Multiobjective Optimal Design of Interior Permanent Magnet Synchronous Motors Considering Improved Core Loss Formula

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Abstract: This paper describes the optimal design of Interior Permanent Magnet Synchronous Motors for which two objective functions regarding motor efficiency and weight are used. Multiobjective optimization technique is applied to finding the optimal solution in this case. An optimal design method that determines both the noninferior solution set and the best compromise solution employing a modified genetic algorithm is proposed. The proposed algorithm adopts the structure of the conventional genetic algorithm, but fitness value and convergence criterion are redefined and some major parameters of the algorithm are adjusted to the multiobjective optimization. In order to predict the motor performance more accurately, a core loss formula is derived considering the flux variation due to the stator currents as well as that due to the magnet.

Keywords: Multiobjective optimal design, Noninferior solution set, Interior Permanent Magnet Synchronous Motor, Genetic algorithm, Core loss

I . INTRODUCTION

There are many conflicting design objectives in the optimal design of electric machines, so multiobjective optimization technique is required to meet design purposes. The conventional approach to these kinds of the problem is the conversion of the problem to the single objective optimization problem using the appropriate method, such as weighting method or constraint method. Recent researches on the multiobjective optimization focus on the methods based on the fuzzy reasoning[1~3], but almost all of them can be regarded as the same class as the conventional one. Few papers deal with the multiobjective optimization strictly, except for the distinguishable research by G.H.Kim[4].

Multiobjective optimization problems, in general, have many solutions. The solution set of the problem is called as a noninferior solution set. Therefore, in order to apply this method to the optimal design of electric machines, auxiliary steps are necessary to find the best compromise solution.

The proposed optimal design algorithm consists of two parts of which each finds the noninferior solution set and the best compromise solution, respectively. In this paper, the algorithm is implemented by a modified genetic algorithm.

The core loss of the Permanent Magnet Motors is, in contrast to the Induction Motor, not a no-load loss because the air gap flux of the motor varies according to the stator current[5,6]. So, the core loss formula considering the flux variation due to the stator current is required to make the more accurate performance predictions possible. In this paper, a formula for the eddy current loss is derived considering the flux variation in the tooth and yoke due to the stator current as well as the magnet.

The proposed optimal design algorithm is applied to the design of the Interior Permanent Magnet Synchronous Motor for which two objective functions regarding motor efficiency and weight are used. And the dimensions, parameters and characteristics of the optimally designed motor are compared with those of the prototype.

II . OPTIMIZATION ALGORITHM

A. Formulation of the Problem

Multiobjective optimization problem with p objectives, n decision variables, and m constraints is formulated as

$$\text{Minimize} \quad f(x). \tag{1}$$

$$\text{s.t} \quad \begin{aligned} &x_i \geq 0, \quad i = 1, \cdots, n \\ &g_j(x) \leq 0, \quad j = 1, \cdots, m \end{aligned}$$

$$\text{where} \quad \begin{aligned} &x = (x_1, \cdots, x_n) \\ &f(x) = (f_1(x), \cdots, f_p(x)) \end{aligned}$$

The solutions must satisfy the noninferiority condition. Noninferiority can be defined in the following way[7];

A feasible solution is noninferior if there exists no other feasible solution that will yield an improvement in one objective without causing a degradation in at least one other objective.

In general, many solutions exist satisfying the noninferiority condition. So, it is necessary to find the best compromise solution among the noninferior solutions. The criterion for the best compromise solution differs according to the design purposes. If a certain objective function is important among others, the solution will exist in the neighborhood of its optimum. Or, if some objective functions have critical limits, the solution will be determined accord-

ing to the limits. But, providing all the objective functions have the equal importance, an adequate criterion for the best compromise solution can be given by the min-max optimum of the relative difference from the global optima of the objective functions

$$v(x^*) = \begin{matrix} \min & \max \\ x \in X_f & k \in K \end{matrix} \left\{ \frac{|f_k(x) - f_k^*|}{|f_k^*|} \right\}. \quad (2)$$

where X_f : feasible region in decision space

f_k^* : optimal solution of kth objective function

$K = \{1, \cdots, p\}$

B. The Modified Genetic Algorithm

The modified genetic algorithm is a solution method to the vector optimization problem. The algorithm searches both the noninferior solution set and the best compromise solution. Algorithm for searching the noninferior solution set has the same flow as the conventional genetic algorithm, except for the following modifications;

a) *Fitness values are high and the same for all the points satisfying the noninferiority condition, and low ones otherwise.*

b) *Convergence criterion is to be satisfied if no further update of noninferior solution set is done during the predetermined number of iterations.*

The flowchart of the modified genetic algorithm for the vector optimization problem is shown in Fig.1.

C. Numerical Example

A numerical example which has two objective functions is given in this section. Functions $f_1(x)$ and $f_2(x)$ given by (3) are 2-dimensional quadratic functions each of which

has only one minimum 100 at the point (5,5) and (8,8), respectively.

$$\text{minimize} \quad \begin{aligned} f_1(x_1,x_2) &= 2\sum_{i=1}^{2}(x_i - 5)^2 + 100 \\ f_2(x_1,x_2) &= \sum_{i=1}^{2}(x_i - 8)^2 + 100 \end{aligned} \quad (3)$$

$$\text{s.t} \quad (x_1 - 6)^2 + (x_2 - 6)^2 - 4 \leq 0$$

Constraint is given by a circle centered at (6,6) with radius 2. So, the feasible region in decision space is the interior and the boundary of the circle. Careful investigation shows that the noninferior solution set in decision space and objective space are the line connecting the minimum point (5,5) and (7.414,7.414) in the feasible region and the curve connecting the corresponding minimum point, respectively.

Important algorithm parameters used in this paper are given in Table 1 and the results are presented in Table 2. Fig.2 shows the initial population which is chosen randomly, the solution set in decision space obtained at the 50th, 400th generation, and the final results, the noninferior solution set in decision and objective space obtained at the 1572nd generation. The number of the solutions obtained at each generation are 372, 625, 696, respectively. It can be seen in Fig.2 that the noninferior solution set converges to the one discussed above with the generation and finally the noninferior solution set and minimum of each function is very close to the analytic one. From the results it can be concluded that sufficiently large number of noninferior solutions can be found by using the proposed algorithm with relatively small number of iterations.

Fig.1. Flowchart of the modified genetic algorithm

TABLE 1
IMPORTANT PARAMETERS FOR THE ALGORITHM

high fitness value	0.75
low fitness value	0.25
the number of initial population	200
the number of iterations for convergence criterion	200
probability of crossover	0.5
probability of mutation	0.05

TABLE 2
OPTIMIZATION RESULTS OF THE EXAMPLE

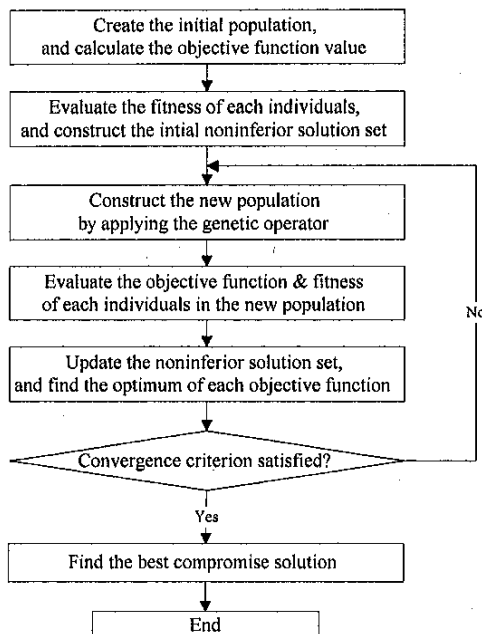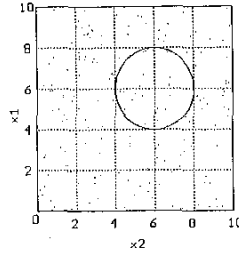|	
the number of noninferior solutions	696
the number of total iterations	1572
minimum of $f_1(x)$ at the point	100.000 (5.005,5.005)
minimum of $f_2(x)$ at the point	100.697 (7.409,7.409)
the best compromise solution the value of $f_1(x)$ the value of $f_2(x)$	(5.992,5.992) 103.938 108.063

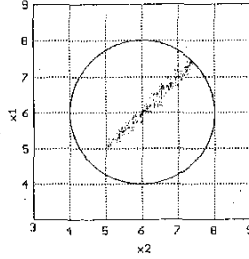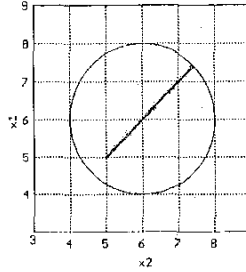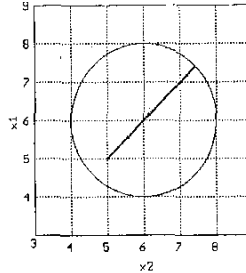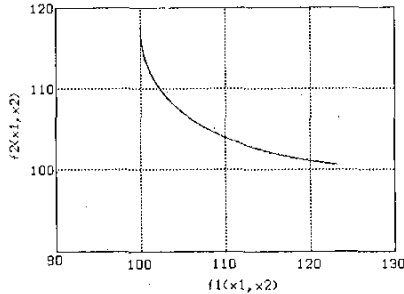(a) Initial population and feasible region in decision space

(b) Noninferior solution set in decision space at 50th generation

(c) Noninferior solution set in decision space at 400th generation

(d) Noninferior solution set in decision space at 1572nd generation

(e) Noninferior solution set in objective space at 1572nd generation

Fig.2. Noninferior solution set in decision space and objective space

III. DESIGN OBJECTIVES OF THE MOTOR

A. Derivation of Core Loss formula

The core loss consists of the eddy current and the hys-

teresis losses and can be calculated from the flux densities and the rate of change of them in the stator teeth and yoke. Analysis of the air gap flux densities due to the magnet, d-axis and q-axis currents of the Interior Permanent Magnet Motor shown in Fig.3 is presented in [8]. Eddy current and hysteresis losses can be decomposed into the terms in the stator teeth and that in the yoke. Tooth flux density B_t can be calculated considering the displacement of the magnet edge from the center of the tooth according to the rotation of the rotor as shown in Fig.4(a) and can be given by

$$B_t = \frac{1}{w_t l_r} \int_{\omega_s t/p-\beta/2}^{\omega_s t/p+\beta/2} B_g(p\theta) l_r r_s \, d\theta. \tag{4}$$

where r_s : stator bore radius

l_r : stator axial length

p : the number of pole pairs

w_t : tooth width

B_g : air gap flux density

ω_s : source angular frequency

β : slot pitch in mechanical angle

The eddy current loss is proportional to the square of the time derivative of the tooth flux density. So, the eddy current loss per unit tooth volume P_{elt} can be given as follows.

$$P_{elt} = \frac{k_{elt}}{T} \int_0^T \left(\frac{dB_t}{dt}\right)^2 dt$$

$$\approx \frac{k_{elt}\omega_s^2}{\pi} \left\{ \frac{2}{p\beta} \left(\left(B_{gm} + B_{dm} - \hat{B}_d \cos\alpha_t\right)^2 + \hat{B}_q^2 \sin^2\alpha_t \right) \right. \tag{5}$$

$$\left. + \hat{B}_d^2 \left(\frac{\alpha_t - \sin 2\alpha_t}{2}\right) + \hat{B}_q^2 \left(\frac{\alpha_t + \sin 2\alpha_t}{2} + w(p^2\beta^2 - 1)/p\beta^2\right) \right\}$$

where k_{elt} : eddy current loss coefficient in teeth

B_{gm} : air gap flux density due to the magnet

B_{dm} : induced flux density at the rotor surface

$\hat{B}_d$: peak flux density due to d-axis current

$\hat{B}_q$: peak flux density due to q-axis current

w : web width in mechanical angle

α : magnet pole arc angle

$\alpha_t = p(\alpha - \beta/2)$

The stator yoke flux density B_y and the eddy current loss per unit yoke volume P_{ely} can be calculated in the same manner considering Fig.4(b).
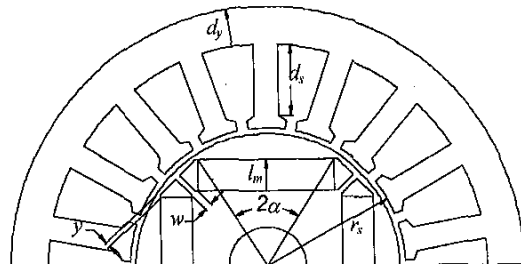

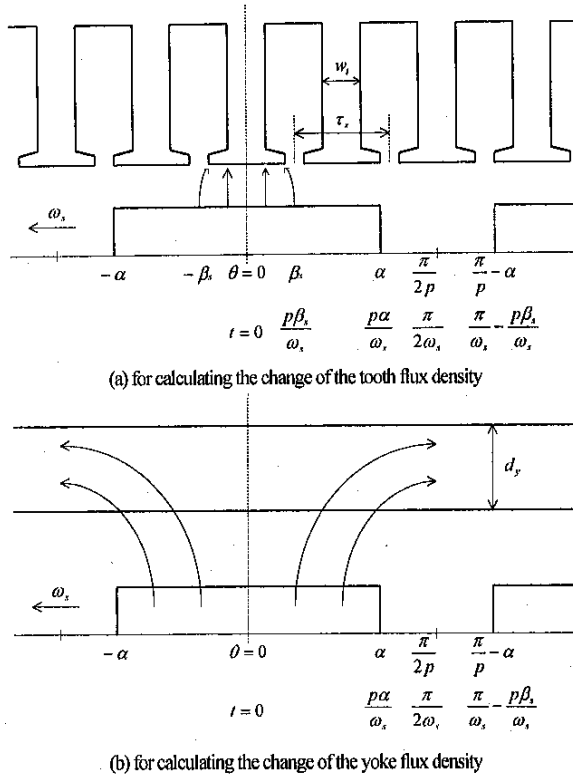Fig.3. Cross-section of the Permanent Magnet Synchronous Motor

(a) for calculating the change of the tooth flux density

(b) for calculating the change of the yoke flux density

Fig.4. Simplified model for calculating the change of the flux density

$$B_y = \frac{1}{d_y l_r} \int_0^{\omega_s i/p} B_g(p\theta) l_r r_s \, d\theta \qquad (6)$$

where d_y : stator yoke depth

$$P_{ely} = \frac{k_{ely}}{T} \int_0^T \left(\frac{dB_y}{dt}\right)^2 dt$$

$$= \frac{k_{ely}}{\pi} \left(\frac{r_s \omega_s}{p d_y}\right)^2 \left\{ 2p\alpha \left(B_{gm} + B_{dm}\right)^2 + \hat{B}_d^2 \left(p\alpha + \frac{\sin 2p\alpha}{2}\right) \right.$$

$$\left. + \hat{B}_q^2 \left(\frac{2p\alpha - \sin 2p\alpha + p\delta + \sin p\delta}{2}\right) - 4\left(B_{gm} + B_{dm}\right)\hat{B}_d \sin p\alpha \right\} \qquad (7)$$

where k_{ely} : eddy current loss coefficient in yoke

Hysteresis loss per unit volume can be obtained from the maximum flux density in the teeth and yoke, respectively.

$$P_{hlt} = k_{hlt} \, \omega_s \, \hat{B}_t^2$$

$$P_{hly} = k_{hly} \, \omega_s \, \hat{B}_y^2 \qquad (8)$$

where k_{hlt} : hysteresis loss coefficient in teeth

k_{hly} : hysteresis loss coefficient in yoke

Stator teeth and yoke volume is given by

$$V_{teeth} = w_t \, d_s \, l_r \, S_n$$

$$V_{yoke} = 2\pi\left(r_s + d_s + d_y/2\right) d_y \, l_r \qquad (9)$$

where d_s : stator slot depth

S_n : the number of the stator slots

Thus the overall core loss is expressed as

$$P_{cl} = V_{teeth}\left(P_{elt} + P_{hlt}\right) + V_{yoke}\left(P_{ely} + P_{hly}\right). \qquad (10)$$

Core loss formula introduced in this paper can be verified by comparing the calculated core loss values with the experimental ones of the prototype motor as shown in Fig.5. It is very difficult to measure the core loss directly, so is inferred from the load test as follows: if the temperature of the motor has stabilized, the stator resistance, that is, the stator winding loss, remains constant. So measuring the stator current and resistance, the input and output power, and the speed of the motor after the motor temperature has stabilized, the total loss, the stator winding loss, and the mechanical loss can be calculated. From these values, the core loss can be obtained.

Fig.5 shows the variation of the core loss data at the rated speed 2500 [rpm] of the motor according to the load conditions. Calculated values agree well with the experimental ones. Slight discrepancy between them may arise mainly from the difficulty in the separation of the core loss from the total loss and from the inaccuracy of the core loss coefficients of the core material.

B. Objective Functions

Design objectives are the efficiency and the weight of the motor. Maximum efficiency is obtained by minimizing the motor loss when the motor output is considered to be constant at the rated value. So the motor loss and weight are taken as the objective functions which are to be minimized. The motor loss consists of the stator winding loss, core loss, and mechanical loss. Other loss components are neglected because of their little contribution to the overall loss.

The stator winding loss can be given by

$$P_{sw} = 3 R_s \, I_s^2$$

$$= \frac{36\rho_c N_s^2 I_s^2}{\pi f_s d_s \left(r_s + d_s\right)}\left(l_r + \frac{\pi r_s k_\sigma}{p}\right). \qquad (11)$$

where ρ_c : resistivity of wire

I_s : stator phase current

N_s : the number of stator winding turns/phase

f_s : stator slot fill factor

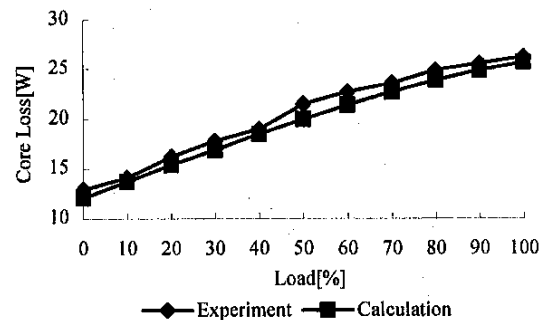k_σ : overhang coefficient of stator winding

Fig.5. Comparison between the calculated and experimental results of the stator core loss

From (10) and (11), the overall motor losses are given as follows. Mechanical loss P_{ml} can be considered as a constant value assuming the fixed motor speed.

$$f_{loss} = P_{sw} + P_{cl} + P_{ml} \qquad (12)$$

Active material motor weight is the sum of the stator, rotor, magnet and winding weight. The rotor volume is confined to the parts that participate in the energy conversion assuming the weight of the remaining parts is not significantly varied. This is because the variation of the design parameters is restricted due to the rotor geometry constraints. It is also assumed that the density of the shaft material is the same as the rotor core material.

$$f_{weight} = \rho_i l_r \left(\pi r_s d_s + \pi d_y (r_s + d_s) \right)$$
$$+ \rho_i l_r \left(\pi r_s^2 - l_m (4 w_m + 2 l_m) \right) + 4 \rho_m w_m l_m l_r \qquad (13)$$
$$+ \rho_w \pi f_s d_s (r_s + d_s) \left(l_r + k_\sigma \frac{2 \pi r_s}{p} \right)$$

where l_m : magnet thickness

w_m : magnet width

ρ_i : density of the steel core

ρ_m : density of the magnet

ρ_w : density of the wire

C. Decision Variables and Constraints

Since the motor losses and the weight are represented as a function of design parameters of the motor, several design parameters can be selected as the decision variables. The decision variables chosen in this paper are the number of the stator winding turns, the stator bore radius, the stator axial length, the stator yoke depth, the stator slot depth, the magnet thickness, and the pole arc angle.

The decision variables are restricted within the range determined by the constraints. The constraints come from the rotor geometry, that is, the condition that the magnets can be inserted in the rotor as shown in Fig.3, and the electrical and magnetic characteristics of the motor such as the limits of the flux density, current density and magnet protection against demagnetization. The output power, i.e., motor torque and the speed, is considered to be the same as the prototype motor for comparison purposes of the prototype with the optimally designed motor.

IV. RESULTS

The proposed optimal design algorithm is applied to the design of the Interior Permanent Magnet Synchronous Motor of 3 phase, 4 pole, 2500rpm and 600 W ratings. The noninferior solution set in objective space is given in Fig.6. The set is not connected, which reveals that the feasible region of the problem is not convex. The optimal design results are given in Tables 3, 4 and 5. The former shows the solution corresponding to the minimum of each objective function, and the latter the comparison of the design parameters between the optimally designed motor and the prototype. The values of the design parameters of the op-

timally designed motor come from the best compromise solution defined in (2). The optimally designed motor shows higher efficiency but larger weight than the prototype. But since the rotor dimension of the optimally designed motor is decreased compared with that of the prototype, it can be deduced that the optimally designed motor could have an enhanced servo performances because of the reduction of the rotor inertia.
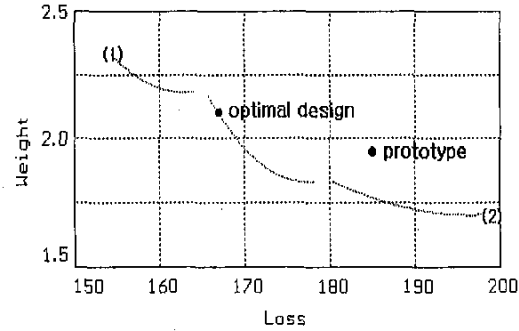

Fig.6. The noninferior solution set in objective space of
the optimal design problem of
the Interior Permanent Magnet Synchronous Motor.

Table 3
MINIMUM OF EACH OBJECTIVE FUNCTION

	(1) minimum loss solution	(2) minimum weight solution
loss [W]	154.43	198.42
weight [kg]	2.33	1.72
the number of winding turns	432	492
stator bore radius [mm]	17.49	19.92
stator axial length [mm]	49.24	42.82
stator slot depth [mm]	14.31	14.17
stator yoke depth [mm]	7.74	5.45
magnet thickness [mm]	3.47	4.15
magnet pole arc angle [deg]	35.62	31.67

Table 4
COMPARISON OF THE DESIGN PARAMETERS BETWEEN THE
OPTIMALLY DESIGNED MOTOR AND THE PROTOTYPE

	prototype	optimally designed motor
loss [W]	185.58	167.58
weight [kg]	1.95	2.07
the number of winding turns	480	444
stator bore radius [mm]	20.50	18.54
stator axial length [mm]	41.60	48.00
stator slot depth [mm]	13.25	13.30
stator yoke depth [mm]	5.80	7.71
magnet thickness [mm]	4.00	3.34
magnet pole arc angle [deg]	34.35	37.26

Table 5
DESIGN PARAMETERS OF THE OPTIMALLY DESIGNED
MOTORS HAVING THE SAME WEIGHT AND LOSS WITH THE
PROTOTYPE, RESPECTIVELY

	same weight motor	same loss motor
loss [W]	**168.79**	185.38
weight [kg]	1.95	**1.76**
the number of winding turns	450	486
stator bore radius [mm]	18.92	18.05
stator axial length [mm]	44.96	48.53
stator slot depth [mm]	14.15	13.94
stator yoke depth [mm]	6.32	5.52
magnet thickness [mm]	3.45	3.35
magnet pole arc angle [deg]	37.09	33.85

V. CONCLUSION

In this paper, multiobjective optimal design algorithm using modified genetic algorithm is presented. The algorithm is derived from the conventional one by redefining the fitness values and the convergence criterion. The main feature of the proposed algorithm is its ability to find the noninferior solution set in a single routine of calculation. And the best compromise solution can be found from the set to meet the design purposes. The effectiveness of the algorithm is verified by the application to the 2 dimensional quadratic function problem, which reveals that the result obtained from the proposed algorithm agrees well with the analytic one.

Also, the improved core loss formula is derived considering the flux variation due to the stator current as well as the magnet in order to predict the motor performance more accurately. Calculated performance is compared with the experimental results. Finally, the proposed algorithm is applied to the optimal design of the Interior Permanent Magnet Synchronous Motor which has two design objectives, i.e., minimizing motor loss and weight. Through the results of the motor design, it can be concluded that the algorithm will provide the appropriate results to meet the design purposes.

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VII. BIOGRAPHIES

Dong-Hyeok Cho was born in Mokpo, Korea in 1973. He received the B.A. degree in 1995 and the M.S. degree in 1997 in Electrical Engineering from Seoul National University.
His research field is the analysis and the design of IPMSM and optimization of electromagnetic devices using stochastic algorithms such as genetic algorithm. He is now in the Ph.D. course at the same university.

Hyun-Kyo Jung received the B.S., M.S., and Ph.D. degrees in electrical engineering from Seoul National University, Seoul, Korea in 1979, 1981, and 1984, respectively. and he worked as a faculty at Kangwon National University from 1985 to 1994 and joined Polytechnic University in NewYork from 1987 to 1989. Then he has been teaching at the School of Electrical Engineering, Seoul National University since 1994. His present interests cover the various fields of design and analysis of electric machinery and field analysis of magnetic field system especially with FEM.

Dong-Joon Sim was born in Jeonju, Korea, in1967. He received the B.S., M.S., and Ph.D. degrees in electrical engineering from Seoul National University, Seoul, Korea in 1990, 1992, and 1997, respectively. He is currently a research engineer in Research Institute of the Industrial Science and Technology (RIST) founded by Pohang Steel & Iron Company(POSCO). His areas of interest include the application of electromagnetism to steel production processes as well as the design of electric machines.

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