Fundamentals of Evolutionary Multi-Objective Optimization

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Chapter 1

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The solution of optimization problems having two or more (often conflicting) criteria has become relatively common in a wide variety of application areas. Such problems are called "multi-objective" and their solution has raised an important amount of research within Operations Research, particularly in the last 35 years [52]. In spite of the large number of mathematical programming methods available for solving multi-objective optimization problems, such methods tend to have a rather limited applicability (e.g., when dealing with differentiable objective functions, or with convex Pareto fronts). This has motivated the use of alternative solution approaches such as evolutionary algorithms.

The use of evolutionary algorithms for solving multi-objective optimization problems was originally hinted at the late 1960s [65], but the first actual implementation of a multi-objective evolutionary algorithm (MOEA) was not produced until 1985 [68]. However, this area, which is now called "evolutionary multi-objective optimization," or EMO) has experienced a very important growth, mainly in the last 15 years [8, 15].

This chapter presents a basic introduction to EMO, focusing on its main concepts, the most popular algorithms in current use, and some of its applications. The remainder of this chapter is organized as follows. In Section 1.1, we provide some basic concepts from multi-objective optimization. The use of evolutionary algorithms in multi-objective optimization is motivated in Section 1.2. Some of the main topics of research which are currently attracting a lot of attention in the EMO field are briefly discussed in Section 1.3. A set of sample applications of MOEAs is provided in Section 1.4. Some of the main topics of research in the EMO field that currently attract a lot of attention are briefly discussed in Section 1.5. Finally, some conclusions are provided in Section 1.6.

1.1 Basic Concepts

We are interested in the solution of multi-objective optimization problems (MOPs) of the form:

$$\text{minimize } [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})] \tag{1.1}$$

subject to the m inequality constraints:

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \ldots, m \tag{1.2}$$

and the p equality constraints:

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \ldots, p \tag{1.3}$$

where k is the number of objective functions $f_i : \mathbb{R}^n \to \mathbb{R}$. We call $\vec{x} = [x_1, x_2, \ldots, x_n]^T$ the vector of decision variables. We wish to determine from among the set $\mathcal{F}$ of all vectors which satisfy (1.2) and (1.3) the particular set of values $x_1^*, x_2^*, \ldots, x_n^*$ which yield the optimum values of all the objective functions.

1.1.1 Pareto optimality

It is rarely the case that there is a single point that simultaneously optimizes all the objective functions.[1] Therefore, we normally look for "trade-offs", rather than single solutions when dealing with multi-objective optimization problems. The notion of "optimality" normally adopted in this case is the one originally proposed by Francis Ysidro Edgeworth [20] and later generalized by Vilfredo Pareto [57]. Although some authors call this notion *Edgeworth-Pareto optimality*, we will use the most commonly adopted term: *Pareto optimality*.

We say that a vector of decision variables $\vec{x}^* \in \mathcal{F}$ is *Pareto optimal* if there does not exist another $\vec{x} \in \mathcal{F}$ such that $f_i(\vec{x}) \leq f_i(\vec{x}^*)$ for all $i = 1, \ldots, k$ and $f_j(\vec{x}) < f_j(\vec{x}^*)$ for at least one j (assuming minimization).

In words, this definition says that $\vec{x}^*$ is Pareto optimal if there exists no feasible vector of decision variables $\vec{x} \in \mathcal{F}$ which would decrease some criterion without causing a simultaneous increase in at least one other criterion. It is worth noting that the use of this concept normally produces a set of solutions called the *Pareto optimal set*. The vectors $\vec{x}^*$ correspoding to the solutions included in the Pareto optimal set are called *nondominated*. The image of the Pareto optimal set under the objective functions is called *Pareto front*.

1.2 Use of Evolutionary Algorithms

The idea of using techniques based on the emulation of the mechanism of natural selection (described in Darwin's evolutionary theory) to solve problems can be traced back to the early 1930s [25]. However, it was not until the 1960s that the three main techniques based on this notion were developed: genetic algorithms [35], evolution strategies [70] and evolutionary programming [26]. These approaches, which are now collectively denominated "evolutionary algorithms," have been very effective for single-objective optimization [30, 71, 27].

[1] In fact, this situation only arises when there is no conflict among the objectives, which would make unnecessary the development of special solution methods, since this single solution could be reached after the sequential optimization of all the objectives, considered separately.

The basic operation of an evolutionary algorithm (EA) is the following. First, they generate a set of possible solutions (called a "population") to the problem at hand. Such a population is normally generated in a random manner. Each solution in the population (called an "individual") encodes all the decision variables of the problem. In order to assess their suitability, a fitness function must be defined. Such a fitness function is a variation of the objective function of the problem that we wish to solve. Then, a selection mechanism must be applied in order to decide which individuals will "mate." This selection process is normally based on the fitness contribution of each individual (i.e., the fittest individuals have a higher probability of being selected). Upon mating, a set of "offspring" are generated. Such offspring are "mutated" (this operator produces a small random change, with a low probability, on the contents of an individual), and constitute the population to be evaluated at the following iteration (called a "generation"). This process is repeated until reaching a stopping condition (normally, a maximum number of generations).

EAs are considered a good choice for solving multi-objective optimization problems because they adopt a population of solutions, which allows them (if properly manipulated) to find several elements of the Pareto optimal set in a single run. This contrasts with mathematical programming methods, which normally generate a single nondominated solution per execution. Additionally, EAs tend to be less susceptible to the discontinuity and the shape of the Pareto front, which is an important advantage over traditional mathematical programming methods [21].

Multi-objective Evolutionary Algorithms (MOEAs) extend a traditional evolutionary algorithm in two main aspects:

- The selection mechanism. In this case, the aim is to select nondominated solutions, and to consider all the nondominated solutions in a population as equally good.
- A diversity maintenance mechanism. This is necessary to avoid convergence to a single solution, which is something that will eventually happen with an EA (because of stochastic noise) if run for a sufficiently long time.

Regarding selection, although in their early days, several MOEAs relied on aggregating functions [34] and relatively simple population-based approaches [68], today, most of them adopt some form of *Pareto ranking*. This approach was originally proposed by David E. Goldberg [30], and it sorts the population of an EA based on Pareto dominance, such that all nondominated individuals are assigned the same rank (or importance). The aim is that all nondominated individuals get the same probability of being selected, and that such probability is higher than the one corresponding to individuals which are dominated. Although conceptually simple, this sort of selection mechanism allows for a wide variety of possible implementations [8, 15].

A number of methods have been proposed in the literature to maintain diversity in an EA. Such approaches include fitness sharing and niching [32, 16], clustering [78, 84], geographically-based schemes [42], and the use of entropy [39, 12], among others. Additionally, some researchers have proposed the use of mating restriction schemes [72, 84]. Furthermore, the use of relaxed forms of Pareto dominance has also become relatively popular in recent years, mainly as an archiving technique which encourages diversity, while allowing the archive to regulate convergence (see for example, ϵ-dominance [45]).

A third component of modern MOEAs is elitism, which normally consists of using an external archive (called a "secondary population") that can (or cannot) interact in different ways with the main (or "primary") population of the MOEA. The main purpose of this archive is to store all the nondominated solutions generated throughout the search process, while removing those that become dominated later in the search (called local nondominated solutions). The approximation of the Pareto optimal set produced by a MOEA is thus the final contents of this archive.

1.3 Multi-Objective Evolutionary Algorithms

Despite the considerable volume of literature on MOEAs that is currently available[2], very few algorithms are used by a significant number of researchers around the world. The following are, from the author's perspective, the most representative MOEAs in current use:

[2]The author maintains the EMOO repository, which, as of December 2008, contains over 3,600 bibliographic references on evolutionary multi-objective optimization. The EMOO repository is available at: `http://delta.cs.cinvestav.mx/~ccoello/EMOO/`

1. **Strength Pareto Evolutionary Algorithm** (SPEA): This MOEA was conceived as the merge of several algorithms developed during the 1990s [84]. It adopts an external archive (called the external nondominated set), which stores the nondominated solutions previously generated, and participates in the selection process (together with the main population). For each individual in this archive, a *strength* value is computed. This strength is proportional to the number of solutions which a certain individual dominates. In SPEA, the fitness of each member of the current population is computed according to the strengths of all external nondominated solutions that dominate it. Since the external nondominated set can grow too much, this could reduce the selection process and could slow down the search. In order to avoid this, SPEA adopts a technique that prunes the contents of the external nondominated set so that its size remains below a certain (pre-defined) threshold. For that sake, the authors use a clustering technique.

2. **Strength Pareto Evolutionary Algorithm 2** (SPEA2): This approach has three main differences with respect to its predecessor [83]: (1) it incorporates a fine-grained fitness assignment strategy which takes into account for each individual the number of individuals that dominate it and the number of individuals by which it is dominated; (2) it uses a nearest neighbor density estimation technique which guides the search more efficiently, and (3) it has an enhanced archive truncation method that guarantees the preservation of boundary solutions.

3. **Pareto Archived Evolution Strategy** (PAES): This is perhaps the most simple MOEA than one can conceive, and was introduced by Knowles and Corne [44]. It consists of a (1+1) evolution strategy (i.e., a single parent that generates a single offspring) in combination with a historical archive that stores the nondominated solutions previously found. This archive is used as a reference set against which each mutated individual is being compared. Such (external) archive adopts a crowding procedure that divides objective function space in a recursive manner. Then, each solution is placed in a certain grid location based on the values of its objectives (which are used as its "coordinates" or "geographical location"). A map of such a grid is maintained, indicating the number of solutions that reside in each grid location. When a new nondominated solution is ready to be stored in the archive, but there is no room for them

(the size of the external archive is bounded), a check is made on the grid location to which the solution would belong. If this grid location is less densely populated than the most densely populated grid location, then a solution (randomly chosen) from this heavily populated grid location is deleted to allow the storage of the newcomer. This aims to redistribute solutions, favoring the less densely populated regions of the Pareto front. Since the procedure is adaptive, no extra parameters are required (except for the number of divisions of the objective space).

4. **Nondominated Sorting Genetic Algorithm II** (NSGA-II): This is a heavily revised version of the Nondominated Sorting Genetic Algorithm (NSGA), which was introduced in the mid 1990s [74]. The NSGA-II adopts a more efficient ranking procedure than its predecessor. Additionally, it estimates the density of solutions surrounding a particular solution in the population by computing the average distance of two points on either side of this point along each of the objectives of the problem. This value is the so-called *crowding distance*. During selection, the NSGA-II uses a crowded-comparison operator which takes into consideration both the nondomination rank of an individual in the population and its crowding distance (i.e., nondominated solutions are preferred over dominated solutions, but between two solutions with the same nondomination rank, the one that resides in the less crowded region is preferred). The NSGA-II does not use an external archive as most of the modern MOEAs in current use. Instead, the elitist mechanism of the NSGA-II consists of combining the best parents with the best offspring obtained (i.e., a $(\mu+\lambda)$-selection). Due to its clever mechanisms, the NSGA-II is much more efficient (computationally speaking) than its predecessor, and its performance is so good that it has become very popular in the last few years, triggering a significant number of applications, and becoming some sort of landmark against which new MOEAs have to be compared in order to merit publication.

5. **Pareto Envelope-based Selection Algorithm** (PESA): This algorithm was proposed by Corne et al. [11], and uses a small internal population and a larger external (or secondary) population. PESA adopts the same adaptive grid from PAES to maintain diversity. However, its selection mechanism is based on the crowding measure used by

the aformentioned grid. This same crowding measure is used to decide what solutions to introduce into the external population (i.e., the archive of nondominated vectors found along the evolutionary process). Therefore, in PESA, the external memory plays a crucial role in the algorithm since it determines not only the diversity scheme, but also the selection performed by the method. There is also a revised version of this algorithm, called PESA-II [10], which is identical to PESA, except for the fact that region-based selection is used in this case. In region-based selection, the unit of selection is a hyperbox rather than an individual. The procedure consists of selecting (using any of the traditional selection techniques [31]) a hyperbox and then randomly selecting an individual within such hyperbox. The main motivation of this approach is to reduce the computational costs associated with traditional MOEAs (i.e., those based on Pareto ranking).

Many other MOEAs have been proposed in the specialized literature (see for example [9, 81, 17]), but they will not be discussed here due to obvious space limitations. A more interesting issue, however, is to devise which sort of MOEA will become predominant in the next few years. Efficiency is, for example, a concern nowadays, and several approaches have been developed in order to improve the efficiency of MOEAs (see for example [37, 41]). There is also an interesting trend consisting on designing MOEAs based on a performance measure (see for example [82, 3]). However, no clear trend exists today, from the author's perspective, that seems to attract the interest of a significant portion of the EMO community, regarding algorithmic design.

1.4 Applications

Today, there exists a very important volume of applications of MOEAs in a wide variety of domains. Next, we will provide a brief list of sample applications classified in three large groups: engineering, industrial and scientific. Specific areas within each of these large groups are also identified.

By far, engineering applications are the most popular in the current EMO literature. This is not surprising if we consider that engineering disciplines normally have problems with better understood mathematical models, which facilitates the use of MOEAs. A representative sample of engineering applications is the following:

- Electrical engineering [63, 1]
- Hydraulic engineering [62, 51]
- Structural engineering [56, 58]
- Aeronautical engineering [38, 47]
- Robotics [2, 77]
- Control [5, 6]
- Telecommunications [59, 75]
- Civil engineering [22, 36]
- Transport engineering [50, 73]

Industrial applications are the second most popular in the EMO literature. A representative sample of industrial applications of MOEAs is the following:

- Design and manufacture [33, 19]
- Scheduling [40, 29]
- Management [53, 64]

Finally, there are several EMO papers devoted to scientific applications. For obvious reasons, computer science applications are the most popular in the EMO literature. A representative sample of scientific applications is the following:

- Chemistry [60, 76]

- Physics [67, 61]
- Medicine [28, 24]
- Computer science [14, 48]

This sample of applications should give at least a rough idea of the increasing interest of researchers for adopting MOEAs in practically all types of disciplines.

1.5 Current Challenges

The existence of challenging, but solvable problems, is a key issue to preserve the interest in a research discipline. Although EMO is a discipline in which a very important amount of research has been conducted, mainly within the last ten years, several interesting problems still remain open. Additionally, the research conducted so far has also led to new, and intriguing topics. The following is a small sample of open problems that currently attract a significant amount of research within EMO:

- **Scalability**: In spite of the popularity of MOEAs in a plethora of applications, it is known that Pareto ranking is doomed to fail as we increase the number of objectives, and it is also known that with about ten objectives, it behaves like random sampling [43]. The reason is that most of the individuals in a population will become non-dominated, as the number of objectives increases. In order to deal with this problem, researchers have proposed selection schemes different from Pareto ranking [23, 18], as well as mechanisms that allow to reduce the number of objectives of a problem [4, 49]). However, there is still a lot of work to be done in this regard, and this is currently a very active research area.

- **Incorporation of user's preferences**: It is normally the case, that the user does not need the entire Pareto front of a problem, but only a certain portion of it. For example, solutions lying at the extreme parts of the Pareto front are unlikely necessary since they represent the best value for one objective, but the worst for the others. Thus, if the user has at least a rough idea of the sort of trade-offs that aims to find it is desirable to be able to explore in more detail only the nondominated solutions within the neighborhood of such trade-offs. This is possible, if we use, for example, biased versions of Pareto ranking [13] or some multi-criteria decision making technique, from the many developed in Operations Research [7]. Nevertheless, this area has not been very actively pursued by EMO researchers, in spite of its usefulness.

- **Parallelism**: Although the use of parallel MOEAs is relatively common in certain disciplines such as aeronautical engineering [55], the lack of serious research in this area is remarkable [8, 79]. Thus, it is expected to see much more research around this topic in the next few years, for example, related to algorithmic design, the role of local search in parallel MOEAs and convergence analysis, among others.

- **Theoretical Foundations**: Although an important effort has been made in recent years to develop theoretical work related to MOEAs, in areas such as convergence [66, 80], archiving [69], algorithm complexity [54], and run-time analysis [46], a lot of work still remains to be done in this regard.

1.6 Conclusions

In this chapter, we have provided some basic concepts related to evolutionary multi-objective optimization, as well as a short description of the main multi-objective evolutionary algorithms in current use. The main application areas of such algorithms have also been included, in order to provide a better idea of their wide applicability and of the increasing interest to use them.

In the last part of the chapter, we provided a short discussion of some challenging topics that are currently very active within this research area. The main objective of this chapter is to serve as a general (although brief) overview of the EMO field. Its main aim is to motivate researchers and newcomers from different areas, who have to deal with multi-objective problems, to consider MOEAs as a viable choice.

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