.3 Defining fuzzy sets for ATO controller

The four main fuzzy sets are defined as follows:

2.3.1 Safety performance indices, TS: One method for ensuring train safety is to run the train below the target speed as specified by the automatic train protection device (ATP). The variable for measuring safety is defined as follows:

$$v_{safe} = v - TargetSpeed \tag{1}$$

where v is the current velocity of the train. *TargetSpeed* is the desired target speed specified by the ATP. It forms a well-defined boundary to the 'space' within which the ATO controller is free to operate. It accounts for the ATO and ATP system tolerances. In this implementation, the *TargetSpeed* without civil speed restriction is chosen to be 80km/h. The fuzzy performance indices are defined as follows:

$$Safe\ (TS\_S)$$

$$\mu_{TS\_S}(v_{safe}) = \left\{ \begin{array}{cc} 1 & v_{safe}<-2KPH \\ \mathrm{int}\,r(-2KPM,1,0,0,v_{safe}) & -2KPH<v_{safe}<0 \\ 0 & v_{safe}>0 \end{array} \right\}$$

$$Danger\ (TS\_D)$$

$$\mu_{TS\_D}(v_{safe}) = \left\{ \begin{array}{cc} 0 & v_{safe}<-2KPH \\ \mathrm{int}\,r(-2KPM,0,0,1,v_{safe}) & -2KPH<v_{safe}<0 \\ 1 & v_{safe}>0 \end{array} \right\} \tag{2}$$

where KPH is the multiplication constant which converts km/h to m/s. As in Fig. 2, $Intr(x_1, y_1, x_3, y_3, x)$ is the interpolating function that interpolates the value of μ for x between (x_1, y_1) and (x_3, y_3) as follows:

$$\mathrm{int}\,r(x_1, y_1, x_3, y_3, x) = \frac{x - x_1}{x_3 - x_1}(y_3 - y_1) + y_1 \tag{3}$$

which means that, when the train runs under 78km/h (normal case), it is free to operate without any limitation. When its speed exceeds the 78km/h, the rule will try to pull back the train.

2.3.2 Punctuality performance indices, P: One of the most important requirements in the controller design is meeting the arrival time of the train. The variable for measuring punctuality is defined as follows:

$$t_{est} = t_1 + t_2 - available\ time\ to\ schedule \qquad (4)$$

where t_1 is the estimated coasting time according to current velocity and t_2 is the estimated braking time according to current velocity. $t_1 + t_2$ is the estimated time needed before reaching the destination, and 'available time to schedule' represents the time left on the schedule. Their difference t_{est} is the indication of train punctuality. The fuzzy performance indices are defined as follows:

$$early\ (P\_E)$$

$$\mu_{P\_E}(t_{est}) = \left\{ \begin{array}{cc} 1 & t_{est}<-20 \\ \mathrm{int}\,r(-20,1,10,0,t_{est}) & -20<t_{est}<10 \\ 0 & t_{est}>10 \end{array} \right\}$$

$$late\ (P\_L)$$

$$\mu_{P\_L}(t_{est}) = \left\{ \begin{array}{cc} 0 & t_{est}<-20 \\ \mathrm{int}\,r(-20,0,10,1,t_{est}) & -20<t_{est}<10 \\ 1 & t_{est}>10 \end{array} \right\} \qquad (5)$$

2.3.3 Energy saving performance indices, AC: One method for conserving energy is to encourage coasting. To achieve this goal, an estimate is made on the maximum time t_1 allowed for coasting before the train must apply braking to stop at the destination. Defining the energy saving performance index AC, which checks on the available coasting time as follows:

$$short\ (AC\_S)$$

$$\mu_{AC\_S}(t_1) = \left\{ \begin{array}{cc} 1 & t_1 < 2 \\ \mathrm{int}\,r(2,1,6,0,t_1) & 2 < t_1 < 6 \\ 0 & t_1 > 6 \end{array} \right\}$$

$$long\ (AC\_L)$$

$$\mu_{AC\_L}(t_1) = \left\{ \begin{array}{cc} 0 & t_1 < 2 \\ \mathrm{int}\,r(2,0,6,1,t_1) & 2 < t_1 < 6 \\ 1 & t_1 > 6 \end{array} \right\} \qquad (6)$$

2.3.4 Passenger comfort performance indices, CT: Passenger comfort may be improved by reducing the number of transitions of train command status during the journey. The status changes frequently when a train is trailing closely behind another train during peak periods. This paper adopts one of the most obvious methods, which tries to minimise the transitions by forcing the train to coast for at least a time period before allowing it to power up.

The performance indices for passenger comfort can therefore be based on the actual length of coasting time t_c. The performance indices are defined as follows:

$$short\ (CT\_S)$$

$$\mu_{CT\_S}(tc) = \left\{ \begin{array}{cc} 1 & tc < 20 \\ \mathrm{int}\,r(20,1,30,0,tc) & 20 < tc < 30 \\ 0 & tc > 30 \end{array} \right\}$$

$$long\ (CT\_L)$$

$$\mu_{CT\_L}(tc) = \left\{ \begin{array}{cc} 0 & tc < 20 \\ \mathrm{int}\,r(20,0,30,1,tc) & 20 < tc < 30 \\ 1 & tc > 30 \end{array} \right\} \qquad (7)$$

3 Proposed DE based tuning of fuzzy ATO controller

The DE algorithm is a promising candidate for minimising real-valued and multimodal objective functions. It is conceptually simple and easy to use and has good convergence. By using float-point string, the dynamic range of the DE's search space is greatly increased as compared with the genetic algorithms. Higher resolution and wider range have been achieved. In Appendix 7.1, the basic concepts of the DE algorithm and the most widely used variants are reviewed.

As mentioned in Section 2.3, there are four main sets of fuzzy membership functions in the fuzzy ATO controller, which are used for choosing the next status command from braking, powering, and coasting. Every set of membership functions is determined by four parameters (x_1, x_2, x_3, x_4) as shown in the example of Fig. 2, for the safety terformance indices TS (TS_safe and TS_danger). Three similar sets of membership functions are used for representing the three other performance indices. Altogether, 16 fuzzy parameters are contained in a vector $X_{i,G}$, $i = 0, 1, 2, ..., 15$.

Different fuzzy parameters will generate completely different train performance. These parameters can initially be formulated by human experience. Subsequent parametric tuning will however be an exhausting and time-consuming task because the current operating condition is unknown and related information is diverse. Therefore, a modified DE algorithm is proposed for optimising $X_{i,G}$.

3.1 Algorithmic details

To obtain faster convergence and better results, the standard algorithm is modified to make it more robust and suitable for tuning the fuzzy ATO controller. Simulation results show that the train performance in terms of punctuality, energy saving and riding comfort has been greatly improved. For detail information of the algorithm, please refer to Appendix 8.1.

3.2 Formulation of objective function

The aim of the proposed tuning algorithm is to achieve good train performance with due regard to energy usage, passenger riding comfort and punctuality. The multiobjective function can be represented by the following performance index (cost):

$$Z(P,E,J) = \min\left(k_1\cdot(T-T_0)^2 + k_2\cdot\left(\sum_{t=0}^{T} E(t)\right)^2 + k_3\cdot\left(\sum_{t=0}^{T} J(t)\right)^2\right)$$

$$\sum_{i=0}^{t} E(t) = V * \int_t i(t)$$

$$J(t) = M\int_t |a(t+1) - a(t)| \qquad (8)$$

where V is the train voltage, which is retrieved from previous electrical network analysis; and i is the train current, which is calculated from the current train control status. The value of E is positive if the train is powering, and negative if train is braking. $J(t)$ represents the fluctuation of train control status, M is the train weight, which will change slightly according to the number of passengers, a is the train acceleration, T and T_0, respectively denote the actual arrival time and scheduled arrive time. k_1, k_2 and k_3 are positive constants used for compromising among the three objectives. They are adjusted to give different priority on energy usage, punctuality and passenger comfort. Usually, these three objectives are in conflict with each other and need to be collectively optimised according to predefined priority. For example, the priority on achieving energy savings should be set lower when there is requirement to minimise journey time during peak periods. k_1 should then be increased to improve the priority of punctuality accordingly.

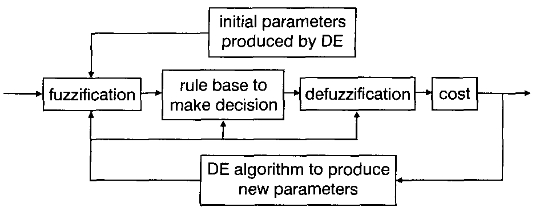

Fig. 3 *Scheme for fuzzy logic controller based on DE*

3.3 Steps of tuning process

The structure of the proposed the DE algorithm is shown in Fig. 3, whose main steps are as follows:

Step 1: In the first generation, generate the set of parameters $X_{i,G}$ randomly for evaluating the fuzzy ATO controller membership functions. Provide also the initial values of F and CR.

Step 2: Using the current parameters $X_{i,G}$, simulate the train performance until it reaches the destination and calculate the performance index as defined in eqn. 8.

Step 3: Repeat step 2 for the entire NP population.

Step 4: Generate the new parameters $X_{i,G}$ according to eqns. 10–13. Modify CR and F according to eqns. 14 and 15. Then revisit steps 2 and 3.

Step 5: Repeat step 4 until satisfied parameters $X_{i,G}$ are obtained or the predetermined maximum generation is reached.

4 Simulation results and discussion

Parameters in this modified differential evolution are chosen as follows: population size $NP = 100$, maximum permitted generations number $MG = 70$, initial crossover probability $CR^{(0)} = 1$, initial amplification probability $F^{(0)} = 0.3$

A track in a typical MRT line between the two adjacent stations *MRB* and *RFP* is chosen for an interstation optimisation to demonstrate the validity of the proposed method. The total length is 923m and the maximum safety speed is 80km/h. There is a civil speed limitation of 60km/h from 370m to 490m. The gradient profile of simulated railway section is given in Fig. 10. Three cases with different time schedules are studied in the simulation. The scheduled time in Figs. 4 and 5, 6 and 7, 8 and 9 are 100s, 90s and 0s, respectively. It is noted in Figs. 4–9 that stages 1, 2, 3 and 4 represent the running modes of motoring, coasting, braking to target speed and braking to stop, respectively.

From Table 1 and Figs. 4–9, the train performance has been much improved after tuning with the DE-based tuning method. In the normal and off-peak schedules, the proposed method not only ensures that the train reaches its destination on time but also improves the energy and reduces jerk factors. It is shown that there is a rapid upward change of gradient near RFP station in the

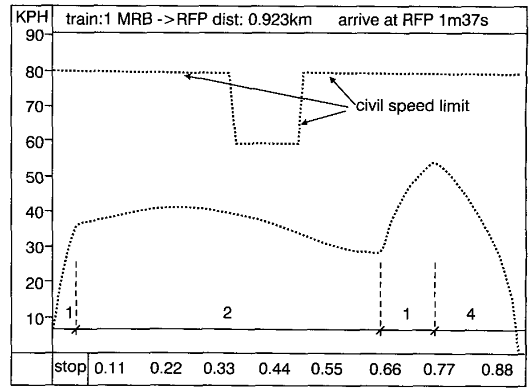

Fig. 4 *Velocity profile of fuzzy ATO controller under loose schedule*

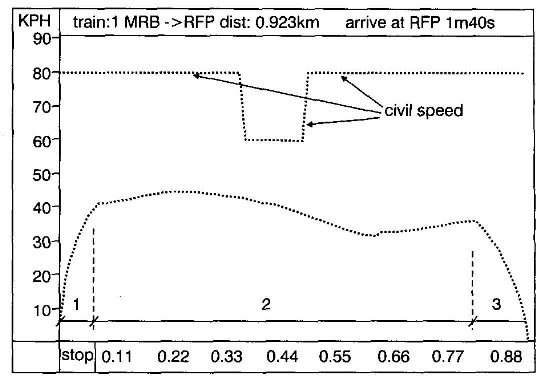

Fig. 5 *Velocity profile of DE-based fuzzy ATO under loose schedule*

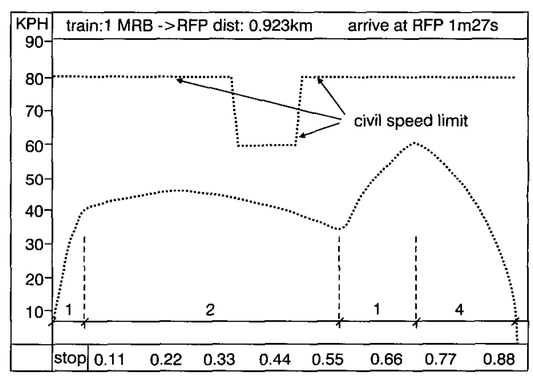

Fig. 6 *Velocity profile of fuzzy ATO controller under normal schedule*

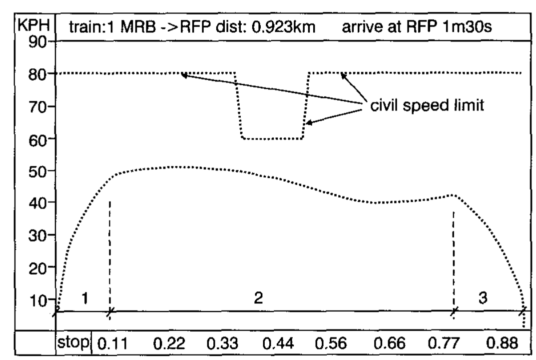

Fig. 7 *Velocity profile of DE-based fuzzy ATO under normal schedule*

Table 1: Comparison of train performance under different time schedules

	Schedule	Arrive time	Jerk factor	Energy factor	Cost
fuzzy ATO control	100	97	7.77×10^4	2.78×10^4	9.50
DE-basedcontrol	100	100	6.56×10^4	2.61×10^4	6.58
fuzzy ATO control	90	87	8.30×10^4	2.36×10^4	9.46
DE-basedcontrol	90	90	7.45×10^4	2.33×10^4	6.86
fuzzy ATO control	0	76	9.15×10^4	1.78×10^4	51.9
DE-basedcontrol	0	72	1.22×10^4	0.69×10^4	30.8

gradient profile, which leads to unreasonable acceleration just before braking (Figs. 4 and 6). In Figs. 5 and 7, it is shown, after tuning, that unreasonable acceleration is avoided by acquiring higher initial velocity after the first acceleration. In the tight schedule, the train with the DE-based control is accelerated immediately after leaving a civil speed limit. Therefore, the train runs faster after tuning. The average speed between the two stations in Fig. 8 is 43.7 (= 923*3.6/76) km/h, while the average speed in Fig. 9 is 46.15 (= 923*3.6/72) km/h. The time taken has been shortened by 4s after tuning. Taking into account the short distance between these two stations, the time saving is significant. The energy has also been reduced by more than 50% after tuning. This is mainly due to the triggering off of regenerative braking by the DE based fuzzy ATO controller. Table 2 gives an analysis of energy component. Assume all the regenerated energy is fully absorbed by other train, the total cost has been reduced to only two-thirds after tuning. The jerk factor is, however, increased, which is caused mainly by motoring after civil speed limit. This is inevitable because the most important task for train under tight schedule is to catch up with the scheduled time. In addition, the maximum acceleration rate is set to be $1.1 m/s^2$, thus most passengers would not feel any discomfort during motoring.

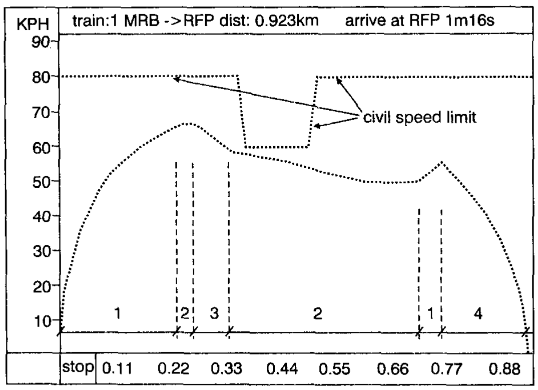

Fig. 8 *Velocity profile of fuzzy controller under tight schedule*

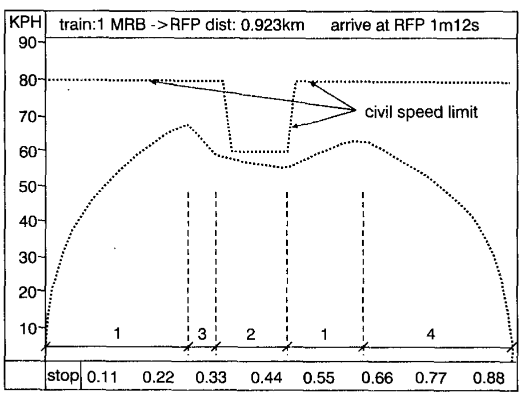

Fig. 9 *Velocity profile of DE-based fuzzy ATO under tight schedule*

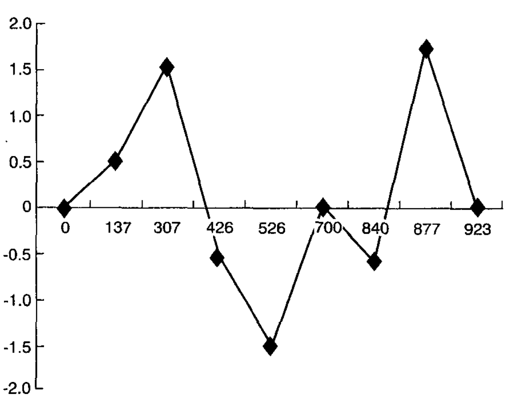

Fig. 10 *Gradient profile of simulated railway section*

Table 2: Energy component

	Schedule time	Total energy consumed	Positive energy	Negative energy
DE-based	100	26103	42017	−15914
Fuzzy ATO based	100	27817	52817	−25000
DE-based	90	23388	43169	−19780
Fuzzy ATO based	90	23618	54033	−23618
DE-based	0	6960	57061	−50101
Fuzzy ATO based	0	17840	64035	−46187

Figs. 11 and 12 illustrate the convergence of the modified DE algorithm. Fig. 11 shows the performance index of the best parameters in each generation and the dashed lines in the Figure represent the performance index of the fuzzy ATO controller before tuning. It can be observed that, after less than 30 generations, the generated parameters are already very close to the optimal solution. This confirms the excellent convergence property of this proposed method. Fig. 12 shows the average performance index in each generation.

In this paper, the ATO control is optimised for each interstation run between two stations. This algorithm can easily be expanded for a practical implementation on multi-station control. Fuzzy membership functions are optimised and stored for each interstation ATO control, which are then retrieved for controlling each interstation run. A platform of master–slave fuzzy controllers [7] can be used for firing the relevant interstation ATO controllers according to different train running conditions.

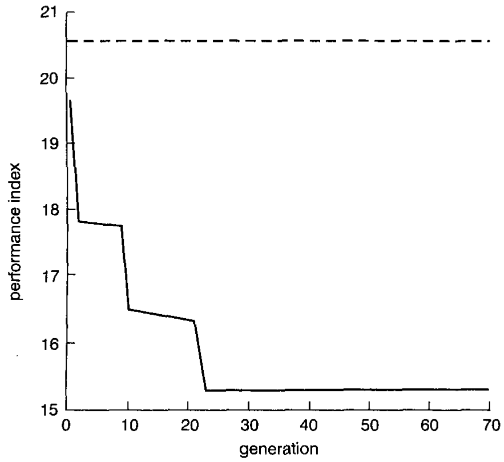

Fig. 11 *Convergence of DE-based control method*
Convergence of best performance index in each generation

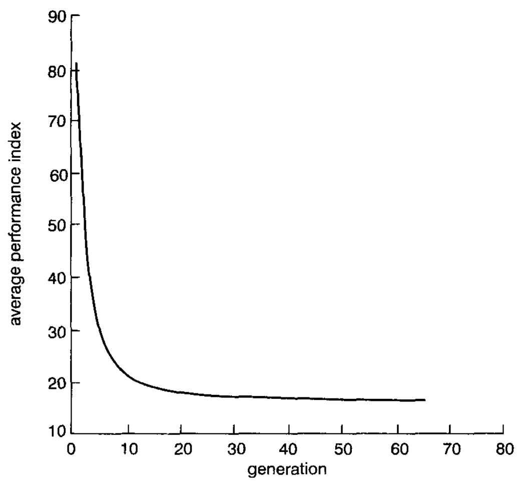

Fig. 12 *Convergence of DE-based control method*
Convergence of average performance index in each generation

5 Conclusion

A DE-based fuzzy ATO controller is proposed in this paper, which automatically fine tunes the fuzzy membership functions used in the controller. The tuning minimises multiobjective performance indices, takes into account of different factors like interstation distance, rapidly changing gradient profiles, and schedules. Through simulations, the proposed tuning method is shown to improve greatly the performance of the fuzzy ATO controller. This method is simple to implement, and, more importantly, very effective. It can also be applied to other configurations of fuzzy controllers with little algorithmic changes. The proposed method is applicable to both the timetable-based and headway-based mass transit systems by simply assigning different priorities to different objectives.

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7 Appendix

7.1 DE basic concepts and variants

The DE algorithm has been a promising candidate for minimising real-valued and multimodal objective functions [5, 6]. It is a parallel direct-search method, which utilises NP D-dimensional parameter vectors:

$$\underline{X}_{i,G} \qquad i = 0, 1, 2, \ldots, NP - 1 \tag{9}$$

as the population for each generation G (i.e. for each iteration of the minimisation). NP does not change during the minimisation process. The initial population is chosen randomly and should be able to cover the entire parameter space uniformly. Basically, the DE algorithm generates new parameter vectors by adding weighted difference between two population vectors to a third vector. If the resulting vector yields a lower objective function value than a predetermined population member does, the newly generated vector will replace the vector which has been compared in the previous generation; otherwise, the old vector is retained. This basic principle is, however, extended being applying to the practical variants of the DE algorithm. For example, adding more than one weighted difference vector to it can perturb an existing vector. In most cases, it is also worthwhile mixing the parameters of the old vector with those of the perturbed one before comparing the objective function values. The following variant of the DE algorithm [3, 4] has been proved the most useful in the present application.

For each vector $X_{i,G}$, $i = 0, 1, 2, \ldots, NP - 1$ a perturbed vector $V_{i,G+1}$ is generated according to the following:

$$\underline{V}_{i,G+1} = \underline{X}_{r1,G} + F \cdot (\underline{X}_{r2,G} - \underline{X}_{r3,G}) \tag{10}$$

where r_1, r_2, $r_3 \in [0, NP - 1]$ are mutually different integers and $F > 0$.

The randomly chosen integers r_1, r_2 and r_3 also have to be different from the running index i. F is a real and constant factor $\in [0, 2]$ which amplifies the differential variation $(X_{r2,G} - X_{r3,G})$. This scheme requires that the vector to be perturbed is randomly chosen and that the perturbation consists of one weighted difference vector only.

To increase the potential diversity of the perturbed parameter vectors, a crossover probability CR is introduced to calculate the new vector,

$$\underline{U}_{i,G+1} = \left(u_{i0,G+1}, u_{i1,G+1}, \ldots, u_{i(D-1),G+1}\right) \tag{11}$$

$U_{i,G+1}$ is a D-dimensional parameter vector in the $(G + 1)$th generation and $u_{ij,G+1}$ is its elements of the following form:

$$u_{ij,G+1} = \left\{ \begin{array}{r} v_{ij,G+1} \text{ for } j = \langle n \rangle_D, \langle n+1 \rangle_D, \ldots, \\ \langle n+L-1 \rangle_D \\ x_{ij,G} \text{ for all other } j \in [0, D-1] \end{array} \right\} \tag{12}$$

where the acute bracket $< >_D$ denotes a module function with modulus D.

The starting index n in eqn. 12 is a randomly chosen integer for the interval $[0, D-1]$. The integer L, which denotes the number of parameters to be exchanged, is drawn from the interval $[1, D]$. The crossover probability CR is taken from the interval [0, 1] and constitutes a control variable in the design process (see eqn. 13). The random decisions for both n and L are updated for each newly generated vector $U_{i,G+1}$.

To decide whether or not it should become a member of generation $G + 1$, the new vector $U_{i,G+1}$ is compared with $X_{i,G}$. If vector $U_{i,G+1}$ yields a smaller objective function value than $X_{i,G}$, $X_{i,G+1}$ is set to $U_{i,G+1}$; otherwise, the old value $X_{i,G}$ is retained.

It has been shown that the DE algorithm is a promising candidate for minimising real-valued, multimodal objective functions [5, 6]. Besides its good convergence properties, the DE algorithm is very simple to understand and implement. Its good performance has been manifested clearly in the first to fifth De Jong functions and other testbeds [6]. The next Section will show how to use a modified version of the DE algorithm to fine tune the fuzzy membership functions in the ATO controller.

8 Appendix

8.1 DE algorithmic details

For each vector $X_{i,G}$, $i = 0, 1, 2, \ldots, NP-1$, a perturbed vector $V_{i,G+1}$ can be generated by programming Eqns. (9)-(11) in C codes as follows:

$$\begin{aligned}
&L = 0; n = \mathrm{int}(rand() \cdot D)\\
&do\{L = L + 1;\\
&v_{i,n} = x_{r1,n} + F \cdot (x_{r2,n} - x_{r3,n})\\
&n = (n+1)\%D;\\
&\}while(rand() < CR\&\&L < D)
\end{aligned} \tag{13}$$

where *rand*() is supposed to generate a random number $\in [0, 1]$.

In the above typical DE algorithm, the most significant control variables are F and CR. Their values directly affect the system convergence and convergence speed. If $CR = 1$, all the parameters in the vector $V_{i,G+1}$, $i \in [0, D]$ will be replaced by a newly generated vector according to eqn. 10. If $CR < 1$, only some of the parameters will be replaced. F is an amplification factor, which controls the amplification of the differential variation $(Xr_{2,G} - X_{r3,G})$. These two parameters are similar to the crossover probability and mutation probability in a genetic algorithm. According to the computational mechanism of a typical genetic algorithm [8], the probability of crossover should be decreased and the probability of mutation should be increased during the solution process in order to increase the computational efficiency and the opportunity to find the optimal solution.

In this paper, the basic DE algorithm is modified by adjusting F and CR with a basic idea from refined genetic algorithm [9]. Initial probabilities for crossover and amplification are selected. For every generation thereafter, CR is linearly decreased while F is linearly increased. At the same time, the limits on these probabilities are set in advance so that they do not go beyond the permitted intervals. In the present application, the permitted interval for CR is between 0.7 and 1, and F is between 0.3 and 0.5. CR and F are changed from generation to generation according to the following equations:

$$CR^{(t)} = CR^{(t-1)} - (CR^{(0)} - 0.7)/MG \tag{14}$$

$$F^{(t)} = F^{(t-1)} + (0.5 - F^{(0)})/MG \tag{15}$$

where t denotes the generation number (i.e. the iteration number), $CR^{(0)}$ and $F^{(0)}$ denote the initial values of the crossover probability and the amplification probability, $CR^{(t)}$ and $F^{(t)}$ denote the crossover probability and the amplification probability at the tth generation. MG is the maximum permitted generation number.