Constraint-handling through a multiobjective optimization technique

Carlos A. Coello Coello*
Laboratorio Nacional de Informática Avanzada
Rébsamen 80, A.P. 696
Xalapa, Veracruz, México 91090
e-mail: ccoello@xalapa.lania.mx
Phone: +52 (28) 18-13-02
Fax: +52 (28) 18-15-08

SUMMARY

Even when genetic algorithms (GAs) have been quite successful in a wide range of applications [6, 1], their use in constrained optimization problems raises several issues to which a considerable amount of research has been devoted in the last few years. From these issues, one of the most important ones is how to incorporate constraints of any sort (linear, non-linear, equality or inequality) into the fitness function as to guide the search properly. Due to the nature of the problems for which the GA is more suitable, it is normally quite difficult (or even impossible) to know the shape of the search space, and therefore is not easy to produce special operators and/or to explore it efficiently, unless we severely constraint the range of applications for which such approach will be useful.

For several years, practitioners have used penalty functions to incorporate constraints (particularly inequality constraints) into the fitness function, and there have been a lot of successful applications of this approach in all engineering fields [8]. However, penalty functions have some well-known limitations [9], from which the most remarkable is the difficulty to define good penalty factors. These penalty factors are normally generated by trial and error, although their definition may severely affect the results produced by the GA [9].

The idea of using multiobjective optimization techniques to handle constraints is not new, since there are at least three approaches reported in the literature since 1994 [7, 11, 2]. The main idea is to redefine the single-objective optimization of f as a multiobjective optimization problem in which we will have $m+1$ objectives, where m is the number of constraints. Then, we can apply any multiobjective optimization technique [4, 5] to the new vector $\bar{v} = (f, f_1, \ldots, f_m)$, where $f_1, \ldots, f_m$ are the original constraints of the problem. An ideal solution $\mathbf{X}$ would thus have $f_i(\mathbf{X})$=0 for $1 \leq i \leq m$ and $f(\mathbf{X}) \leq \mathbf{f}(\mathbf{Y})$ for all feasible $\mathbf{Y}$ (assuming minimization).

The use of an evolutionary multiobjective optimization technique in this domain is not straightforward, because the number of objectives increases as we increase the number of constraints and there are not many evolutionary multiobjective optimization techniques reported in the literature that have been actually tested with more than a few objectives (normally no more than 5) [3]. Furthermore, the minimization of constraint violation while minimizing the value of the objective function is not as simple as it might seem. For example, if we concentrate first in just finding a feasible solution so that we can later concentrate in optimizing the objective function, then we would be sampling points in the feasible space at random and it would be later very difficult to approach the region where the optimum resides.

We have recently proposed a population-based approach similar to VEGA to handle constraints [4]. This technique does not use dominance to impose an order on the constraints based on their violation (like in the case of COMOGA [11]) which is a more expensive process (in terms of CPU time) that also requires additional parameters. The proposed approach does not rank individuals, but it uses instead different fitness functions for each of the sub-population allocated (whose number depends on the number of constraints) depending on the feasibility of the individuals contained within each of them. This is easier to implement, does not require special operators to preserve feasiblity (like in the case of Parmee and Purchase's approach [7]), makes unnecessary the use of a sharing function to preserve diversity (like with traditional multiobjective optimization techniques), and does not require extra parameters to control the mixture of fea-

*Please send all correspondence to: PO Box 60326-394, Houston, Texas 77205.

sible and infeasible individuals (like in the case of COMOGA [11]). Although VEGA is known to have difficulties in multiobjective optimization problems due to the fact that it tries to find individuals that excel only in one dimension regardless of the others (the so-called "middling" problem [10, 5]), that drawback turns out to be an advantage in this context, because what we want to find are precisely solutions that are completely feasible, instead of good compromises that may not satisfy one of the constraints.

We are currently experimenting with a second approach in which we use a Pareto-ranking scheme similar to MOGA to classify the population based on nondominance, and our first results are very encouraging.

Our experiments suggest that exploring the use of evolutionary multiobjective optimization techniques to handle constraint is not only an interesting scientific exercise, but could also be seen as a viable alternative to the use of penalty functions for single-objective optimization. Additionally, this might be an interesting domain to test evolutionary multiobjective optimization techniques, since comparisons are straightforward, although any new approach will probably have to be adapted to comply with the special conditions imposed by a constrained single-objective optimization problem.

Some of the issues that deserve attention are: suitability of evolutionary multiobjective optimization techniques to handle a large number of objectives, use of sharing in this domain (how to define σ_{share}), parallelization, and efficiency (both in terms of CPU time and quality of the solutions found).

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