

# Evolutionary Multi-Objective Optimization: Some Current Research Trends and Topics that Remain to be Explored

Carlos A. Coello Coello\*  
CINVESTAV-IPN  
Evolutionary Computation Group  
Departamento de Computación  
Av. Instituto Politécnico Nacional No. 2508  
Col. San Pedro Zacatenco  
México, D. F. 07300  
ccoello@cs.cinvestav.mx

## Abstract

This paper provides a short review of some of the main topics in which the current research in evolutionary multi-objective optimization is being focused. The topics discussed include new algorithms, efficiency, relaxed forms of dominance, scalability, and alternative metaheuristics. This discussion motivates some further topics which, from the author's perspective, constitute good potential areas for future research, namely, constraint-handling techniques, incorporation of user's preferences and parameter control. This information is expected to be useful for those interested in pursuing research in this area.

## 1 Introduction

Evolutionary algorithms (EAs) are a population-based metaheuristic inspired on the “survival of the fittest” principle, whose use has become increasingly popular over the last three decades, mainly for optimization and classification tasks [64, 48]. This popularity has given rise to a series of subdisciplines within the so-called evolutionary computation area. One of the subdisciplines that has experienced one of the fastest growth is evolutionary multi-objective optimization (EMO), which refers to the use of EAs for solving multi-objective problems (MOPs). A MOP has two or more (usually conflicting) objective functions that we wish to optimize simultaneously. Because of

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\*The author is also associated to the UMI-LAFMIA 3175 CNRS.

their nature, MOPs normally have several solutions rather than a single one<sup>1</sup> (like in global optimization). Thus, the use of the population to conduct the search presents the advantage of allowing us to generate several solutions after a single run. Additionally, because of their heuristic nature, EAs are less susceptible to the specific features of a MOP (e.g., continuity) than mathematical programming techniques, and therefore their increasing popularity within different domains, mainly during the last 15 years [21, 31, 17, 112].

The first implementation of a multi-objective evolutionary algorithm (MOEA) dates back to the mid-1980s [136, 137]. Since then, many other MOEAs have been proposed, and an important number of publications have been released.<sup>2</sup> Readers interested in the historical development of this field, should refer to [19].

After 23 years of existence, EMO is now experiencing growing pains. With no doubt, this is a very popular discipline, but at the same time, it seems less friendly to newcomers. Producing original contributions has apparently become harder (e.g., at the level of a PhD thesis), and a lot of “work by analogy” is now commonly seen in a number of publications. This has led to some EMO researchers to raise an important question: **will we continue to do research in EMO during the next few years?** This is precisely the focus of this paper, in which we will briefly discuss some of the topics that are currently the main focus of research in EMO and that, from the author’s perspective, represent promising research venues for the years to come. Thus, the main hypothesis of this paper is that there still exist enough research topics for both novice and advanced researchers, if one looks carefully within the (now overwhelming) EMO literature. The main goal of this paper is precisely to provide some hints to get relatively quickly to these promising research topics.

The remainder of this paper is organized as follows. Section 2 presents some basic concepts on multi-objective optimization, which are provided in order to make this paper self-contained. The topics that, from the author’s perspective, are more representative of the current research trends in the area are discussed in Section 3. Section 4 presents some additional topics that we believe that are worth exploring in the future. Finally, Section 5 presents our conclusions.

## 2 Basic Concepts

We are interested in solving problems of the type<sup>3</sup>:

$$\text{minimize } \vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), \dots, f_k(\vec{x})] \quad (1)$$

subject to:

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \dots, m \quad (2)$$

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<sup>1</sup>A MOP will have a single solution only if the objectives have no conflict among them, in which case there is no need to use any sort of special approach, since the sequential optimization of each of the objectives, considered separately, will lead us to this single solution.

<sup>2</sup>The author maintains the EMO repository, which currently contains over 3400 bibliographical references, plus public-domain software, and a small database of EMO researchers. The EMO repository is located at: <http://delta.cs.cinvestav.mx/~ccoello/EMO>

<sup>3</sup>Without loss of generality, we will assume only minimization problems.

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \dots, p \quad (3)$$

where  $\vec{x} = [x_1, x_2, \dots, x_n]^T$  is the vector of decision variables,  $f_i : \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $i = 1, \dots, k$  are the objective functions and  $g_i, h_j : \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $i = 1, \dots, m, j = 1, \dots, p$  are the constraint functions of the problem.

Now, we will provide some definitions that are required in order to make this paper more understandable.

**Definition 1.** Given two vectors  $\vec{u}, \vec{v} \in \mathbb{R}^k$ , we say that  $\vec{u} \leq \vec{v}$  if  $u_i \leq v_i$  for  $i = 1, \dots, k$ , and that  $\vec{u} < \vec{v}$  if  $\vec{u} \leq \vec{v}$  and  $\vec{u} \neq \vec{v}$ .

**Definition 2.** Given two vectors  $\vec{u}, \vec{v} \in \mathbb{R}^k$ , we say that  $\vec{u}$  **dominates**  $\vec{v}$  (denoted by  $\vec{u} \prec \vec{v}$ ) iff  $\vec{u} < \vec{v}$ .

**Definition 3.** We say that a vector of decision variables  $\vec{x}^* \in \mathcal{F}$  ( $\mathcal{F}$  is the feasible region) is **Pareto optimum** if there does not exist another  $\vec{x} \in \mathcal{F}$  such that  $\vec{f}(\vec{x}) \prec \vec{f}(\vec{x}^*)$ .

**Definition 4.** The **Pareto Optimal Set**  $\mathcal{P}^*$  is defined by:

$$\mathcal{P}^* = \{\vec{x} \in \mathcal{F} | \vec{x} \text{ is Pareto optimum}\}$$

The vectors  $\vec{x}^*$  corresponding to the solutions included in the Pareto optimal set are called *nondominated*.

**Definition 5.** The **Pareto Front**  $\mathcal{PF}^*$  is defined by:

$$\mathcal{PF}^* = \{\vec{f}(\vec{x}) \in \mathbb{R}^n | \vec{x} \in \mathcal{P}^*\}$$

We thus wish to determine the Pareto optimal set from the set  $\mathcal{F}$  of all the decision variable vectors that satisfy (2) and (3).

### 3 Some of the Current Research Trends

Based on an analysis of a sample of the specialized literature, we have selected the following list of topics, which seem to be representative of the main current research trends in EMO:

1. New algorithms
2. Efficiency
3. Relaxed forms of dominance
4. Scalability
5. Alternative metaheuristics

Each of these topics will be briefly discussed next.

### 3.1 New Algorithms

In the early days of EMO, the design of new algorithms was a hot topic. However, from the many MOEAs that have been proposed in the specialized literature since Schaffer’s *Vector Evaluated Genetic Algorithm* (VEGA) [137] (published in 1985), few have become widely used in the EMO community. The most popular nonelitist<sup>4</sup> MOEAs were: Multi-Objective Genetic Algorithm (MOGA) [58], Niche-Pareto Genetic Algorithm (NPGA) [73], and Nondominated Sorting Genetic Algorithm (NSGA) [142].

Although some notions of elitism had already been contemplated by some EMO researchers since the mid-1990s (see for example [75, 122]), it was until the publication of the Strength Pareto Evolutionary Algorithm (SPEA) [169] in the late 1990s, that elitist MOEAs became common. Although several elitist MOEAs exist, few have become widely used (see for example [92, 167]), and from them, one has become extremely popular: the Nondominated Sorting Genetic Algorithm-II (NSGA-II) [36]. In fact, the popularity of this algorithm has created a new trend within EMO to propose mechanisms that improve (e.g., for a certain class of problems) its performance (see for example [6, 82, 120, 94]).

It is important to note that MOEAs normally modify EAs in two ways: (1) they incorporate a selection mechanism based on Pareto optimality, and (2) they adopt a diversity preservation mechanism that avoids that the entire population converges to a single solution (as would normally occur because of the stochastic nature of EAs). Diversity preservation mechanisms have also evolved over the years, from naive fitness sharing schemes in which an individual is penalized for sharing the same “niche” with other individuals from the population (a niche is defined either in decision or in objective function space by adopting a certain niche radius from each individual, whose value is normally defined by the user) [65, 33]. Over the years, other (more elaborate) schemes have been proposed: clustering [169], the adaptive grid [92, 89], the crowded-comparison operator [36], and entropy [27, 52, 53], among others.

In spite of the previously indicated trends within this area, the design of algorithms is still an active area of research, although it is now much less popular than before. One of the current trends within this area is to adopt a selection mechanism based on some performance measure. For example, the Indicator-Based Evolutionary Algorithm (IBEA) [166] is intended to be adapted to the user’s preferences by formalizing such preferences in terms of continuous generalizations of the dominance relation. This is a nice idea, since it avoids the need to provide an explicit diversity preservation mechanism. In order to achieve this, the optimization goal of IBEA is defined in terms of a binary performance measure (e.g., the additive  $\epsilon$ -indicator [171]). Recently, the same authors introduced the Set Preference Algorithm for Multiobjective Optimization (SPAM) [170], which consists of a hillclimber based on the same idea of IBEA, but which turns out to be more general, since it is not restricted to a single binary performance measure (several of such performance measures can be used in sequence, and any type of set preference relation is acceptable). Within a similar line of thought,

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<sup>4</sup>Elitism is an operator that retains the best solution from the population of an EA and passes it intact to the next generation. In EMO, elitism, however, involves ALL the nondominated solutions from the population, and is normally implemented using an external archive that filters solutions, such that only solutions that are nondominated with respect to all the previously evaluated populations are retained.

but without explicitly considering the incorporation of user’s preferences, the S Metric Selection Evolutionary Multiobjective Optimization Algorithm (SMS-EMOA) [49, 9] adopts a selection operator based on the hypervolume measure (also known as S metric [168, 165]). There have also been multi-objective extensions of successful single-objective evolutionary optimizer, such as CMA-ES [76, 77], which is invariant to rotation in its two versions (single- and multi-objective).

Obviously, other types of MOEAs may also be developed inspired, for example, on concepts from mathematical programming (see for example the Nash Genetic Algorithm [140] and the  $\epsilon$ -constraint Cultural Differential Evolution [98]), or on existing (single-objective) EAs (see for example the Multiobjective Cellular Genetic Algorithm [117, 118] and the micro Genetic Algorithm for Multiobjective Optimization [23, 150]). Clearly, much remains to be done regarding algorithm design, and a new generation of MOEAs is expected to arise in the future.

## 3.2 Efficiency

Several EMO researchers have addressed efficiency issues<sup>5</sup> (see for example [73, 92, 36, 79]). If focused on algorithm design, one gets the impression that little can be done to improve efficiency, since the computational efficiency bounds of nondominance checking have been known for over thirty years [96]. Nevertheless, this is normally assumed by researchers, but few detailed studies of MOEA’s algorithmic complexity and of the algorithms used to extract nondominated solutions from a set are currently available in the specialized literature (see for example [131, 161]).

Interestingly, most EMO researchers have focused on an apparently easier way of increasing efficiency: the reduction of the number of individuals that are used for determining nondominance. Perhaps the first attempt to reduce the number of individuals involved in the Pareto ranking process of a MOEA is the selection mechanism of the Niche-Pareto Genetic Algorithm (NPGA) [73]. The NPGA uses binary tournament selection. However, instead of comparing fitness directly between two individuals (randomly chosen from the population), in this case a small sample of the population is randomly chosen (e.g., 10% of the total population size). Then, each of the two individuals participating in the tournament are compared with respect to the sample. If one of them turns out to be nondominated (with respect to the sample) and the other is dominated, then the nondominated individual wins the tournament and is selected as a parent. In any other case (i.e., both individuals are nondominated or both are dominated), the individual with less neighbors in its niche wins. Since the sample randomly chosen is smaller than the total population size, the NPGA never ranks an individual with respect to the entire population. This results in a faster algorithm. Another remarkable work in the same direction of the NPGA is the improved ranking procedure proposed by Jensen [79], which significantly reduces the computational complexity of the NSGA-II [36]. However, this approach is based on an algorithm that, as indicated before, is sensitive to the number of objectives [79]. There have also been proposals in which a very small population size is adopted, based on the concept of the micro-genetic algorithm [95],

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<sup>5</sup>By *efficiency*, we refer here to any sort of process that reduces the number of instructions performed in an algorithm (a MOEA in our case), without modifying the outcome produced by such algorithm.

in which no more than five individuals are used in the population [23]. This sort of MOEA requires, however, of clever reinitialization schemes in order to avoid getting stuck during the search.

Nowadays, a more common research trend has been to focus on the design of MOEAs that reduce the number of objective function evaluations performed, under the assumption that such evaluations may be very expensive in some real-world applications (clearly, much more expensive than a Pareto ranking scheme). For that sake, EMO researchers have been adopting techniques such as surrogate models, which have long been used in engineering (see for example [159, 83, 154, 88, 126]). The main idea of surrogate models is to build an approximate model of the problem, which is cheap (computationally speaking) to evaluate. Their main problem is that these models evidently have errors with respect to the original function to be optimized and, sometimes, such an error may be very significant. Also, some of the current MOEAs that adopt this sort of scheme can only be applied to problems of low dimensionality (e.g., parEGO [88]). Another possible approach is to use previously gathered knowledge (e.g., based on previous evaluations of the fitness function), in order to adapt the recombination and mutation operators so that we can sample offspring in promising areas of the search space (this is the idea of cultural algorithms [130], which have been scarcely considered for multi-objective optimization [22]). Knowledge of past evaluations can also be used to build an empirical model that approximates the fitness function to optimize. This approximation can then be used to predict promising new solutions at a smaller evaluation cost than that of the original problem (see for example [88, 80]). It is also possible to use fitness inheritance in order to reduce the number of evaluations of the objective functions. Fitness inheritance [141] works as follows: when assigning fitness to an individual, sometimes the objective function is evaluated as usual, but the rest of the time, the fitness of an individual is assigned as the average of the fitnesses of its parents, thus avoiding a fitness function evaluation based on the assumption of similarity of the individual to its parents. Fitness inheritance has been extended for multi-objective optimization by a few researchers (see for example [15, 128]). For a more thorough discussion on the different knowledge incorporation schemes that have been adopted in MOEAs, the interested reader is referred to [99].

It is worth noting, however, that other approaches are possible, by using hybrid schemes. For example, in [71], a MOEA is used to produce a rough approximation of the Pareto front, and then a local search scheme based on rough sets theory is adopted to rebuild the missing portions of the Pareto front. In [133], a similar scheme is proposed, but using scatter search as the local search engine, instead. Clearly, the use of powerful local search schemes hybridized with MOEAs that can produce rough approximations of the Pareto front with a reduced number of evaluations [157], or with MOEAs that use special operators to accelerate convergence [2, 3], is a very promising research topic.

### 3.3 Relaxed Forms of Dominance

In recent years, some researchers have proposed the use of relaxed forms of Pareto dominance as a way of regulating convergence of a MOEA [93]. From these proposals, the most popular is the so-called  $\epsilon$ -dominance, which was introduced in [102].

This mechanism acts as an archiving strategy to ensure both properties of convergence towards the Pareto optimal set and properties of diversity among the solutions found. The idea is to use a set of boxes to cover the Pareto front, where the size of such boxes is defined by a user-defined parameter (called  $\epsilon$ ). Within each box, it is only allowed a single nondominated solution to be retained (e.g., the one closest to the lower lefthand corner, if both objectives are being minimized). Thus, by using a large value of  $\epsilon$ , the user can speed up convergence, but at the sake of sacrificing the quality of the Pareto front approximation obtained. Conversely, if a high-quality approximation of the front is required, then a small value of  $\epsilon$  must be adopted instead. The definition of  $\epsilon$ , is then, quite important. Unfortunately, it is not straightforward to find the most appropriate value of  $\epsilon$  that produces a certain (required) number of nondominated solution within an archive, when nothing is known in advance about the shape of the Pareto front. Also, to correlate the number of nondominated solutions desired with the value of  $\epsilon$  chosen is not easy, and normally some preliminary runs are required in order to estimate the appropriate value. This makes difficult to compare approaches that adopt  $\epsilon$  with respect to MOEAs that do not use this concept. Additionally, because of its nature, this mechanism eliminates certain portions of the Pareto front (e.g., almost straight segments and the extremes of the Pareto front), which may be undesirable in some cases [153]. This, however, can be (at least partially) compensated by using geometrical assumptions about the possible shapes of the Pareto front, and adopting boxes of varying size (see for example [72]).

Several modern MOEAs have adopted the concept of  $\epsilon$ -dominance (see for example [34, 113, 35, 134]), and, mainly because of its nice mathematical properties, its use has become relatively popular in the last few years. However, much more work on this topic is expected to be developed in the years to come, both from a pragmatic and from a theoretical point of view.

### 3.4 Scalability

For several years, most EMO research focused on solving MOPs with only two or three objectives, and it was assumed that scaling such MOEAs to a larger number of objectives would be straightforward. However, several EMO researchers have found this assumption to be wrong [87, 74, 155]. One of the reasons for this is that the proportion of nondominated solutions in a population increases rapidly with the number of objectives. Indeed, in [54], it is shown that this number goes to infinity when the number of objectives approaches to infinity. This implies that in the presence of many objectives the selection of new solutions is carried out almost at random since a large number of the solutions are equally good in the Pareto sense [91]. This has made scalability an important research topic [125, 124, 37, 38].

Currently, there are mainly two approaches to deal with problems involving many objectives: 1) to adopt relaxed forms of Pareto optimality by proposing an optimality relation that yields a solution ordering finer than that yielded by Pareto optimality (see for example [54, 55, 37, 144]) and 2) to reduce the number of objectives of the original MOP, thus lowering the dimensionality to a reasonable value that can be handled by standard MOEAs [135, 13]. Although the second of these types of approaches seems to be an attractive choice, the difficulties commonly associated with dimensionality

reductions has made relaxed forms of Pareto optimality more popular in the literature [106]. Because of its relevance, an important increase of research in this area is expected to occur in the coming years.

It is worth noting, however, that until recently, the focus of scalability studies has been high dimensionality in objective function space, but scalability in decision variable space is also worth studying [43, 119].

### 3.5 Alternative Metaheuristics

Relatively recently, several other biologically-inspired metaheuristics have been adapted to solve MOPs [21, 26]:

- **Artificial Immune Systems:** From a computational perspective, our immune system can be seen as a distributed intelligent system, which is able to learn and retrieve knowledge previously acquired, in order to solve recognition and classification tasks [116]. Because of these features, researchers have developed computational models of our immune system and have used them for a variety of tasks, including classification, pattern recognition, and optimization [29, 116, 121]. Several multi-objective extensions of artificial immune systems have been proposed in the specialized literature (see for example [108, 107, 20, 59, 16]). Also, combinations of artificial immune systems and another type of approach have been proposed, aiming to solve specific types of MOPs (e.g., [148, 147], in which the aim is to solve bi-objective flowshop scheduling problems). However, from the author's perspective, the potential of multi-objective artificial immune systems for solving classification and pattern recognition problems has not been fully exploited yet [163].
- **Ant Colony Optimization:** This is a metaheuristic inspired on the foraging behavior of real ants. It is a distributed, stochastic search procedure based on the indirect communication of a set (called "colony") of artificial ants, which mediate using artificial pheromone trails. These pheromone trails can be seen as distributed information which is used by the ants to construct their solutions to the problem at hand. Such pheromone trails are modified during the algorithm's execution, such that they reflect the search experience acquired by the ants. This metaheuristic is intended for solving difficult (both static and dynamic) combinatorial optimization problems, in which solutions can be generated through the use of a construction procedure [25, 41, 10, 42]. There are several multi-objective extensions of ant colony optimization (ACO) algorithms (see for example [132, 78, 7, 66, 39, 40, 61]), but as multi-objective combinatorial optimization becomes more attractive for EMO researchers [45, 60], it is expected that more multi-objective ACO approaches (and hybrids of ACO algorithms with MOEAs and other metaheuristics) are proposed in the near future.
- **Particle Swarm Optimization:** This metaheuristic is inspired on the choreography of a bird flock which aim to find food [84, 86]. It can be seen as a distributed behavioral algorithm that performs (in its more general version) a multidimensional search. The implementation of the algorithm adopts a population of par-

ticles, whose behavior is affected by either the best local (i.e., within a certain neighborhood) or the best global individual. Particle swarm optimization (PSO) has been successfully used for both continuous nonlinear and discrete binary optimization [44, 85, 86, 50, 51]. An important number of multi-objective versions of PSO currently exist (see for example [114, 105, 24, 143, 4, 127, 129]). However, until relatively recently, most of the research had concentrated on producing new variations of existing algorithms, rather than on studying other (more interesting) topics, such as the role of the main components of PSO in multi-objective optimization. Some recent research in that direction has shown that certain components that had been traditionally disregarded (e.g., the leader selection mechanism and the parameters of the flight formula) play a key role in the performance of a multi-objective PSO algorithm [12, 151]. This opens new paths for future research within this area.

- **Scatter Search:** This approach was originally conceived as an extension of a heuristic called surrogate constraint relaxation, which was designed for solving integer programming problems [62]. The main idea of this approach is to adopt a series of different initializations to generate solutions. A reference set of solutions (the best found so far) is adopted, and then such solutions are “diversified” in order to generate new solutions within the neighborhood of the contents of the reference set. This sort of simple procedure is repeated until no further improvements to the contents of the reference set are detected. In the mid-1990s, some further mechanisms were added to the original scatter search algorithm, which allowed its extension to solve nonlinear, binary and permutation optimization problems [63]. These new applications triggered an important amount of research in the last few years [97, 109]. Multi-objective extensions of scatter search are relatively recent, but have been steadily increasing [162, 5, 8, 119]. Scatter search has a lot of potential for hybrid approaches, such as memetic MOEAs [90], since it can act as a powerful local search engine for tasks such as generating missing parts of a Pareto front [133]. Because of its flexibility and ease of use, scatter search is expected to become more commonly adopted in the near future, particularly when designing hybrid MOEAs that rely heavily on good local search engines.

## 4 What Else Remains to be Done

Other topics that, from the author’s perspective, are worth exploring within the next few years are the following:

1. **Constraint-handling:** One of the research areas that has attracted a lot of interest in recent years has been the use of multi-objective optimization concepts to design constraint-handling mechanisms for (single-objective) EAs (see for example [145, 70, 160, 156, 111]). Interestingly, however, relatively few research has been done regarding the design of constraint-handling mechanisms for MOEAs (see for example [67, 123, 158, 69]), in spite of the importance of constraints in real-world applications of MOEAs. Most of the current work has

focused on extending the Pareto optimality relation in order to incorporate constraints (e.g., giving preference to feasibility over dominance, such that an infeasible solution is discarded even if it is nondominated). Also, the use of penalty functions that “punish” a solution for not being feasible are easy to incorporate into a MOEA [18]. However, topics such as the design of constraint-handling mechanisms for dealing with equality constraints,<sup>6</sup> the design of scalable test functions that incorporate constraints of different types (linear, nonlinear, equality, inequality), and the study of mechanisms that allow an efficient exploration of constrained search spaces in MOPs remain practically unexplored.

2. **Incorporation of user’s preferences:** In practical applications of MOEAs, users are normally not interested in a large number of nondominated solutions. Instead, they are usually only interested in a few types of trade-offs among the objectives (e.g., perhaps only the solutions around the “knee” of the Pareto front are of interest to the user). Thus, if such user’s preferences are incorporated into the selection mechanism of a MOEA, the search can be much more efficient (e.g., one can zoom in a certain region of the Pareto front and evolve the population only towards the area of interest) and the results more meaningful. Although some research has been done in this direction (see for example [28, 81, 156, 11]), it is still relatively uncommon to report results of a MOEA that incorporates user’s preferences. It is thus important that EMO researchers get closer to the extensive work done in Operations Research in this regard (see for example [57]).
3. **Parameter control:** The design of mechanisms that allow an automated control of the parameters of a MOEA (by using, for example, online adaptation [46, 47] or self-adaptation [110], so that the MOEA can adapt its parameters without any human intervention) has been scarcely explored by EMO researchers [100, 146, 14, 1, 164, 150, 32]. This is clearly a very challenging topic, due to the high nonlinear interaction among the parameters of an EA [30]. The goal of a parameterless MOEA is rarely discussed in the EMO literature [150], and alternative (perhaps more viable) schemes such as the use of internal restarts (in other words, the use of information from previous runs to improve performance of subsequent runs) is also scarcely addressed [30]. Additionally, studies that show the effect of the parameters of a MOEA in its performance are still lacking in the specialized literature (see for example [149]), and are a key aspect of algorithmic design.

Several other topics that are also very promising research paths will not be discussed due to obvious space limitations (for example, runtime analysis of MOEAs<sup>7</sup>

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<sup>6</sup>When dealing with equality constraints, the optimum lies on the boundary between the feasible and the infeasible regions. Therefore, the use of approaches that always favor feasible solutions over the infeasible ones are not effective in this case.

<sup>7</sup>Runtime analysis addresses the question of how long a certain algorithm takes to find the optimal solution for a specific problem or a class of problems.

[104, 103], archiving techniques<sup>8</sup> [115, 139, 89, 72] and convergence analysis<sup>9</sup> [101, 152, 138], just to name a few), but they serve as a good indicator of a healthy research field in which many things remain to be done.

## 5 Conclusions

This paper has attempted to provide a summary of the main topics in which EMO researchers are currently working, and which, from the author's perspective, provide several interesting challenges for the years to come. This aims to provide a quick reference for those interested in start doing research in this field, so that they can get a very general picture of the current state of the area.

At the end of the paper, a few other topics are briefly discussed. Such topics also offer the potential to become very popular research areas within a few more years, and have remained relatively unexplored so far, thus offering important opportunities for newcomers. Hopefully, this general overview of the current and future status of the field will serve to maintain and increase the interest of researchers and practitioners in EMO, since such is the main goal of this paper.

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## References

- [1] Hussein A. Abbass. The Self-Adaptive Pareto Differential Evolution Algorithm. In *Congress on Evolutionary Computation (CEC'2002)*, volume 1, pages 831–836, Piscataway, New Jersey, May 2002. IEEE Service Center.
- [2] Salem F. Adra, Ian Griffin, and Peter J. Fleming. An Informed Convergence Accelerator for Evolutionary Multiobjective Optimiser. In Dirk Thierens, editor, *2007 Genetic and Evolutionary Computation Conference (GECCO'2007)*, volume 1, pages 734–740, London, UK, July 2007. ACM Press.
- [3] Salem Fawaz Adra. *Improving Convergence, Diversity and Pertinency in Multiobjective Optimisation*. PhD thesis, Department of Automatic Control and Systems Engineering, The University of Sheffield, UK, October 2007.
- [4] Julio E. Alvarez-Benitez, Richard M. Everson, and Jonathan E. Fieldsend. A MOPSO Algorithm Based Exclusively on Pareto Dominance Concepts. In

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<sup>8</sup>This refers to the use of special data structures for an efficient storage and retrieval of nondominated solutions (e.g., quadrees [115, 68], red-black trees [56], etc.).

<sup>9</sup>This refers to providing mathematical proofs of convergence of a MOEA under certain conditions.

- Carlos A. Coello Coello, Arturo Hernández Aguirre, and Eckart Zitzler, editors, *Evolutionary Multi-Criterion Optimization. Third International Conference, EMO 2005*, pages 459–473, Guanajuato, México, March 2005. Springer. Lecture Notes in Computer Science Vol. 3410.
- [5] Joao A. Vasconcelos, Joao H.R.D. Maciel, and Roberta O. Parreiras. Scatter Search Techniques Applied to Electromagnetic Problems. *IEEE Transactions on Magnetics*, 41(5):1804–1807, May 2005.
- [6] Meghna Babbar, Ashvin Lakshmikantha, and David E. Goldberg. A Modified NSGA-II to Solve Noisy Multiobjective Problems. In James Foster, editor, *2003 Genetic and Evolutionary Computation Conference. Late-Breaking Papers*, pages 21–27, Chicago, Illinois, USA, July 2003. AAAI.
- [7] B. Barán and M. Schaerer. A Multiobjective Ant Colony System for Vehicle Routing Problem with Time Windows. In *Proceedings of the 21st IASTED International Conference on Applied Informatics*, pages 97–102, Innsbruck, Austria, February 2003. IASTED.
- [8] Ricardo P. Beausoleil. “MOSS” multiobjective scatter search applied to non-linear multiple criteria optimization. *European Journal of Operational Research*, 169(2):426–449, March 2006.
- [9] Nicola Beume, Boris Naujoks, and Michael Emmerich. SMS-EMOA: Multiobjective selection based on dominated hypervolume. *European Journal of Operational Research*, 181(3):1653–1669, 16 September 2007.
- [10] Eric Bonabeau, Marco Dorigo, and Guy Theraulaz. *Swarm Intelligence. From Natural to Artificial Systems*. Oxford University Press, New York, 1999.
- [11] Jürgen Branke and Kalyanmoy Deb. Integrating User Preferences into Evolutionary Multi-Objective Optimization. In Yaochu Jin, editor, *Knowledge Incorporation in Evolutionary Computation*, pages 461–477. Springer, Berlin Heidelberg, 2005. ISBN 3-540-22902-7.
- [12] Jürgen Branke and Sanaz Mostaghim. About Selecting the Personal Best in Multi-Objective Particle Swarm Optimization. In Thomas Philip Runarsson, Hans-Georg Beyer, Edmund Burke, Juan J. Merelo-Guervós, L. Darrell Whitley, and Xin Yao, editors, *Parallel Problem Solving from Nature - PPSN IX, 9th International Conference*, pages 523–532. Springer. Lecture Notes in Computer Science Vol. 4193, Reykjavik, Iceland, September 2006.
- [13] Dimo Brockhoff and Eckart Zitzler. Are All Objectives Necessary? On Dimensionality Reduction in Evolutionary Multiobjective Optimization. In Thomas Philip Runarsson, Hans-Georg Beyer, Edmund Burke, Juan J. Merelo-Guervós, L. Darrell Whitley, and Xin Yao, editors, *Parallel Problem Solving from Nature - PPSN IX, 9th International Conference*, pages 533–542. Springer. Lecture Notes in Computer Science Vol. 4193, Reykjavik, Iceland, September 2006.

- [14] Dirk Büche, Gianfranco Guidati, Peter Stoll, and Petros Koumoursakos. Self-Organizing Maps for Pareto Optimization of Airfoils. In Juan Julián Merelo Guervós, Panagiotis Adamidis, Hans-Georg Beyer, José-Luis Fernández-Villanás, and Hans-Paul Schwefel, editors, *Parallel Problem Solving from Nature—PPSN VII*, pages 122–131, Granada, Spain, September 2002. Springer-Verlag. Lecture Notes in Computer Science No. 2439.
- [15] Lam T. Bui, Hussein A. Abbass, and Daryl Essam. Fitness Inheritance For Noisy Evolutionary Multi-Objective Optimization. In Hans-Georg Beyer et al., editor, *2005 Genetic and Evolutionary Computation Conference (GECCO'2005)*, volume 1, pages 779–785, New York, USA, June 2005. ACM Press.
- [16] Felipe Campelo, Frederico G. Guimarães, and Hajime Igarashi. Overview of Artificial Immune Systems for Multi-Objective Optimization. In Shigeru Obayashi, Kalyanmoy Deb, Carlo Poloni, Tomoyuki Hiroyasu, and Tadahiko Murata, editors, *Evolutionary Multi-Criterion Optimization, 4th International Conference, EMO 2007*, pages 937–951, Matshushima, Japan, March 2007. Springer. Lecture Notes in Computer Science Vol. 4403.
- [17] Carlos A. Coello Coello. An Updated Survey of GA-Based Multiobjective Optimization Techniques. *ACM Computing Surveys*, 32(2):109–143, June 2000.
- [18] Carlos A. Coello Coello. Theoretical and Numerical Constraint-Handling Techniques used with Evolutionary Algorithms: A Survey of the State of the Art. *Computer Methods in Applied Mechanics and Engineering*, 191(11–12):1245–1287, January 2002.
- [19] Carlos A. Coello Coello. Evolutionary multiobjective optimization: A historical view of the field. *IEEE Computational Intelligence Magazine*, 1(1):28–36, February 2006.
- [20] Carlos A. Coello Coello and Nareli Cruz Cortés. Solving Multiobjective Optimization Problems using an Artificial Immune System. *Genetic Programming and Evolvable Machines*, 6(2):163–190, June 2005.
- [21] Carlos A. Coello Coello, Gary B. Lamont, and David A. Van Veldhuizen. *Evolutionary Algorithms for Solving Multi-Objective Problems*. Springer, New York, second edition, September 2007. ISBN 978-0-387-33254-3.
- [22] Carlos A. Coello Coello and Ricardo Landa Becerra. Evolutionary Multiobjective Optimization using a Cultural Algorithm. In *2003 IEEE Swarm Intelligence Symposium Proceedings*, pages 6–13, Indianapolis, Indiana, USA, April 2003. IEEE Service Center.
- [23] Carlos A. Coello Coello and Gregorio Toscano Pulido. Multiobjective Optimization using a Micro-Genetic Algorithm. In Lee Spector, Erik D. Goodman, Annie Wu, W.B. Langdon, Hans-Michael Voigt, Mitsuo Gen, Sandip Sen, Marco Dorigo, Shahram Pezeshk, Max H. Garzon, and Edmund Burke,

- editors, *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO'2001)*, pages 274–282, San Francisco, California, 2001. Morgan Kaufmann Publishers.
- [24] Carlos A. Coello Coello, Gregorio Toscano Pulido, and Maximino Salazar Lechuga. Handling Multiple Objectives With Particle Swarm Optimization. *IEEE Transactions on Evolutionary Computation*, 8(3):256–279, June 2004.
- [25] A. Colomi, M. Dorigo, and V. Maniezzo. Distributed Optimization by Ant Colonies. In F. J. Varela and P. Bourguine, editors, *Proceedings of the First European Conference on Artificial Life*, pages 134–142. MIT Press, Cambridge, MA, 1992.
- [26] David Corne, Marco Dorigo, and Fred Glover, editors. *New Ideas in Optimization*. McGraw-Hill, London, 1999.
- [27] Xunxue Cui, Miao Li, and Tingjian Fang. Study of Population Diversity of Multiobjective Evolutionary Algorithm Based on Immune and Entropy Principles. In *Proceedings of the Congress on Evolutionary Computation 2001 (CEC'2001)*, volume 2, pages 1316–1321, Piscataway, New Jersey, May 2001. IEEE Service Center.
- [28] Dragan Cvetković and Ian C. Parmee. Preferences and their Application in Evolutionary Multiobjective Optimisation. *IEEE Transactions on Evolutionary Computation*, 6(1):42–57, February 2002.
- [29] Dipankar Dasgupta, editor. *Artificial Immune Systems and Their Applications*. Springer-Verlag, Berlin, 1999.
- [30] Kenneth De Jong. Parameter Setting in EAs: a 30 Year Perspective. In Fernando G. Lobo, Cláudio F. Lima, and Zbigniew Michalewicz, editors, *Parameter Setting in Evolutionary Algorithms*, pages 1–18. Springer-Verlag, Berlin, 2007.
- [31] Kalyanmoy Deb. *Multi-Objective Optimization using Evolutionary Algorithms*. John Wiley & Sons, Chichester, UK, 2001. ISBN 0-471-87339-X.
- [32] Kalyanmoy Deb. Evolutionary Multi-Objective Optimization Without Additional Parameters. In Fernando G. Lobo, Cláudio F. Lima, and Zbigniew Michalewicz, editors, *Parameter Setting in Evolutionary Algorithms*, pages 241–257. Springer-Verlag, Berlin, 2007.
- [33] Kalyanmoy Deb and David E. Goldberg. An Investigation of Niche and Species Formation in Genetic Function Optimization. In J. David Schaffer, editor, *Proceedings of the Third International Conference on Genetic Algorithms*, pages 42–50, San Mateo, California, June 1989. George Mason University, Morgan Kaufmann Publishers.

- [34] Kalyanmoy Deb, Manikanth Mohan, and Shikhar Mishra. Towards a Quick Computation of Well-Spread Pareto-Optimal Solutions. In Carlos M. Fonseca, Peter J. Fleming, Eckart Zitzler, Kalyanmoy Deb, and Lothar Thiele, editors, *Evolutionary Multi-Criterion Optimization. Second International Conference, EMO 2003*, pages 222–236, Faro, Portugal, April 2003. Springer. Lecture Notes in Computer Science. Volume 2632.
- [35] Kalyanmoy Deb, Manikanth Mohan, and Shikhar Mishra. Evaluating the  $\epsilon$ -Domination Based Multi-Objective Evolutionary Algorithm for a Quick Computation of Pareto-Optimal Solutions. *Evolutionary Computation*, 13(4):501–525, Winter 2005.
- [36] Kalyanmoy Deb, Amrit Pratap, Sameer Agarwal, and T. Meyarivan. A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, 6(2):182–197, April 2002.
- [37] Francesco di Pierro. *Many-Objective Evolutionary Algorithms and Applications to Water Resources Engineering*. PhD thesis, School of Engineering, Computer Science and Mathematics, UK, August 2006.
- [38] Francesco di Pierro, Shoon-Thiam Khu, and Dragan A. Savić. An Investigation on Preference Order Ranking Scheme for Multiobjective Evolutionary Optimization. *IEEE Transactions on Evolutionary Computation*, 11(1):17–45, February 2007.
- [39] Karl Doerner, Walter J. Gutjahr, Richard F. Hartl, Christine Strauss, and Christian Stummer. Pareto Ant Colony Optimization: A Metaheuristic Approach to Multiobjective Portfolio Selection. *Annals of Operations Research*, 131(1–4):79–99, October 2004.
- [40] K.F. Doerner, W.J. Gutjahr, R.F. Hartl, C. Strauss, and C. Stummer. Pareto ant colony optimization with ILP preprocessing in multiobjective portfolio selection. *European Journal of Operational Research*, 171(3):830–841, June 2006.
- [41] Marco Dorigo and Gianni Di Caro. The Ant Colony Optimization Meta-Heuristic. In David Corne, Marco Dorigo, and Fred Glover, editors, *New Ideas in Optimization*, pages 11–32, London, 1999. McGraw-Hill.
- [42] Marco Dorigo and Thomas Stützle. *Ant Colony Optimization*. The MIT Press, 2004. ISBN 0-262-04219-3.
- [43] Juan J. Durillo, Antonio J. Nebro, Carlos A. Coello Coello, Francisco Luna, and Enrique Alba. A Comparative Study of the Effect of Parameter Scalability in Multi-Objective Metaheuristics. In *2008 Congress on Evolutionary Computation (CEC'2008)*, pages 1893–1900, Hong Kong, June 2008. IEEE Service Center.
- [44] R.C. Eberhart and Y. Shi. Comparison between Genetic Algorithms and Particle Swarm Optimization. In V. W. Porto, N. Saravanan, D. Waagen, and A.E.

- Eibe, editors, *Proceedings of the Seventh Annual Conference on Evolutionary Programming*, pages 611–619. Springer-Verlag, March 1998.
- [45] Matthias Ehrgott and Xavier Gandibleux. Multiobjective Combinatorial Optimization—Theory, Methodology, and Applications. In Matthias Ehrgott and Xavier Gandibleux, editors, *Multiple Criteria Optimization: State of the Art Annotated Bibliographic Surveys*, pages 369–444. Kluwer Academic Publishers, Boston, 2002.
- [46] A.E. Eiben, R. Hinterding, and Z. Michalewicz. Parameter control in evolutionary algorithms. *IEEE Transactions on Evolutionary Computation*, 3(2):124–141, 1999.
- [47] A.E. Eiben, Z. Michalewicz, M. Schoenauer, and J.E. Smith. Parameter Control in Evolutionary Algorithms. In Fernando G. Lobo, Cláudio F. Lima, and Zbigniew Michalewicz, editors, *Parameter Setting in Evolutionary Algorithms*, pages 19–46. Springer-Verlag, Berlin, 2007.
- [48] A.E. Eiben and J.E. Smith. *Introduction to Evolutionary Computing*. Springer, Berlin, 2003. ISBN 3-540-40184-9.
- [49] Michael Emmerich, Nicola Beume, and Boris Naujoks. An EMO Algorithm Using the Hypervolume Measure as Selection Criterion. In Carlos A. Coello Coello, Arturo Hernández Aguirre, and Eckart Zitzler, editors, *Evolutionary Multi-Criterion Optimization. Third International Conference, EMO 2005*, pages 62–76, Guanajuato, México, March 2005. Springer. Lecture Notes in Computer Science Vol. 3410.
- [50] Andries P. Engelbrecht. *Computational Intelligence: An Introduction*. John Wiley & Sons, 2003. ISBN 0-47084-870-7.
- [51] Andries P. Engelbrecht. *Fundamentals of Computational Swarm Intelligence*. John Wiley & Sons, Ltd, 2005. ISBN 0-470-09191-6.
- [52] Ali Farhang-Mehr and Shapour Azarm. Diversity Assessment of Pareto Optimal Solution Sets: An Entropy Approach. In *Congress on Evolutionary Computation (CEC'2002)*, volume 1, pages 723–728, Piscataway, New Jersey, May 2002. IEEE Service Center.
- [53] Ali Farhang-Mehr and Shapour Azarm. Entropy-based multi-objective genetic algorithm for design optimization. *Structural and Multidisciplinary Optimization*, 24(5):351–361, November 2002.
- [54] M. Farina and P. Amato. On the Optimal Solution Definition for Many-criteria Optimization Problems. In *Proceedings of the NAFIPS-FLINT International Conference'2002*, pages 233–238, Piscataway, New Jersey, June 2002. IEEE Service Center.

- [55] M. Farina and P. Amato. A fuzzy definition of “optimality” for many-criteria optimization problems. *IEEE Transactions on Systems, Man, and Cybernetics Part A—Systems and Humans*, 34(3):315–326, May 2004.
- [56] Jonathan E. Fieldsend, Richard M. Everson, and Sameer Singh. Using Unconstrained Elite Archives for Multiobjective Optimization. *IEEE Transactions on Evolutionary Computation*, 7(3):305–323, June 2003.
- [57] José Figueira, Vincent Mousseau, and Bernard Roy, editors. *Multiple Criteria Decision Analysis. State of the Art Surveys*. Springer, New York, USA, 2005. ISBN 978-0387-23067-2.
- [58] Carlos M. Fonseca and Peter J. Fleming. Genetic Algorithms for Multiobjective Optimization: Formulation, Discussion and Generalization. In Stephanie Forrest, editor, *Proceedings of the Fifth International Conference on Genetic Algorithms*, pages 416–423, San Mateo, California, 1993. University of Illinois at Urbana-Champaign, Morgan Kaufman Publishers.
- [59] Fabio Freschi and Maurizio Repetto. VIS: an artificial immune network for multi-objective optimization. *Engineering Optimization*, 38(8):975–996, December 2006.
- [60] Xavier Gandibleux and Matthias Ehrgott. 1984-2004 – 20 Years of Multiobjective Metaheuristics. But What About the Solution of Combinatorial Problems with Multiple Objectives? In Carlos A. Coello Coello, Arturo Hernández Aguirre, and Eckart Zitzler, editors, *Evolutionary Multi-Criterion Optimization. Third International Conference, EMO 2005*, pages 33–46, Guanajuato, México, March 2005. Springer. Lecture Notes in Computer Science Vol. 3410.
- [61] C. García-Martínez, O. Cordón, and F. Herrera. A taxonomy and an empirical analysis of multiple objective ant colony optimization algorithms for the bi-criteria TSP. *European Journal of Operational Research*, 180(1):116–148, July 2007.
- [62] Fred Glover. Heuristics for integer programming using surrogate constraints. *Decision Sciences*, 8:156–166, 1977.
- [63] Fred Glover. Tabu search for nonlinear and parametric optimization (with links to genetic algorithms). *Discrete Applied Mathematics*, 49:231–255, 1994.
- [64] David E. Goldberg. *Genetic Algorithms in Search, Optimization and Machine Learning*. Addison-Wesley Publishing Company, Reading, Massachusetts, 1989.
- [65] David E. Goldberg and Jon Richardson. Genetic algorithm with sharing for multimodal function optimization. In John J. Grefenstette, editor, *Genetic Algorithms and Their Applications: Proceedings of the Second International Conference on Genetic Algorithms*, pages 41–49, Hillsdale, New Jersey, 1987. Lawrence Erlbaum.

- [66] Michael Guntsch and Martin Middendorf. Solving Multi-criteria Optimization Problems with Population-Based ACO. In Carlos M. Fonseca, Peter J. Fleming, Eckart Zitzler, Kalyanmoy Deb, and Lothar Thiele, editors, *Evolutionary Multi-Criterion Optimization. Second International Conference, EMO 2003*, pages 464–478, Faro, Portugal, April 2003. Springer. Lecture Notes in Computer Science. Volume 2632.
- [67] Himanshu Gupta and Kalyanmoy Deb. Handling Constraints in Robust Multi-Objective Optimization. In *2005 IEEE Congress on Evolutionary Computation (CEC'2005)*, volume 1, pages 25–32, Edinburgh, Scotland, September 2005. IEEE Service Center.
- [68] W. Habenicht. Quad trees: A data structure for discrete vector optimization problems. In *Lecture Notes in Economics and Mathematical Systems No. 209*, pages 136–145, 1982.
- [69] Ken Harada, Jun Sakuma, Isao Ono, and Shigenobu Kobayashi. Constraint-Handling Method for Multi-objective Function Optimization: Pareto Descent Repair Operator. In Shigeru Obayashi, Kalyanmoy Deb, Carlo Poloni, Tomoyuki Hiroyasu, and Tadahiko Murata, editors, *Evolutionary Multi-Criterion Optimization, 4th International Conference, EMO 2007*, pages 156–170, Matsushima, Japan, March 2007. Springer. Lecture Notes in Computer Science Vol. 4403.
- [70] Arturo Hernández Aguirre, Salvador Botello Rionda, Giovanni Lizárraga Lizárraga, and Carlos Coello Coello. IS-PAES: Multiobjective Optimization with Efficient Constraint Handling. In Tadeusz Burczyński and Andrzej Osyczka, editors, *IUTAM Symposium on Evolutionary Methods in Mechanics*, pages 111–120. Kluwer Academic Publishers, Dordrecht/Boston/London, 2004. ISBN 1-4020-2266-2.
- [71] Alfredo G. Hernández-Díaz, Luis V. Santana-Quintero, Carlos Coello Coello, Rafael Caballero, and Julián Molina. A New Proposal for Multi-Objective Optimization using Differential Evolution and Rough Sets Theory. In Maarten Keijzer et al., editor, *2006 Genetic and Evolutionary Computation Conference (GECCO'2006)*, volume 1, pages 675–682, Seattle, Washington, USA, July 2006. ACM Press. ISBN 1-59593-186-4.
- [72] Alfredo G. Hernández-Díaz, Luis V. Santana-Quintero, Carlos A. Coello Coello, and Julián Molina. Pareto-adaptive  $\epsilon$ -dominance. *Evolutionary Computation*, 15(4):493–517, Winter 2007.
- [73] Jeffrey Horn, Nicholas Nafpliotis, and David E. Goldberg. A Niche Pareto Genetic Algorithm for Multiobjective Optimization. In *Proceedings of the First IEEE Conference on Evolutionary Computation, IEEE World Congress on Computational Intelligence*, volume 1, pages 82–87, Piscataway, New Jersey, June 1994. IEEE Service Center.

- [74] Evan J. Hughes. Evolutionary Many-Objective Optimisation: Many Once or One Many? In *2005 IEEE Congress on Evolutionary Computation (CEC'2005)*, volume 1, pages 222–227, Edinburgh, Scotland, September 2005. IEEE Service Center.
- [75] Phil Husbans. Distributed Coevolutionary Genetic Algorithms for multi-Criteria and Multi-Constraint Optimisation. In Terence C. Fogarty, editor, *Evolutionary Computing. AIS Workshop. Selected Papers*, Lecture Notes in Computer Science Vol. 865, pages 150–165. Springer Verlag, April 1994.
- [76] Christian Igel, Nikolaus Hansen, and Stefan Roth. Covariance Matrix Adaptation for Multi-objective Optimization. *Evolutionary Computation*, 15(1):1–28, Spring 2007.
- [77] Christian Igel, Thorsten Suttrop, and Nikolaus Hansen. Steady-State Selection and Efficient Covariance Matrix Update in the Multi-objective CMA-ES. In Shigeru Obayashi, Kalyanmoy Deb, Carlo Poloni, Tomoyuki Hiroyasu, and Tadahiko Murata, editors, *Evolutionary Multi-Criterion Optimization, 4th International Conference, EMO 2007*, pages 171–185, Matshushima, Japan, March 2007. Springer. Lecture Notes in Computer Science Vol. 4403.
- [78] Steffen Iredi, Daniel Merkle, and Martin Middendorf. Bi-Criterion Optimization with Multi Colony Ant Algorithms. In Eckart Zitzler, Kalyanmoy Deb, Lothar Thiele, Carlos A. Coello Coello, and David Corne, editors, *First International Conference on Evolutionary Multi-Criterion Optimization*, pages 359–372. Springer-Verlag. Lecture Notes in Computer Science No. 1993, 2001.
- [79] Mikkel T. Jensen. Reducing the Run-Time Complexity of Multiobjective EAs: The NSGA-II and Other Algorithms. *IEEE Transactions on Evolutionary Computation*, 7(5):503–515, October 2003.
- [80] Yaochu Jin. A comprehensive survey of fitness approximation in evolutionary computation. *Soft Computing*, 9(1):3–12, 2005.
- [81] Yaochu Jin and Bernhard Sendhoff. Incorporation of Fuzzy Preferences into Evolutionary Multiobjective Optimization. In W.B. Langdon, E. Cantú-Paz, K. Mathias, R. Roy, D. Davis, R. Poli, K. Balakrishnan, V. Honavar, G. Rudolph, J. Wegener, L. Bull, M.A. Potter, A.C. Schultz, J.F. Miller, E. Burke, and N. Jonoska, editors, *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO'2002)*, page 683, San Francisco, California, July 2002. Morgan Kaufmann Publishers.
- [82] Nicolas Jozefowicz, Frédéric Semet, and El-Ghazali Talbi. Enhancements of NSGA II and Its Application to the Vehicle Routing Problem with Route Balancing. In El-Ghazali Talbi, Pierre Liardet, Pierre Collet, Evelyne Lutton, and Marc Schoenauer, editors, *Artificial Evolution, 7th International Conference, Evolution Artificielle, EA 2005*, pages 131–142. Springer. Lecture Notes in Computer Science Vol. 3871, Lille, France, October 2005.

- [83] Marios K. Karakasis and Kyriakos C. Giannakoglou. Metamodel-Assisted Multi-Objective Evolutionary Optimization. In R. Schilling, W. Haase, J. Periaux, H. Baier, and G. Bueda, editors, *EUROGEN 2005. Evolutionary Methods for Design, Optimization and Control with Applications to Industrial Problems*, Munich, Germany, 2005.
- [84] James Kennedy and Russell C. Eberhart. Particle Swarm Optimization. In *Proceedings of the 1995 IEEE International Conference on Neural Networks*, pages 1942–1948, Piscataway, New Jersey, 1995. IEEE Service Center.
- [85] James Kennedy and Russell C. Eberhart. A Discrete Binary Version of the Particle Swarm Algorithm. In *Proceedings of the 1997 IEEE Conference on Systems, Man, and Cybernetics*, pages 4104–4109, Piscataway, New Jersey, 1997. IEEE Service Center.
- [86] James Kennedy and Russell C. Eberhart. *Swarm Intelligence*. Morgan Kaufmann Publishers, San Francisco, California, 2001.
- [87] V. Khare, X. Yao, and K. Deb. Performance Scaling of Multi-objective Evolutionary Algorithms. In Carlos M. Fonseca, Peter J. Fleming, Eckart Zitzler, Kalyanmoy Deb, and Lothar Thiele, editors, *Evolutionary Multi-Criterion Optimization. Second International Conference, EMO 2003*, pages 376–390, Faro, Portugal, April 2003. Springer. Lecture Notes in Computer Science. Volume 2632.
- [88] Joshua Knowles. ParEGO: A Hybrid Algorithm With On-Line Landscape Approximation for Expensive Multiobjective Optimization Problems. *IEEE Transactions on Evolutionary Computation*, 10(1):50–66, February 2006.
- [89] Joshua Knowles and David Corne. Properties of an Adaptive Archiving Algorithm for Storing Nondominated Vectors. *IEEE Transactions on Evolutionary Computation*, 7(2):100–116, April 2003.
- [90] Joshua Knowles and David Corne. Memetic Algorithms for Multiobjective Optimization: Issues, Methods and Prospects. In William E. Hart, N. Krasnogor, and J.E. Smith, editors, *Recent Advances in Memetic Algorithms*, pages 313–352. Springer. Studies in Fuzziness and Soft Computing, Vol. 166, 2005.
- [91] Joshua Knowles and David Corne. Quantifying the Effects of Objective Space Dimension in Evolutionary Multiobjective Optimization. In Shigeru Obayashi, Kalyanmoy Deb, Carlo Poloni, Tomoyuki Hiroyasu, and Tadahiko Murata, editors, *Evolutionary Multi-Criterion Optimization, 4th International Conference, EMO 2007*, pages 757–771, Matshushima, Japan, March 2007. Springer. Lecture Notes in Computer Science Vol. 4403.
- [92] Joshua D. Knowles and David W. Corne. Approximating the Nondominated Front Using the Pareto Archived Evolution Strategy. *Evolutionary Computation*, 8(2):149–172, 2000.

- [93] Ikeda Kokolo, Kita Hajime, and Kobayashi Shigenobu. Failure of Pareto-based MOEAs: Does Non-dominated Really Mean Near to Optimal? In *Proceedings of the Congress on Evolutionary Computation 2001 (CEC'2001)*, volume 2, pages 957–962, Piscataway, New Jersey, May 2001. IEEE Service Center.
- [94] Mario Köppen and Kaori Yoshida. Substitute Distance Assignments in NSGA-II for Handling Many-Objective Optimization Problems. In Shigeru Obayashi, Kalyanmoy Deb, Carlo Poloni, Tomoyuki Hiroyasu, and Tadahiko Murata, editors, *Evolutionary Multi-Criterion Optimization, 4th International Conference, EMO 2007*, pages 727–741, Matshushima, Japan, March 2007. Springer. Lecture Notes in Computer Science Vol. 4403.
- [95] Kalmanje Krishnakumar. Micro-genetic algorithms for stationary and non-stationary function optimization. In *SPIE Proceedings: Intelligent Control and Adaptive Systems*, volume 1196, pages 289–296, 1989.
- [96] H.T. Kung, F. Luccio, and F.P. Preparata. On finding the maxima of a set of vectors. *Journal of the Association for Computing Machinery*, 22(4):469–476, 1975.
- [97] Manuel Laguna and Rafael Martí. *Scatter Search : Methodology and Implementations in C*. Kluwer Academic Publishers, 2003. ISBN 1-402-07376-3.
- [98] Ricardo Landa Becerra and Carlos A. Coello Coello. Solving Hard Multiobjective Optimization Problems Using  $\varepsilon$ -Constraint with Cultured Differential Evolution. In Thomas Philip Runarsson, Hans-Georg Beyer, Edmund Burke, Juan J. Merelo-Guervós, L. Darrell Whitley, and Xin Yao, editors, *Parallel Problem Solving from Nature - PPSN IX, 9th International Conference*, pages 543–552. Springer. Lecture Notes in Computer Science Vol. 4193, Reykjavik, Iceland, September 2006.
- [99] Ricardo Landa-Becerra, Luis V. Santana-Quintero, and Carlos A. Coello Coello. Knowledge Incorporation in Multi-Objective Evolutionary Algorithms. In Ashish Ghosh, Satchidananda Dehuri, and Susmita Ghosh, editors, *Multi-objective Evolutionary Algorithms for Knowledge Discovery from Data Bases*, pages 23–46. Springer, Berlin, 2008.
- [100] Marco Laumanns, Günter Rudolph, and Hans-Paul Schwefel. Mutation Control and Convergence in Evolutionary Multi-Objective Optimization. In *Proceedings of the 7th International Mendel Conference on Soft Computing (MENDEL 2001)*, Brno, Czech Republic, June 2001.
- [101] Marco Laumanns, Lothar Thiele, Kalyanmoy Deb, and Eckart Zitzler. On the Convergence and Diversity-Preservation Properties of Multi-Objective Evolutionary Algorithms. Technical Report 108, Computer Engineering and Networks Laboratory (TIK), Swiss Federal Institute of Technology (ETH) Zurich, Gloristrasse 35, CH-8092 Zurich, Switzerland, May 2001.

- [102] Marco Laumanns, Lothar Thiele, Kalyanmoy Deb, and Eckart Zitzler. Combining Convergence and Diversity in Evolutionary Multi-objective Optimization. *Evolutionary Computation*, 10(3):263–282, Fall 2002.
- [103] Marco Laumanns, Lothar Thiele, and Eckart Zitzler. Running Time Analysis of Evolutionary Algorithms on a Simplified Multiobjective Knapsack Problem. *Natural Computing*, 3(1):37–51, 2004.
- [104] Marco Laumanns, Lothar Thiele, and Eckart Zitzler. Running Time Analysis of Multiobjective Evolutionary Algorithms on Pseudo-Boolean Functions. *IEEE Transactions on Evolutionary Computation*, 8(2):170–182, April 2004.
- [105] Xiaodong Li. A Non-dominated Sorting Particle Swarm Optimizer for Multiobjective Optimization. In Erick Cantú-Paz et al., editor, *Genetic and Evolutionary Computation—GECCO 2003. Proceedings, Part I*, pages 37–48. Springer. Lecture Notes in Computer Science Vol. 2723, July 2003.
- [106] Antonio López Jaimes, Carlos A. Coello Coello, and Debrup Chakraborty. Objective Reduction Using a Feature Selection Technique. In *2008 Genetic and Evolutionary Computation Conference (GECCO'2008)*, pages 674–680, Atlanta, USA, July 2008. ACM Press. ISBN 978-1-60558-131-6.
- [107] Guan-Chun Luh and Chung-Huei Chueh. Multi-objective optimal design of truss structure with immune algorithm. *Computers and Structures*, 82:829–844, 2004.
- [108] Guan-Chun Luh, Chung-Huei Chueh, and Wei-Wen Liu. MOIA: Multi-Objective Immune Algorithm. *Engineering Optimization*, 35(2):143–164, April 2003.
- [109] Rafael Martí. Scatter Search—Wellsprings and Challenges. *European Journal of Operational Research*, 169:351–358, 2006.
- [110] Silja Meyer-Nieberg and Hans-Georg Beyer. Self-Adaptation in Evolutionary Algorithms. In Fernando G. Lobo, Cláudio F. Lima, and Zbigniew Michalewicz, editors, *Parameter Setting in Evolutionary Algorithms*, pages 47–75. Springer-Verlag, Berlin, 2007.
- [111] Efrén Mezura-Montes and Carlos A. Coello Coello. Constrained Optimization via Multiobjective Evolutionary Algorithms. In Joshua Knowles, David Corne, and Kalyanmoy Deb, editors, *Multi-Objective Problem Solving from Nature: From Concepts to Applications*, pages 53–75. Springer, Berlin, 2008. ISBN 978-3-540-72963-1.
- [112] Kaisa M. Miettinen. *Nonlinear Multiobjective Optimization*. Kluwer Academic Publishers, Boston, Massachusetts, 1999.
- [113] Sanaz Mostaghim and Jürgen Teich. The Role of  $\varepsilon$ -dominance in Multi Objective Particle Swarm Optimization Methods. In *Proceedings of the 2003 Congress on Evolutionary Computation (CEC'2003)*, volume 3, pages 1764–1771, Canberra, Australia, December 2003. IEEE Press.

- [114] Sanaz Mostaghim and Jürgen Teich. Strategies for Finding Good Local Guides in Multi-objective Particle Swarm Optimization (MOPSO). In *2003 IEEE Swarm Intelligence Symposium Proceedings*, pages 26–33, Indianapolis, Indiana, USA, April 2003. IEEE Service Center.
- [115] Sanaz Mostaghim, Jürgen Teich, and Ambrish Tyagi. Comparison of Data Structures for Storing Pareto-sets in MOEAs. In *Congress on Evolutionary Computation (CEC'2002)*, volume 1, pages 843–848, Piscataway, New Jersey, May 2002. IEEE Service Center.
- [116] Leandro N. de Castro and Jonathan Timmis. *An Introduction to Artificial Immune Systems: A New Computational Intelligence Paradigm*. Springer, London, 2002. ISBN 1-85233-594-7.
- [117] Antonio J. Nebro, Juan J. Durillo, Francisco Luna, Bernabé Dorronsoro, and Enrique Alba. A Cellular Genetic Algorithm for Multiobjective Optimization. In David A. Pelta and Natalio Krasnogor, editors, *Proceedings of the Workshop on Nature Inspired Cooperative Strategies for Optimization (NICSO 2006)*, pages 25–36, Granada, Spain, 2006.
- [118] Antonio J. Nebro, Juan J. Durillo, Francisco Luna, Bernabé Dorronsoro, and Enrique Alba. Design Issues in a Multiobjective Cellular Genetic Algorithm. In Shigeru Obayashi, Kalyanmoy Deb, Carlo Poloni, Tomoyuki Hiroyasu, and Tadahiko Murata, editors, *Evolutionary Multi-Criterion Optimization, 4th International Conference, EMO 2007*, pages 126–140, Matshushima, Japan, March 2007. Springer. Lecture Notes in Computer Science Vol. 4403.
- [119] Antonio J. Nebro, Francisco Luna, Enrique Alba, Bernabé Dorronsoro, Juan J. Durillo, and Andreas Beham. AbYSS: Adapting Scatter Search to Multiobjective Optimization. *IEEE Transactions on Evolutionary Computation*, 12(4):439–457, August 2008.
- [120] Yusuke Nojima, Kaname Narukawa, Shiori Kaige, and Hisao Ishibuchi. Effects of Removing Overlapping Solutions on the Performance of the NSGA-II Algorithm. In Carlos A. Coello Coello, Arturo Hernández Aguirre, and Eckart Zitzler, editors, *Evolutionary Multi-Criterion Optimization. Third International Conference, EMO 2005*, pages 341–354, Guanajuato, México, March 2005. Springer. Lecture Notes in Computer Science Vol. 3410.
- [121] Leandro Nunes de Castro and F. J. Von Zuben. Learning and Optimization Using the Clonal Selection Principle. *IEEE Transactions on Evolutionary Computation*, 6(3):239–251, 2002.
- [122] Andrzej Osyczka and Sourav Kundu. A Genetic Algorithm Approach to Multicriteria Network Optimization Problems. In *Proceedings of the 20th International Conference on Computers and Industrial Engineering*, pages 329–332, Kyongju, Korea, October 1996.

- [123] Akira Oyama, Koji Shimoyama, and Kozo Fujii. New constraint-handling method for multi-objective and multi-constraint evolutionary optimization. *Transactions of the Japan Society for Aeronautical and Space Sciences*, 50(167):56–62, May 2007.
- [124] Robic C. Purshouse and Peter J. Fleming. On the Evolutionary Optimization of Many Conflicting Objectives. *IEEE Transactions on Evolutionary Algorithms*, 11(6):770–784, December 2007.
- [125] Robin Charles Purshouse. *On the Evolutionary Optimisation of Many Objectives*. PhD thesis, Department of Automatic Control and Systems Engineering, The University of Sheffield, Sheffield, UK, September 2003.
- [126] Tapabrata Ray and Warren Smith. A surrogate assisted parallel multiobjective evolutionary algorithm for robust engineering design. *Engineering Optimization*, 38(8):997–1011, December 2006.
- [127] Margarita Reyes Sierra and Carlos A. Coello Coello. Improving PSO-Based Multi-objective Optimization Using Crowding, Mutation and  $\epsilon$ -Dominance. In Carlos A. Coello Coello, Arturo Hernández Aguirre, and Eckart Zitzler, editors, *Evolutionary Multi-Criterion Optimization. Third International Conference, EMO 2005*, pages 505–519, Guanajuato, México, March 2005. Springer. Lecture Notes in Computer Science Vol. 3410.
- [128] Margarita Reyes Sierra and Carlos A. Coello Coello. A Study of Fitness Inheritance and Approximation Techniques for Multi-Objective Particle Swarm Optimization. In *2005 IEEE Congress on Evolutionary Computation (CEC'2005)*, volume 1, pages 65–72, Edinburgh, Scotland, September 2005. IEEE Service Center.
- [129] Margarita Reyes-Sierra and Carlos A. Coello Coello. Multi-Objective Particle Swarm Optimizers: A Survey of the State-of-the-Art. *International Journal of Computational Intelligence Research*, 2(3):287–308, 2006.
- [130] Robert G. Reynolds, Zbigniew Michalewicz, and M. Cavaretta. Using cultural algorithms for constraint handling in GENOCOP. In J. R. McDonnell, R. G. Reynolds, and D. B. Fogel, editors, *Proceedings of the Fourth Annual Conference on Evolutionary Programming*, pages 298–305. MIT Press, Cambridge, Massachusetts, 1995.
- [131] Greg Rohling. *Multiple Objective Evolutionary Algorithms for Independent, Computationally Expensive Objective Evaluations*. PhD thesis, School of Electrical and Computer Engineering, November 2004.
- [132] Carlos Eduardo Mariano Romero and Eduardo Morales Manzanares. MOAQ an Ant-Q Algorithm for Multiple Objective Optimization Problems. In W. Banzhaf, J. Daida, A. E. Eiben, M. H. Garzon, V. Honavar, M. Jakiela, and R. E. Smith, editors, *Genetic and Evolutionary Computing Conference (GECCO 99)*, volume 1, pages 894–901, San Francisco, California, July 1999. Morgan Kaufmann.

- [133] Luis V. Santana-Quintero, Noel Ramírez, and Carlos Coello Coello. A Multi-objective Particle Swarm Optimizer Hybridized with Scatter Search. In Alexander Gelbukh and Carlos Alberto Reyes-Garcia, editors, *MICAI 2006: Advances in Artificial Intelligence, 5th Mexican International Conference on Artificial Intelligence*, pages 294–304. Springer, Lecture Notes in Artificial Intelligence Vol. 4293, Apizaco, Mexico, November 2006.
- [134] Luis Vicente Santana-Quintero and Carlos A. Coello Coello. An Algorithm Based on Differential Evolution for Multi-Objective Problems. *International Journal of Computational Intelligence Research*, 1(2):151–169, 2005.
- [135] Dhish Kumar Saxena and Kalyanmoy Deb. Non-linear Dimensionality Reduction Procedures for Certain Large-Dimensional Multi-objective Optimization Problems: Employing Correntropy and a Novel Maximum Variance Unfolding. In Shigeru Obayashi, Kalyanmoy Deb, Carlo Poloni, Tomoyuki Hiroyasu, and Tadahiko Murata, editors, *Evolutionary Multi-Criterion Optimization, 4th International Conference, EMO 2007*, pages 772–787, Matshushima, Japan, March 2007. Springer. Lecture Notes in Computer Science Vol. 4403.
- [136] J. David Schaffer. *Multiple Objective Optimization with Vector Evaluated Genetic Algorithms*. PhD thesis, Vanderbilt University, 1984.
- [137] J. David Schaffer. Multiple Objective Optimization with Vector Evaluated Genetic Algorithms. In *Genetic Algorithms and their Applications: Proceedings of the First International Conference on Genetic Algorithms*, pages 93–100. Lawrence Erlbaum, 1985.
- [138] Oliver Schuetze, Marco Laumanns, Emilia Tantar, Carlos A. Coello Coello, and El ghazali Talbi. Convergence of Stochastic Search Algorithms to Gap-Free Pareto Front Approximations. In Dirk Thierens, editor, *2007 Genetic and Evolutionary Computation Conference (GECCO'2007)*, volume 1, pages 892–899, London, UK, July 2007. ACM Press.
- [139] Oliver Schütze. A New Data Structure for the Nondominance problem in Multi-objective Optimization. In Carlos M. Fonseca, Peter J. Fleming, Eckart Zitzler, Kalyanmoy Deb, and Lothar Thiele, editors, *Evolutionary Multi-Criterion Optimization. Second International Conference, EMO 2003*, pages 509–518, Faro, Portugal, April 2003. Springer. Lecture Notes in Computer Science. Volume 2632.
- [140] M. Sefrioui and J. Periaux. Nash Genetic Algorithms: examples and applications. In *2000 Congress on Evolutionary Computation*, volume 1, pages 509–516, San Diego, California, July 2000. IEEE Service Center.
- [141] Robert E. Smith, B. A. Dike, and S. A. Stegmann. Fitness Inheritance in Genetic Algorithms. In *Proceedings of the 1995 ACM Symposium on Applied Computing*, pages 345–350, Nashville, Tennessee, USA, 1995. ACM Press.

- [142] N. Srinivas and Kalyanmoy Deb. Multiobjective Optimization Using Nondominated Sorting in Genetic Algorithms. *Evolutionary Computation*, 2(3):221–248, Fall 1994.
- [143] Dipti Srinivasan and Tian Hou Seow. Particle Swarm Inspired Evolutionary Algorithm (PS-EA) for Multi-Criteria Optimization Problems. In Ajith Abraham, Lakhmi Jain, and Robert Goldberg, editors, *Evolutionary Multiobjective Optimization: Theoretical Advances And Applications*, pages 147–165. Springer-Verlag, London, 2005. ISBN 1-85233-787-7.
- [144] André Süßflow, Nicole Drechsler, and Rolf Drechsler. Robust Multi-objective Optimization in High Dimensional Spaces. In Shigeru Obayashi, Kalyanmoy Deb, Carlo Poloni, Tomoyuki Hiroyasu, and Tadahiko Murata, editors, *Evolutionary Multi-Criterion Optimization, 4th International Conference, EMO 2007*, pages 715–726, Matshushima, Japan, March 2007. Springer. Lecture Notes in Computer Science Vol. 4403.
- [145] Patrick D. Surry and Nicholas J. Radcliffe. The COMOGA Method: Constrained Optimisation by Multiobjective Genetic Algorithms. *Control and Cybernetics*, 26(3):391–412, 1997.
- [146] K.C. Tan, T.H. Lee, and E.F. Khor. Evolutionary Algorithms with Dynamic Population Size and Local Exploration for Multiobjective Optimization. *IEEE Transactions on Evolutionary Computation*, 5(6):565–588, December 2001.
- [147] R. Tavakkoli-Moghaddam, A.R. Rahimi-Vahed, and A.H. Mirzaei. Solving a multi-objective no-wait flow shop scheduling problem with an immune algorithm. *International Journal of Advanced Manufacturing Technology*, 36(9–10):969–981, April 2008.
- [148] Reza Tavakkoli-Moghaddam, Alireza Rahimi-Vahed, and Ali Hossein Mirzaei. A hybrid multi-objective immune algorithm for a flow shop scheduling problem with bi-objectives: Weighted mean completion time and weighted mean tardiness. *Information Sciences*, 177(22):5072–5090, November 15 2007.
- [149] Gregorio Toscano Pulido. *On the Use of Self-Adaptation and Elitism for Multi-objective Particle Swarm Optimization*. PhD thesis, Computer Science Section, Department of Electrical Engineering, CINVESTAV-IPN, Mexico, September 2005.
- [150] Gregorio Toscano Pulido and Carlos A. Coello Coello. The Micro Genetic Algorithm 2: Towards Online Adaptation in Evolutionary Multiobjective Optimization. In Carlos M. Fonseca, Peter J. Fleming, Eckart Zitzler, Kalyanmoy Deb, and Lothar Thiele, editors, *Evolutionary Multi-Criterion Optimization. Second International Conference, EMO 2003*, pages 252–266, Faro, Portugal, April 2003. Springer. Lecture Notes in Computer Science. Volume 2632.

- [151] Gregorio Toscano-Pulido, Carlos A. Coello Coello, and Luis Vicente Santana-Quintero. EMOPSO: A Multi-Objective Particle Swarm Optimizer with Emphasis on Efficiency. In Shigeru Obayashi, Kalyanmoy Deb, Carlo Poloni, Tomoyuki Hiroyasu, and Tadahiko Murata, editors, *Evolutionary Multi-Criterion Optimization, 4th International Conference, EMO 2007*, pages 272–285, Matsushima, Japan, March 2007. Springer. Lecture Notes in Computer Science Vol. 4403.
- [152] Mario Villalobos-Arias, Carlos A. Coello Coello, and Onésimo Hernández-Lerma. Asymptotic Convergence of Metaheuristics for Multiobjective Optimization Problems. *Soft Computing*, 10(11):1001–1005, September 2006.
- [153] Mario Alberto Villalobos-Arias, Gregorio Toscano Pulido, and Carlos A. Coello Coello. A Proposal to Use Stripes to Maintain Diversity in a Multi-Objective Particle Swarm Optimizer. In *2005 IEEE Swarm Intelligence Symposium (SIS'05)*, pages 22–29, Pasadena, California, USA, June 2005. IEEE Press.
- [154] Ivan Voutchkov and A.J. Keane. Multiobjective Optimization using Surrogates. In I.C. Parmee, editor, *Adaptive Computing in Design and Manufacture 2006. Proceedings of the Seventh International Conference*, pages 167–175, Bristol, UK, April 2006. The Institute for People-centred Computation.
- [155] Tobias Wagner, Nicola Beume, and Boris Naujoks. Pareto-, Aggregation-, and Indicator-Based Methods in Many-Objective Optimization. In Shigeru Obayashi, Kalyanmoy Deb, Carlo Poloni, Tomoyuki Hiroyasu, and Tadahiko Murata, editors, *Evolutionary Multi-Criterion Optimization, 4th International Conference, EMO 2007*, pages 742–756, Matsushima, Japan, March 2007. Springer. Lecture Notes in Computer Science Vol. 4403.
- [156] Jiachuan Wang and Janis P. Terpenney. Interactive Preference Incorporation in Evolutionary Engineering Design. In Yaochu Jin, editor, *Knowledge Incorporation in Evolutionary Computation*, pages 525–543. Springer, Berlin Heidelberg, 2005. ISBN 3-540-22902-7.
- [157] Elizabeth F. Wanner, Frederico G. Guimaraes, Ricardo H.C. Takahashi, and Peter J. Fleming. Local Search with Quadratic Approximations into Memetic Algorithms for Optimization with Multiple Criteria. *Evolutionary Computation*, 16(2):185–224, Summer 2008.
- [158] Yomas G. Woldesembet, Biruk G. Tessema, and Gary G. Yen. Constraint handling in multi-objective evolutionary optimization. In *2007 IEEE Congress on Evolutionary Computation (CEC'2007)*, pages 3077–3084, Singapore, September 2007. IEEE Press.
- [159] Kok Sung Won and Tapabrata Ray. Performance of Kriging and Cokriging based Surrogate Models within the Unified Framework for Surrogate Assisted Optimization. In *2004 Congress on Evolutionary Computation (CEC'2004)*, volume 2, pages 1577–1585, Portland, Oregon, USA, June 2004. IEEE Service Center.

- [160] Wang Yong and Cai Zixing. A Constrained Optimization Evolutionary Algorithm Based on Multiobjective Optimization Techniques. In *2005 IEEE Congress on Evolutionary Computation (CEC'2005)*, volume 2, pages 1081–1087, Edinburgh, Scotland, September 2005. IEEE Service Center.
- [161] Michael A. Yukish. *Algorithms to Identify Pareto Points in Multi-Dimensional Data Sets*. PhD thesis, College of Engineering, Pennsylvania State University, USA, August 2004.
- [162] R. Romero Zaliz, I. Zwir, and E. Ruspini. Generalized Analysis of Promoters: A Method for DNA Sequence Description. In Carlos A. Coello Coello and Gary B. Lamont, editors, *Applications of Multi-Objective Evolutionary Algorithms*, pages 427–449. World Scientific, Singapore, 2004.
- [163] Xiangrong Zhang, Bin Lu, Shuiping Gou, and Licheng Jiao. Immune Multiobjective Optimization Algorithm Using Unsupervised Feature Selection. In Franz Rothlauf et al., editor, *Applications of Evolutionary Computing. EvoWorkshops 2006: EvoBIO, EvoCOMNET, EvoHOT, EvoIASP, EvoINTERACTION, EvoMUSART, and EvoSTOC*, pages 484–494, Budapest, Hungary, April 2006. Springer, Lecture Notes in Computer Science Vol. 3907.
- [164] Zhong-Yao Zhu and Kwong-Sak Leung. Asynchronous Self-Adjustable Island Genetic Algorithm for Multi-Objective Optimization Problems. In *Congress on Evolutionary Computation (CEC'2002)*, volume 1, pages 837–842, Piscataway, New Jersey, May 2002. IEEE Service Center.
- [165] Eckart Zitzler. *Evolutionary Algorithms for Multiobjective Optimization: Methods and Applications*. PhD thesis, Swiss Federal Institute of Technology (ETH), Zurich, Switzerland, November 1999.
- [166] Eckart Zitzler and Simon Künzli. Indicator-based Selection in Multiobjective Search. In Xin Yao et al., editor, *Parallel Problem Solving from Nature - PPSN VIII*, pages 832–842, Birmingham, UK, September 2004. Springer-Verlag. Lecture Notes in Computer Science Vol. 3242.
- [167] Eckart Zitzler, Marco Laumanns, and Lothar Thiele. SPEA2: Improving the Strength Pareto Evolutionary Algorithm. In K. Giannakoglou, D. Tsahalis, J. Periaux, P. Papailou, and T. Fogarty, editors, *EUROGEN 2001. Evolutionary Methods for Design, Optimization and Control with Applications to Industrial Problems*, pages 95–100, Athens, Greece, 2002.
- [168] Eckart Zitzler and Lothar Thiele. Multiobjective Optimization Using Evolutionary Algorithms—A Comparative Study. In A. E. Eiben, editor, *Parallel Problem Solving from Nature V*, pages 292–301, Amsterdam, September 1998. Springer-Verlag.
- [169] Eckart Zitzler and Lothar Thiele. Multiobjective Evolutionary Algorithms: A Comparative Case Study and the Strength Pareto Approach. *IEEE Transactions on Evolutionary Computation*, 3(4):257–271, November 1999.

- [170] Eckart Zitzler, Lothar Thiele, and Johannes Bader. SPAM: Set Preference Algorithm for Multiobjective Optimization. In Günter Rudolph, Thomas Jansen, Simon Lucas, Carlo Poloni, and Nicola Beume, editors, *Parallel Problem Solving from Nature–PPSN X*, pages 847–858. Springer. Lecture Notes in Computer Science Vol. 5199, Dortmund, Alemania, September 2008.
- [171] Eckart Zitzler, Lothar Thiele, Marco Laumanns, Carlos M. Fonseca, and Viviane Grunert da Fonseca. Performance Assessment of Multiobjective Optimizers: An Analysis and Review. *IEEE Transactions on Evolutionary Computation*, 7(2):117–132, April 2003.