Recent Trends in Evolutionary Multiobjective Optimization

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Summary. This chapter presents a brief review of some of the most relevant research currently taking place in evolutionary multiobjective optimization. The main topics covered include algorithms, applications, metrics, test functions, and theory. Some of the most promising future paths of research are also addressed.

1 Introduction

Evolutionary Algorithms (EAs) are heuristics that use natural selection as their search engine to solve problems. The use of EAs for search and optimization tasks has become very popular in the last few years with a constant development of new algorithms, theoretical achivements and novel applications [49, 5, 84]. One of the emergent research areas in which EAs have become increasingly popular is multiobjective optimization. In multiobjective optimization problems, we have two or more objective functions to be optimized at the same time, instead of having only one. As a consequence, there is no unique solution to multiobjective optimization problems, but instead, we aim to find all of the good trade-off solutions available (the so-called Pareto optimal set).

The first implementation of a multi-objective evolutionary algorithm (MOEA) dates back to the mid-1980s [108, 109]. Since then, a considerable amount of research has been done in this area, now known as evolutionary multiobjective optimization (EMO for short). The growing importance of this field is reflected by a significant increment (mainly during the last ten years) of technical papers in international conferences and peer-reviewed journals, books, special sessions at international conferences and interest groups on the Internet [20].[1]

The main motivation for using EAs to solve multiobjective optimization problems is because EAs deal simultaneously with a set of possible solutions (the so-called population) which allows us to find several members of the Pareto optimal set in a single run of the algorithm, instead of having to perform a series of separate runs as in the case of the traditional mathematical programming techniques [83]. Additionally, EAs are less susceptible to the shape or continuity of the Pareto front (e.g., they can easily deal with discontinuous and concave Pareto fronts), whereas these two issues are a real concern for mathematical programming techniques [15, 29, 20].

2 Basic Concepts

The emphasis of this chapter is the solution of multiobjective optimization problems (MOPs) of the form:

$$\text{minimize } [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})] \tag{1}$$

subject to the m inequality constraints:

$$g_i(\mathbf{x}) \leq 0 \quad i = 1, 2, \ldots, m \tag{2}$$

and the p equality constraints:

$$h_i(\mathbf{x}) = 0 \quad i = 1, 2, \ldots, p \tag{3}$$

where k is the number of objective functions $f_i : \mathbb{R}^n \rightarrow \mathbb{R}$. We call $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$ the vector of decision variables. We wish to determine from among the set $\mathcal{F}$ of all vectors which satisfy (2) and (3) the particular set of values $x_1^*, x_2^*, \ldots, x_n^*$ which yield the optimum values of all the objective functions.

2.1 Pareto optimality

It is rarely the case that there is a single point that simultaneously optimizes all the objective functions of a multiobjective optimization problem. Therefore, we normally look for "trade-offs", rather than single solutions when dealing with multiobjective optimization problems. The notion of "optimality" is therefore, different in this case. The most commonly adopted notion of optimality is that originally proposed by Francis Ysidro Edgeworth [37] and later generalized by Vilfredo Pareto [92]. Although some authors call this notion

[1] The author maintains an EMO repository which currently contains over 1450 bibliographical entries at: `http://delta.cs.cinvestav.mx/~ccoello/EMOO`, with a mirror at `http://www.lania.mx/~ccoello/EMOO/`

Edgeworth-Pareto optimality (see for example [114]), we will use the most commonly accepted term: *Pareto optimality.*

We say that a vector of decision variables $\mathbf{x}^* \in \mathcal{F}$ is *Pareto optimal* if there does not exist another $\mathbf{x} \in \mathcal{F}$ such that $f_i(\mathbf{x}) \leq f_i(\mathbf{x}^*)$ for all $i = 1, \ldots, k$ and $f_j(\mathbf{x}) < f_j(\mathbf{x}^*)$ for at least one j.

In words, this definition says that $\mathbf{x}^*$ is Pareto optimal if there exists no feasible vector of decision variables $\mathbf{x} \in \mathcal{F}$ which would decrease some criterion without causing a simultaneous increase in at least one other criterion. Unfortunately, this concept almost always gives not a single solution, but rather a set of solutions called the *Pareto optimal set.* The vectors $\mathbf{x}^*$ correspoding to the solutions included in the Pareto optimal set are called *nondominated.* The image of the Pareto optimal set under the objective functions is called *Pareto front.*

3 Algorithms

The potential of evolutionary algorithms for solving multiobjective optimization problems was hinted as early as the late 1960s by Rosenberg in his PhD thesis [104]. Rosenberg's study contained a suggestion that would have led to multiobjective optimization if he had carried it out as presented. His suggestion was to use multiple *properties* (nearness to some specified chemical composition) in his simulation of the genetics and chemistry of a population of single-celled organisms. Since his actual implementation contained only one single property, the multiobjective approach could not be shown in his work.

The first actual implementation of what it is now called a multi-objective evolutionary algorithm (or MOEA, for short) was Schaffer's *Vector Evaluation Genetic Algorithm* (VEGA), which was introduced in the mid-1980s, mainly aimed for solving problems in machine learning [108, 109, 110]. Since then, a wide variety of algorithms have been proposed in the literature [20, 15, 16].

We can roughly divide MOEAs into the following types:

- Aggregating Functions
- Population-based Approaches
- Pareto-based Approaches

We will briefly discuss each of them in the following subsections.

3.1 Aggregating Functions

Perhaps the most straightforward approach to handle multiple objectives with any technique is to use a combination of all the objectives into a single one using either an addition, multiplication or any other combination of arithmetical operations that we could think of. These techniques are normally known as "aggregating functions", because they combine (or "aggregate") all the objectives of the problem into a single one. In fact, aggregating approaches are the oldest mathematical programming methods for multiobjective optimization, since they can be derived from the Kuhn-Tucker conditions for nondominated solutions [69].

An example of this approach is a linear sum of weights of the form:

$$\min \sum_{i=1}^{k} w_i f_i(\mathbf{x}) \tag{4}$$

where $w_i \geq 0$ are the weighting coefficients representing the relative importance of the k objective functions of our problem. It is usually assumed that

$$\sum_{i=1}^{k} w_i = 1 \tag{5}$$

Aggregating functions may be linear (as the previous example) or nonlinear (e.g., the aggregating functions adopted by game theory [101, 95], goal programming [28, 127], goal attainment [128, 130] and the min-max algorithm [54, 13]). Both types of aggregating functions have been used with evolutionary algorithms in a number of occasions, with relative success.

Aggregating functions have been largely underestimated by EMO researchers mainly because of the well-known limitation of linear aggregating functions (i.e., they cannot generate non-convex portions of the Pareto front regardless of the weight combination used [24]). Note however that nonlinear aggregating functions do not necessarily present such limitation [20]. In fact, even linear aggregating functions can be cleverly defined such that concave Pareto fronts can be generated [63]. However, the EMO community tends to show little interest in new algorithms based on aggregating functions and therefore their relatively low popularity among EMO researchers.

3.2 Population-based Approaches

In these techniques, the population of an EA is used to diversify the search, but the concept of Pareto dominance is not directly incorporated into the selection process. The classical example of this sort of approach is the Vector Evaluated Genetic Algorithm (VEGA), proposed by Schaffer [109]. VEGA basically consists of a simple genetic algorithm with a modified selection mechanism. At each generation, a number of sub-populations are generated by performing proportional selection according to each objective function in turn. Thus, for a problem with k objectives, k sub-populations of size M/k each are generated (assuming a total population size of M). These sub-populations are then shuffled together to obtain a new population of size M, on which the genetic algorithm (GA) applies the crossover and mutation operators.

VEGA has several problems, from which the most serious is that its selection scheme is opposed to the concept of Pareto dominance. If, for example, there is an individual that encodes a good compromise solution for all the objectives, but it is not the best in any of them, it will be discarded. Note however, that such individual should really be preserved because it encodes a Pareto optimal solution. Schaffer suggested some heuristics to deal with this problem. For example, to use a heuristic selection preference approach for nondominated individuals in each generation, to protect individuals that encode Pareto optimal solutions but are not the best in any single objective function. Also, crossbreeding among the "species" could be encouraged by adding some mate selection heuristics instead of using the random mate selection of the traditional GA. Nevertheless, the fact that Pareto dominance is not directly incorporated into the selection process of the algorithm remains as its main disadvantage.

One interesting aspect of VEGA is that despite its drawbacks it remains in current use by some researchers mainly because it is appropriate for problems in which we want the selection process to be biased and in which we have to deal with a large number of objectives (e.g., when handling constraints as objectives in single-objective optimization [12]).

Other researchers have proposed variations of VEGA or other similar population-based approaches (e.g., [87, 103, 112, 126]). Despite the limitations of these approaches, their simplicity has attracted several researchers and we should expect to see more work on population-based approaches in the next few years.

3.3 Pareto-based Approaches

Taking as a basis the main drawbacks of VEGA, Goldberg discussed on pages 199 to 201 of his famous book on genetic algorithms [49] a way of tackling multiobjective problems. His procedure consists of a selection scheme based on the concept of Pareto optimality. Goldberg not only suggested what would become the standard MOEA for several years, but also indicated that stochastic noise would make such algorithm useless unless some special mechanism was adopted to block convergence. Niching or fitness sharing [32] was suggested by Goldberg as a way to maintain diversity and avoid convergence of the GA to a single solution.

Pareto-based approaches can be historically studied as covering two generations. The first generation is characterized by the use of fitness sharing and niching combined with Pareto ranking (as defined by Goldberg or adopting a slight variation). The most representative algorithms from the first generation are the following:

1. **Nondominated Sorting Genetic Algorithm** (NSGA): This algorithm was proposed by Srinivas and Deb [113]. The approach is based on several layers of classifications of the individuals as suggested by Goldberg [49]. Before selection is performed, the population is ranked on the basis of nondomination: all nondominated individuals are classified into one category

(with a dummy fitness value, which is proportional to the population size, to provide an equal reproductive potential for these individuals). To maintain the diversity of the population, these classified individuals are shared with their dummy fitness values. Then this group of classified individuals is ignored and another layer of nondominated individuals is considered. The process continues until all individuals in the population are classified. Stochastic remainder proportionate selection is adopted for this technique. Since individuals in the first front have the maximum fitness value, they always get more copies than the rest of the population. This allows to search for nondominated regions, and results in convergence of the population toward such regions. Sharing, by its part, helps to distribute the population over this region (i.e., the Pareto front of the problem).

2. **Niched-Pareto Genetic Algorithm** (NPGA): Proposed by Horn et al. [58]. The NPGA uses a tournament selection scheme based on Pareto dominance. The basic idea of the algorithm is the following: Two individuals are randomly chosen and compared against a subset from the entire population (typically, around 10% of the population). If one of them is dominated (by the individuals randomly chosen from the population) and the other is not, then the nondominated individual wins. When both competitors are either dominated or nondominated (i.e., there is a tie), the result of the tournament is decided through fitness sharing [51].

3. **Multi-Objective Genetic Algorithm** (MOGA): Proposed by Fonseca and Fleming [44]. In MOGA, the rank of a certain individual corresponds to the number of chromosomes in the current population by which it is dominated. Consider, for example, an individual x_i at generation t, which is dominated by $p_i^{(t)}$ individuals in the current generation.
   The rank of an individual is given by [44]:
   $$\text{rank}(x_i, t) = 1 + p_i^{(t)} \tag{6}$$
   All nondominated individuals are assigned rank 1, while dominated ones are penalized according to the population density of the corresponding region of the trade-off surface.
   Fitness assignment is performed in the following way [44]:
   a) Sort population according to rank.
   b) Assign fitness to individuals by interpolating from the best (rank 1) to the worst (rank $n \leq M$) in the way proposed by Goldberg (1989), according to some function, usually linear, but not necessarily.
   c) Average the fitnesses of individuals with the same rank, so that all of them are sampled at the same rate. This procedure keeps the global population fitness constant while maintaining appropriate selective pressure, as defined by the function used.

The second generation of MOEAs was born with the introduction of the notion of elitism. In the context of multiobjective optimization, elitism usually (althought not necessarily) refers to the use of an external population (also called secondary population) to retain the nondominated individuals. However, the use of this external file raises several questions:

- How does the external file interact with the main population?
- What do we do when the external file is full?
- Do we impose additional criteria to enter the file instead of just using Pareto dominance?

Note that elitism can also be introduced through the use of a $(\mu + \lambda)$-selection in which parents compete with their children and those which are nondominated (and possibly comply with some additional criterion such as providing a better distribution of solutions) are selected for the following generation.

The most representative second generation MOEAs are the following:

1. **Strength Pareto Evolutionary Algorithm** (SPEA): This algorithm was introduced by Zitzler and Thiele [134]. This approach was conceived as a way of integrating different MOEAs. SPEA uses an archive containing nondominated solutions previously found (the so-called external nondominated set). At each generation, nondominated individuals are copied to the external nondominated set. For each individual in this external set, a *strength* value is computed. This strength is similar to the ranking value of MOGA, since it is proportional to the number of solutions to which a certain individual dominates.
   In SPEA, the fitness of each member of the current population is computed according to the strengths of all external nondominated solutions that dominate it. Additionally, a clustering technique called "average linkage method" [85] is used to keep diversity.

2. **Strength Pareto Evolutionary Algorithm 2** (SPEA2): This approach has three main differences with respect to its predecessor [132]: (1) it incorporates a fine-grained fitness assignment strategy which takes into account for each individual the number of individuals that dominate it and the number of individuals by which it is dominated; (2) it uses a nearest neighbor density estimation technique which guides the search more efficiently, and (3) it has an enhanced archive truncation method that guarantees the preservation of boundary solutions.

3. **Pareto Archived Evolution Strategy** (PAES): This algorithm was introduced by Knowles and Corne [67]. PAES consists of a (1+1) evolution strategy (i.e., a single parent that generates a single offspring) in combination with a historical archive that records some of the nondominated solutions previously found. This archive is used as a reference set against

which each mutated individual is being compared. An interesting aspect of this algorithm is its procedure used to maintain diversity which consists of a crowding procedure that divides objective space in a recursive manner. Each solution is placed in a certain grid location based on the values of its objectives (which are used as its "coordinates" or "geographical location"). A map of such grid is maintained, indicating the number of solutions that reside in each grid location. Since the procedure is adaptive, no extra parameters are required (except for the number of divisions of the objective space).

4. **Nondominated Sorting Genetic Algorithm II** (NSGA-II): Deb et al. [30, 31, 33] proposed a revised version of the NSGA [113], called NSGA-II, which is more efficient (computationally speaking), uses elitism and a crowded comparison operator that keeps diversity without specifying any additional parameters. The NSGA-II does not use an external memory as the previous algorithms. Instead, its elitist mechanism consists of combining the best parents with the best offspring obtained (i.e., a $(\mu+\lambda)$-selection).

5. **Niched Pareto Genetic Algorithm 2** (NPGA 2): Erickson et al. [39] proposed a revised version of the NPGA [58] called the NPGA 2. This algorithm uses Pareto ranking but keeps tournament selection (solving ties through fitness sharing as in the original NPGA). In this case, no external memory is used and the elitist mechanism is similar to the one adopted by the NSGA-II. Niche counts in the NPGA 2 are calculated using individuals in the partially filled next generation, rather than using the current generation. This is called continuously updated fitness sharing, and was proposed by Oei et al. [89].

6. **Pareto Envelope-based Selection Algorithm** (PESA): This algorithm was proposed by Corne et al. [22]. This approach uses a small internal population and a larger external (or secondary) population. PESA uses the same hyper-grid division of phenotype (i.e., objective funcion) space adopted by PAES to maintain diversity. However, its selection mechanism is based on the crowding measure used by the hyper-grid previously mentioned. This same crowding measure is used to decide what solutions to introduce into the external population (i.e., the archive of nondominated vectors found along the evolutionary process). Therefore, in PESA, the external memory plays a crucial role in the algorithm since it determines not only the diversity scheme, but also the selection performed by the method. There is also a revised version of this algorithm, called PESA-II [21]. This algorithm is identical to PESA, except for the fact that region-based selection is used in this case. In region-based selection, the unit of selection is a hyperbox rather than an individual. The procedure consists of selecting (using any of the traditional selection techniques [50]) a hy-

perbox and then randomly select an individual within such hyperbox. The main motivation of this approach is to reduce the computational costs associated with traditional MOEAs (i.e., those based on Pareto ranking).

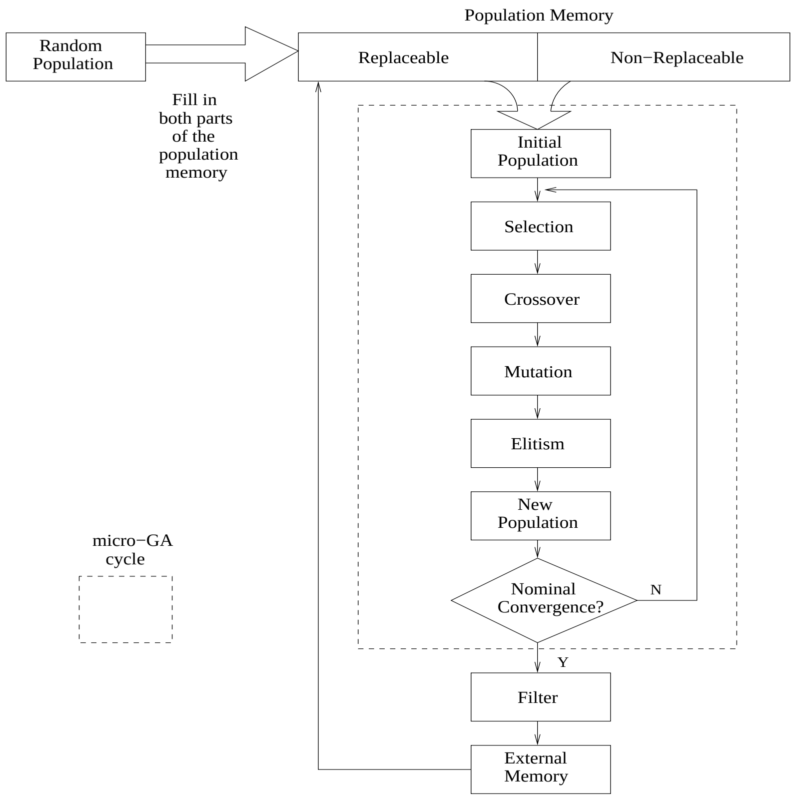

Fig. 1. Diagram that illustrates the way in which the micro-GA for multiobjective optimization works [19].

7. **Micro Genetic Algorithm**: This approach was introduced by Coello Coello & Toscano Pulido [18, 19]. A micro-genetic algorithm is a GA with a small population and a reinitialization process. The way in which the micro-GA works is illustrated in Figure 1. First, a random population is generated. This random population feeds the population memory, which is divided in two parts: a replaceable and a non-replaceable portion. The non-replaceable portion of the population memory never changes during the entire run and is meant to provide the required diversity for the algorithm. In contrast, the replaceable portion experiences changes after each cycle of the micro-GA.

The population of the micro-GA at the beginning of each of its cycles is taken (with a certain probability) from both portions of the population memory so that there is a mixture of randomly generated individuals (non-replaceable portion) and evolved individuals (replaceable portion). During each cycle, the micro-GA undergoes conventional genetic operators. After the micro-GA finishes one cycle, two nondominated vectors are chosen[2] from the final population and they are compared with the contents of the external memory (this memory is initially empty). If either of them (or both) remains as nondominated after comparing it against the vectors in this external memory, then they are included there (i.e., in the external memory). This is the historical archive of nondominated vectors. All dominated vectors contained in the external memory are eliminated. The micro-GA uses then three forms of elitism: (1) it retains nondominated solutions found within the internal cycle of the micro-GA, (2) it uses a replaceable memory whose contents is partially "refreshed" at certain intervals, and (3) it replaces the population of the micro-GA by the nominal solutions produced (i.e., the best solutions found after a full internal cycle of the micro-GA).

EMO researchers are still wondering about the sort of algorithms that will give rise to the third generation, but the emphasis seems to be on algorithmic efficiency [17, 62] and on spatial data structures that improve the efficiency of the storage in the external population [40, 43, 53, 86]. We should also expect to see more work on the true role of elitism in evolutionary multiobjective optimization [98, 78].

4 Applications

An analysis of the evolution of the EMO literature reveals some interesting facts. From the first EMO approach published in 1985 [109] up to the first survey of the area published in 1995 [45], the number of published papers related to EMO is relatively low. However, from 1995 to our days, the increase of EMO-related papers is exponential. Today, the EMO repository registers over 1450 papers, from which a vast majority are applications. The vast number of EMO papers currently available makes it impossible to attempt to produce a detailed review of them in this section. Instead, we will discuss the most popular application fields, indicating some of the specific areas within them in which researchers have focused their main efforts.

Current EMO applications can be roughly classified in three large groups: engineering, industrial and scientific. Some specific areas within each of these groups are indicated next.

[2] This is assuming that there are two or more nondominated vectors. If there is only one, then this vector is the only one selected.

We will start with the engineering applications, which are, by far, the most popular in the literature. This should not be too surprising, since engineering disciplines normally have problems with better understood mathematical models which facilitates the use of evolutionary algorithms. A representative sample of engineering applications is the following (aeronautical engineering seems to be the most popular subdiscipline within this group):

- Electrical engineering [118, 1, 99]
- Hydraulic engineering [102, 39, 46]
- Structural engineering [73, 14, 82]
- Aeronautical engineering [97, 88, 80]
- Robotics [91, 117, 90]
- Control [7, 116, 71]
- Telecommunications [9, 70, 96]
- Civil engineering [42, 6, 65]
- Transport engineering [10, 48, 79]

Industrial applications occupy the second place in popularity in the EMO literature. Within this group, scheduling is the most popular subdiscipline. A representative sample of industrial applications is the following:

- Design and manufacture [4, 100, 107]
- Scheduling [60, 115, 8]
- Management [61, 68, 36]

Finally, we have a variety of scientific applications, from which the most popular are (for obvious reasons) those related to computer science:

- Chemistry [64, 57, 72]
- Physics [93, 95, 52]
- Medicine [26, 2, 74]
- Computer science [25, 47, 38, 81]

The above distribution of applications indicates a strong interest for developing real-world applications of EMO algorithms (something not surprising considering that most real-world problems are of a multiobjective nature). Furthermore, the previous sample of EMO applications should give a general idea of the application areas that have not been explored yet, some of which are mentioned in the following section.

5 Test Functions

One of the fundamental issues when proposing an algorithm is to have a standard methodology to validate it. As part of this methodology, certain test functions (i.e., a benchmark) is required. In the early days of EMO research, very simple unconstrained bi-objective test functions were adopted

[54, 113, 58]. However, in the last few years several researchers have produced an important number of test functions that have become standard in the EMO community [27, 124, 20, 34]. Such test functions present certain difficulties for traditional EAs and mathematical programming techniques used for multi-objective optimization (e.g., multifrontality, disconnected or concave Pareto fronts). Note however that no serious theoretical study has been performed regarding the characteristics that make a multiobjective problem difficult for an MOEA and some apparently "difficult" test functions have been found to be relatively easy for most MOEAs [20].

Today, the transition from two to three objective functions is taking place in the literature, and high-dimensional problems are the current focus of study among EMO researchers [35]. We should expect that more complex test functions appear in the literature in the next few years, emphasizing aspects such as the presence of noise, uncertainty, dynamic objective functions, and epistasis, among other issues [94, 20, 119, 59].

6 Metrics

The definition of appropriate metrics is very important to be able to validate an algorithm. However, when dealing with multiobjective optimization problems, there are several reasons why the qualitative assessment of results becomes difficult. The initial problem is that we will be generating several solutions, instead of only one (we aim to generate as many elements as possible of the Pareto optimal set). The second problem is that the stochastic nature of evolutionary algorithms makes it necessary to perform several runs to assess their performance. Thus, our results have to be validated using statistical analysis tools. Finally, we may be interested in measuring different things. For example, we may be interested in having a robust algorithm that approximates the global Pareto front of a problem consistently, rather than an algorithm that converges to the global Pareto front but only occasionally. Also, we may be interested in analyzing the behavior of an evolutionary algorithm during the evolutionary process, trying to establish its capabilities to keep diversity and to progressively converge to a set of solutions close to the global Pareto front of a problem.

Three are normally the issues to take into consideration to design a good metric in this domain [131]:

1. Minimize the distance of the Pareto front produced by our algorithm with respect to the global Pareto front (assuming we know its location).
2. Maximize the spread of solutions found, so that we can have a distribution of vectors as smooth and uniform as possible.
3. Maximize the number of elements of the Pareto optimal set found.

The research produced in the last few years has included a wide variety of metrics that assess the performance of an MOEA in one of the three aspects previously indicated [20]. Some examples are the following:

1. **Error Ratio** (ER): This metric was proposed by Van Veldhuizen [120] to indicate the percentage of solutions (from the nondominated vectors found so far) that are not members of the true Pareto optimal set:
$$ER = \frac{\sum_{i=1}^{n} e_i}{n}, \tag{7}$$
where n is the number of vectors in the current set of nondominated vectors available; $e_i = 0$ if vector i is a member of the Pareto optimal set, and $e_i = 1$ otherwise. It should then be clear that $ER = 0$ indicates an ideal behavior, since it would mean that all the vectors generated by our MOEA belong to the Pareto optimal set of the problem. This metric addresses the third issue from the list previously provided.

2. **Generational Distance** (GD): The concept of generational distance was introduced by Van Veldhuizen & Lamont [122] as a way of estimating how far are the elements in the set of nondominated vectors found so far from those in the Pareto optimal set and is defined as:
$$GD = \frac{\sqrt{\sum_{i=1}^{n} d_i^2}}{n} \tag{8}$$
where n is the number of vectors in the set of nondominated solutions found so far and d_i is the Euclidean distance (measured in objective space) between each of these and the nearest member of the Pareto optimal set. It should be clear that a value of $GD = 0$ indicates that all the elements generated are in the Pareto optimal set. Therefore, any other value will indicate how "far" we are from the global Pareto front of our problem. This metric addresses the first issue from the list previously provided.

3. **Spacing** (SP): Here, one desires to measure the spread (distribution) of vectors throughout the nondominated vectors found so far. Since the "beginning" and "end" of the current Pareto front found are known, a suitably defined metric judges how well the solutions in such front are distributed. Schott [111] proposed such a metric measuring the range (distance) variance of neighboring vectors in the nondominated vectors found so far. This metric is defined as:
$$S \triangleq \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (\overline{d} - d_i)^2}\ , \tag{9}$$
where $d_i = \min_j(\mid f_1^i(\mathbf{x}) - f_1^j(\mathbf{x}) \mid + \mid f_2^i(\mathbf{x}) - f_2^j(\mathbf{x}) \mid)$, $i, j = 1, \dots, n$, $\overline{d}$ is the mean of all d_i, and n is the number of nondominated vectors found

so far. A value of zero for this metric indicates all members of the Pareto front currently available are equidistantly spaced. This metric addresses the second issue from the list previously provided.

Many other metrics exist (see for example [123, 124, 20, 29]), but some recent theoretical results seem to indicate that they may not be as reliable as we think and further research in this direction is necessary [133, 66, 135].

7 Theory

The weakest aspect of the current EMO research lies on the theoretical foundations of the area. Most of the current research concentrates on proving convergence of MOEAs [105, 106, 55, 56, 121, 76].

However, several research topics are still open. For example:

- Study the structure of fitness landscapes in multiobjective optimization problems [129, 3].
- There are no current attempts to answer a fundamental question: what makes difficult a multiobjective optimization problem for an MOEA?
- Develop a formal framework to analyze and prove convergence of parallel MOEAs.
- We know that if too many objective functions are used, the concept of Pareto dominance will eventually lead us to a situation in which all the individuals in the population will be nondominated. The question is then, what is the theoretical limit for Pareto ranking assuming finite size populations?
- Perform run-time analysis of an MOEA [77].
- It is necessary to provide definitions of robustness, convergence, and diversity (among others) in the context of evolutionary multiobjective optimization that are acceptable by the EMO community at large [75].

8 Promising Paths for Future Research

After providing a general overview of the research currently done in evolutionary multiobjective optimization, it is important to indicate now what are some of the areas and problems that represent the most promising research challenges for the next few years. Some of these promising paths for future research are the following:

- **Incorporation of preferences in MOEAs**: Despite the efforts of some researchers to incorporate user's preferences into MOEAs as to narrow the search, most of the multicriteria decision making techniques developed in Operations Research have not been applied in evolutionary multiobjective optimization [11, 23]. Such incorporation of preferences is very important in real-world applications since the user will only need one Pareto optimal solution and not the whole set as normally assumed by EMO researchers.

- **Dynamic Test Functions**: After tackling static problems with two and three objective functions, the next logical step is to develop MOEAs that can deal with dynamic test functions [41] (i.e., test functions in which the Pareto front moves over time due to the existence of random variables).

- **Highly-Constrained Search Spaces**: There is little work in the current literature regarding the solution of multiobjective problems with highly-constrained search spaces. However, it is rather common to have such problems in real-world applications and it is then necessary to develop novel constraint-handling techniques that can deal with highly-constrained search spaces efficiently.

- **Parallelism**: We should expect more work on parallel MOEAs in the next few years. Currently, there is a noticeable lack of research in this area [20, 125] and it is therefore open to new ideas. It is necessary to have more algorithms, formal models to prove convergence, and more real-world applications that use parallelism.

- **Theoretical Foundations**: It is quite important to develop the theoretical foundations of MOEAs. Although a few steps have been taken regarding proving convergence using Markov Chains (e.g., [105, 106]), much more work remains to be done as indicated in Section 7 (see [20]).

9 Conclusions

This chapter has discussed some of the most relevant research currently taking place in evolutionary multiobjective optimization. The main topics discussed include algorithms, metrics, test functions and theoretical foundations of EMO. The overview provided intends to give the reader a general picture of the current state of the field so that newcomers can analyze the current progress in their areas of interest.

Additionally, we have provided some possible paths of future research that seem promising in the short and medium term. The areas indicated should provide research material for those interested in making contributions in evolutionary multiobjective optimization. The areas described present challenges that are likely to determine the future research directions in this area.

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