Structural Optimization of Laminated Plates with Genetic Algorithms

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Abstract

Laminated plate optimization is a combinatorial problem where the objective is to find the optimal sequence of materials from a given set, along with the respective fiber orientation. Due to manufacturing reasons, the size of each ply can only take a discrete number of values and, together with the available materials, results in an integer programming problem. Therefore, an approach based on Genetic Algorithms seems to present some advantages in the solution of this structural optimization problem. The proposed stiffness maximization approach optimizes the stacking sequence of various plies, with different orientations and materials; the thickness of each ply and the global number of plies are, a priori, assumed. Moreover, a constraint on the global cost is also presented. Genetic Algorithms successfully identify the designs with the desired structural response.

1 INTRODUCTION

Composite materials have received substantial attention as manufacturing materials. Although the high stiffness-to-weight and strength-to-weight properties of composite materials are attractive, their greatest advantage is their ability to be designed to satisfy directional strength and stiffnesses for any particular loading, or multi-loading, of the structure.

In laminated composite structures, each ply has its greatest stiffness and strength properties, along the direction, through which the fibers are oriented in. By orienting each layer at different angles, the structure can be designed for a specific loading environment.

Along with structural performance and weight, cost is an area of great interest when considering optimization studies in structural design. Obviously, reducing the amount of material required for the structure, minimizes the cost of a laminate composite. However, another method for cost reduction is to allow more than one material in the stacking sequence. Thus, it is possible to use layers of low cost material at locations, in the structure, where performance is less important.

In general, the problem of composite laminate stacking sequence optimization has been formulated as a continuous design problem, and solved using gradient based techniques. These methods of solution present several disadvantages:

- Stacking sequence design often involves design variables, which are limited to small discrete sets of values of ply thickness, orientation angle or material type, due to manufacturing or cost limitations; therefore, these methods require the transformation of these variables into continuous variables, in order that a solution might be obtained;
- Converting the continuous solutions back to discrete feasible values, often produces sub-optimal, or even infeasible designs;
- Composite laminate design problems often have discontinuous objective functions, exhibiting multiple designs with similar performances, involving many local optimum designs.

Genetic Algorithms are suitable optimization algorithms for problems with discrete design variables. Its implementation does not require any evaluation of gradients which, together with its easiness of implementation, make it worthwhile investigating. Although, Genetic Algorithms require many function evaluations, which reflect in large computational costs, there are many reported applications of Genetic Algorithms to the design of composite structures. Genetic algorithms have been applied to stacking sequence optimization of composite plates, (Callahan and Weeks, 1992), to stiffened composite panel design (Nagendra et al., 1996), design of laminated composite panels (Hajela, 1990) (Leung and Nevill, 1994) (Fernandes et al., 1998) (Haftka, 1998).

2 LAMINATE STRUCTURAL ANALYSIS

In this work, the equivalent single-layer laminated plate model based on first and third order shear deformation theory is employed to analyze each possible design. This approach allows the reduction of the number of degrees of freedom required to describe the structural response, with sufficient detailed representation, and without excessive computational cost.

In the equivalent single-layer laminate theories, an heterogeneous laminate plate is treated as a statically equivalent single layer, with a complex constitutive behavior; this approach reduces a continuum three-dimensional problem to a two-dimensional problem. Therefore, composite laminates are treated as plate elements.

The development of a plate theory requires the assumption of a certain form of displacement field within the plate. Thus, an appropriate power series expansion of the displacements, in the coordinate system with x_3 normal to the midplane of the plate, is used.

Considering a laminated composite plate of total thickness h, composed of orthotropic layers, a Cartesian coordinate system x_i is defined on the plate, where the x_1x_2 plane coincides with the plate geometric midplane. It is important to note that Latin subscripts run in 1,2,3 and Greek subscripts run in 1,2. Also, summation on repeated subscripts is implied. For compactness, the general assumed static displacement field is,

$$u_i\left(x_\alpha, x_3\right) = u_i^0\left(x_\alpha\right) + x_3\Phi_i\left(x_\alpha\right) \tag{1}$$

where u_i^0 are displacement components in the x_i directions of the middle plane, and Φ_α, are unknown functions that, in some cases, are partially known. In particular, in first order shear plate theory,

$$\Phi_1 = \theta_2 \qquad \Phi_2 = -\theta_1 \qquad \Phi_3 = 0 \tag{2}$$

where θ_α are the „right-hand-rule" rotations of the normal to the middle plane along the x_α axes.

The first order theory leads to constant transverse shear stress, which violates equilibrium at the free surfaces of the plate, and continuity requirements of the interlaminar shear stress. To account for the discrepancy between the constant state of shear strains in the first order theory and the quadratic or higher order distribution of shear strains in the elasticity, shear correction factors are introduced. These factors may be calculated for laminated plates, and many works have addressed the selection of exact, or improved values (Whitney, 1973) (Lardeur, 1990).

Higher order theories involve additional terms in x_3, and may not violate equilibrium at the free surface, therefore yielding a more accurate interlaminar stress distribution. However, they require more computational effort. First order theories are the simplest equivalent to single layer plate theories, and adequately describe the cinematic behavior of most laminates.

Approximating the unknowns with the appropriate interpolation functions (Reddy, 1997), and following the standard displacement finite element procedures, it is possible to obtain the equilibrium system of linear equations in the form,

$$\mathbf{K}\ \mathbf{d} = \mathbf{F} \tag{3}$$

where **K** is the stiffness matrix, **F** is the force vector, and **d** is the vector of unknown functions.

At element level (e), and in local coordinates (ξ,η), the stiffness matrix may be written as

$$\begin{aligned}\mathbf{K}^{(e)} &= \int_{-1}^{+1}\int_{-1}^{+1}\left[\mathbf{B}^T\mathbf{D}\mathbf{B}\right]^{(e)}\left|\mathbf{J}\right| d\xi d\eta \\ &= \int_{-1}^{+1}\int_{-1}^{+1}\begin{bmatrix}\mathbf{B}_M^T\mathbf{D}_M\mathbf{B}_M & \mathbf{B}_M^T\mathbf{D}_{MF}\mathbf{B}_F & 0\\ \mathbf{B}_F^T\mathbf{D}_{FM}\mathbf{B}_M & \mathbf{B}_F^T\mathbf{D}_F\mathbf{B}_F & 0\\ 0 & 0 & \mathbf{B}_C^T\mathbf{D}_C\mathbf{B}_C\end{bmatrix}^{(e)}\left|\mathbf{J}\right| d\xi d\eta\end{aligned} \tag{4}$$

where **B** is the strain displacement matrix, **D** is the elasticity matrix and | **J** | is the Jacobian. The subscripts *M*, *F* and *C* stand for membrane, bending and shear, respectively, and

$$\mathbf{B}^{(e)} = \mathbf{L}\ \mathbf{S}^{(e)} \tag{5}$$

where $\mathbf{S}^{(e)}$ is the shape function matrix, depending on the choice of finite element and laminate theory, and **L** is a matrix of differential operators. On the other hand, the constitutive equation is

$$\begin{Bmatrix}\mathbf{N}\\ \mathbf{M}\\ \mathbf{Q}\end{Bmatrix}^{(e)} = \mathbf{D}^{(e)}\begin{Bmatrix}\boldsymbol{\varepsilon}^0\\ \boldsymbol{\varepsilon}^i\\ \gamma^c\end{Bmatrix}^{(e)} \tag{6}$$

with

$$\mathbf{D}^{(e)} = \begin{bmatrix}\mathbf{D}_M & \mathbf{D}_{MF} & 0\\ \mathbf{D}_{FM} & \mathbf{D}_F & 0\\ 0 & 0 & \mathbf{D}_C\end{bmatrix}^{(e)} \tag{7}$$

The submatrix $\mathbf{D}_M$ relates membrane force resultants **N** to membrane strains ($\boldsymbol{\varepsilon}^0$), $\mathbf{D}_F$ relates generalized moments **M** to generalized curvatures ($\boldsymbol{\varepsilon}^i$), $\mathbf{D}_{MF}$ relates membrane force resultants to generalized curvatures, and generalized moments to membrane strains, and finally, $\mathbf{D}_C$ relates transverse shear resultants **Q** to shear strains ($\boldsymbol{\gamma}^c$).

As each layer may have different properties, the elasticity matrix **D** must be evaluated by summations carried out all over the thickness. Therefore, equivalent single layer theories produce equivalent stiffness matrix, which is a weighted average of the individual layer stiffness through the thickness. Following the conventional procedures of the finite element method, layer stresses can be found from nodal results.

In this work, two first order quadrilateral elements with eight (serendipity family) and nine nodes (Lagrange family), are used for the analysis of general composite laminated plates. Serendipity elements have fewer nodes compared to the Lagrange elements because they do not have interior points. All elements present five degrees of freedom per node. The complete development of these elements is described in (Leal, 1998).

3 GENETIC ALGORITHMS IN LAMINATED PLATE DESIGN

In laminated composite structures, the goal is to find the material for each layer and ply orientation angles that will provide a structure with the best performance, for a given set of loading conditions. Additionally, geometry, manufacturing, cost, and failure constraints may also be considered in the design.

In order to reduce the design space, a symmetric and balanced laminate is considered, such that the global laminate is symmetric relatively to the midplane, with a $-\alpha^{o}$ ply for each $+\alpha^{o}$ ply. Figure 1 shows the laminate structure, where *h* states for the total thickness, and m_i, a_i are, respectively, the material and angle for layer *i*.

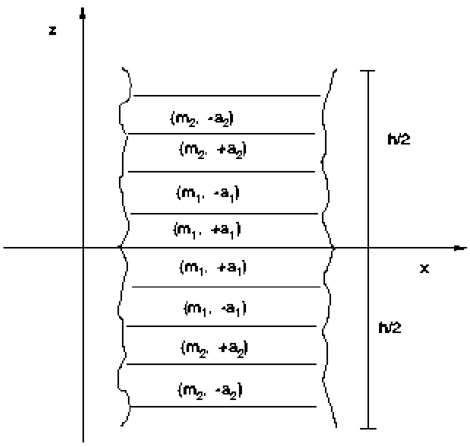

Figure 1: Laminated plate structure

Genetic Algorithms (GAs) are search and optimization algorithms that mimic the process of natural evolution (Goldberg, 1989) (Schwefel, 1985). The basic requirements for building a Genetic Algorithm are:

- Encoding technique;
- Evaluation function;
- Initialization procedure;
- Genetic operators.

In order to illustrate the application of GAs to the laminated plate design problem, let us consider a simple symmetrical problem, consisting on a four sides supported plate with sixteen layers, two materials (Glass/Epoxy (G); Graphite/Epoxy (C)), and seven ply angles (0º, 15º, 30º, 45º, 60º, 75º, 90º). Thus, a feasible solution to the problem must specify the materials in each layer, as well as the angles. Since the problem is symmetrical and balanced, only four design variables need to be specified. For instance, a feasible solution could be [(G, ±30º); (C, ±75º); (G, ±45º); (G, ±60º)]$_s$, that is, the first layer is of glass/epoxy and +30º, the second is of glass/epoxy and –30º (imposed to restrain the search space), the third layer is of graphite/epoxy and +75, and so on. The laminate is symmetric (s subscript) to restrain the search space. It is important to note that, if the angle is 0º or 90º, two identical layers are considered.

3.1 ENCODING TECHNIQUE

Let us consider the following laminate [(G, ±30º); (C, ±75º); (G, ±45º); (G, ±60º)]$_s$. This laminate can be coded as

Design Variable	Material	Angle
1	G	30º
2	C	75º
3	G	45º
4	G	60º

This solution can be seen as a string, where each material is represented by a digit (G-0, C-1), and each angle by a digit ranging from 0 to 6 (0-0º; 1-15º; 2-30º; 3-45º; 4-60º; 5-75º; 6-90º). Therefore, the solution could be represented by the following alternate sequence of materials and angles:

0 2   1 5   0 3   0 4

In this approach, the codification of the Genetic Algorithm requires a binary coding,

Design Variable	Material	Angle
1	0	010
2	1	101
3	0	011
4	0	100

implying that the solution would be represented by the following chromosome

0 0 1 0 1 1 0 1 0 0 1 1 0 1 0 0

3.2 EVALUATION FUNCTION

Each point in the search space, i.e., each chromosome is evaluated in terms of its compliance, i.e., the inverse of stiffness. Such evaluation is performed by a finite element module, which for a given set of materials and angles, produces a value for the objective function. Thus, this evaluation mechanism is the bridge between the bit string manipulator algorithm and the real world.

For the problem under consideration, four biquadratic elements of the serendipity and lagrangean families were considered on the evaluation of the objective function.

A penalty function scheme was used to take into account solutions, which did not observe the constraints of the problem.

3.3 GA PARAMETERS

In this approach, a two-point crossover, uniform mutation and linear ranking have been used. Table 1 presents the parameters values used in this example.

Table1: Genetic Algorithm Parameters

Parameter	**Value**
Population Size	100
Crossover Probability	0.7
Mutation Probability	0.001
Penalty coefficient	1000

Several runs were carried out in order to evaluate the reliability of the solutions.

3.4 RESULTS OF TEST PROBLEM

In this section, the results concerning the design problem are presented. The initial population was randomly generated, and the solutions, which violated the problem constraints, were penalized. Furthermore, each problem instance was replicated 10 times, thus, the values in the tables represent averages. The execution was terminated when convergence to a solution was observed. Cost constraints were considered with the following material relative costs: G – 1 and C – 8. Table 2 presents the results obtained, in terms of the average number of generations and number of function evaluations. Table 3 lists the best solutions obtained for the several instances of *Problem 1*.

Table 2: Results for *Problem 1*

***Problem 1*: 2 materials, 7 angles**	**Number of Generations**	**Number of Function Evaluations**
Without any restriction	19	823
Cost <= 32	19	823
Cost <= 25	21	992
Cost <= 18	21	984
Cost <= 11	23	1119

Table 3: Best solutions for *Problem 1*

***Problem 1*: 2 materials, 7 angles**	**Solution**	**Objective Function (Nm)**
Without restrictions	$[(C, \pm 45^o)_4]_s$	162
Cost <= 32	$[(C, \pm 45^o)_4]_s$	162
Cost <= 25	$[(C, \pm 45^o)_3; (G, \pm 45^o)]_s$	163
Cost <= 18	$[(C, \pm 45^o)_2; (G, \pm 45^o)_2;]_s$	173
Cost <= 11	$[(C, \pm 45^o); (G, \pm 45^o)_3]_s$	208

Figure 2 presents the evolution of the average objective function value along the generations, for the instance of Problem 1 without restrictions.

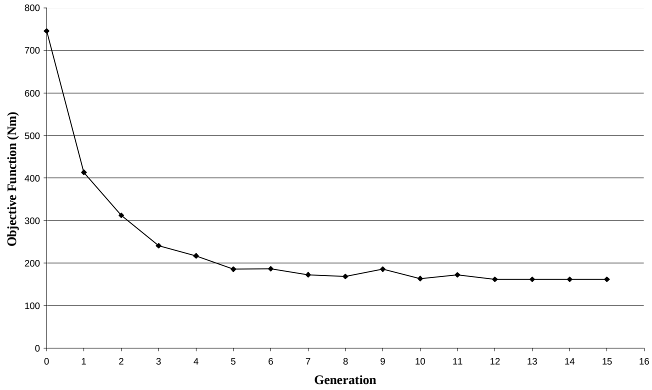

Figure 2: Average objective function value.

4 APPLICATION TO PLATE DESIGN PROBLEMS

In this section, the results for two distinct plate design problems are presented. The problems are: a four sides clamped plate design problem and a four sides supported plate design problem. In both problems, 6 materials and 7 ply angles are considered. The materials are composites of Epoxy, three with fibers reinforcement of Glass (G1, G2, G3), and three with Graphite (C1, C2, C3). Cost constraints are also considered with the following relative costs for the 6 materials: 1, 3, 4, 8, 10, 12, respectively.

Two point crossover, uniform mutation and linear ranking were used, with the same GA parameters presented on table 1, but with a population of 150 chromosomes.

4.1 FOUR SIDES CLAMPED PLATE DESIGN PROBLEM

The results for a four sides clamped plate, in terms of the average number of generations and average number of function evaluations, using four biquadratic elements of the serendipity (*Problem 2-SER.*) and lagrangean (*Problem 2-LAG.*) families for objective function evaluation, are presented on Tables 4 and 5.

Table 4: Results for *Problem 2-SER.*

***Problem 2-SER.*: 6 materials, 7 angles**	**Number of Generations**	**Number of Function Evaluations**
Without restrictions	34	2495
Cost <= 40	37	2452
Cost <= 30	43	3058
Cost <= 20	55	3486
Cost <= 10	28	2203

Table 5: Results for *Problem 2-LAG.*

***Problem 2-LAG.*: 6 materials, 7 angles**	**Number of Generations**	**Number of Function Evaluations**
Without restrictions	40	2605
Cost <= 40	44	2916
Cost <= 30	45	3037
Cost <= 20	40	2928
Cost <= 10	35	2428

Tables 6 and 7 list the best solutions obtained for the several instances of *Problem 2*. Note that, in some cases, two different solutions were obtained in distinct GA executions.

Table 6: Best solutions for *Problem 2-SER.*

***Problem 2-SER.*: 6 materials, 7 angles**	**Solution**	**Obj. Funct. (Nm)**
Without restrictions	$[(C3, \pm90^o); (C3, \pm0^o)_2; (C3, \pm90^o)]_s$ $[(C3, \pm0^o); (C3, \pm90^o)_2; (C3, \pm0^o)]_s$	4.7
Cost <= 40	$[(C3, \pm0^o); (C3, \pm90^o)_2; (G3, \pm0^o)]_s$ $[(C3, \pm90^o); (C3, \pm0^o)_2; (G3, \pm90^o)]_s$	4.8
Cost <= 30	$[(C3, \pm90^o); (C3, \pm0^o); (G2, \pm0^o); (G2, \pm90^o)]_s$ $[(C3, \pm0^o); (C3, \pm90^o); (G2, \pm90^o); (G2, \pm0^o)]_s$	5.1
Cost <= 20	$[(C2, \pm90^o); (C1, \pm0^o); (G1, \pm90^o)_2]_s$	5.8
Cost <= 10	$[(G2, \pm90^o); (G2, \pm0^o)_2; (G1, \pm90^o)]_s$ $[(G2, \pm0^o); (G2, \pm90^o)_2; (G1, \pm0^o)]_s$	10.8

Table 7: Best solutions for *Problem 2-LAG.*

***Problem 2-LAG.*: 6 materials, 7 angles**	**Solution**	**Obj. Funct. (Nm)**
Without restrictions	$[(C3, \pm0^o); (C3, \pm90^o)_2; (C3, \pm0^o)]_s$	4.7
Cost <= 40	$[(C3, \pm0^o); (C3, \pm90^o)_2; (G3, \pm0^o)]_s$	4.8
Cost <= 30	$[(C3, \pm90^o); (C3, \pm0^o); (G2, \pm0^o); (G2, \pm90^o)]_s$	5.1
Cost <= 20	$[(C2, \pm90^o); (C1, \pm0^o); (G1, \pm90^o)_2]_s$ $[(C2, \pm0^o); (C1, \pm90^o); (G1, \pm0^o)_2]_s$	5.8
Cost <= 10	$[(G2, \pm90^o); (G2, \pm0^o)_2; (G1, \pm90^o)]_s$ $[(G2, \pm0^o); (G2, \pm90^o)_2; (G1, \pm0^o)]_s$	10.8

4.2 FOUR SIDES SUPPORTED PLATE DESIGN PROBLEM

The results for a four sides supported plate, in terms of the average number of generations and average number of function evaluations, using four biquadratic elements of the serendipity (*Problem 3-SER.*) and lagrangean (*Problem 3-LAG.*) families for objective function evaluation, are presented on Tables 8 and 9, respectively.

Table 8: Results for *Problem 3-SER.*

***Problem 3-Ser.*: 6 materials, 7 angles**	**Number of Generations**	**Number of Function Evaluations**
Without restrictions	29	1245
Cost <= 40	37	2156
Cost <= 30	42	2614
Cost <= 20	41	2879
Cost <= 10	32	2249

Table 9: Results for *Problem 3-LAG.*

***Problem 3-LAG.*: 6 materials, 7 angles**	**Number of Generations**	**Number of Function Evaluations**
Without restrictions	31	2090
Cost <= 40	39	2244
Cost <= 30	37	2860
Cost <= 20	37	2866
Cost <= 10	31	2262

Tables 10 and 11 list the best solutions obtained for the several instances of *Problem 3*.

Table 10: Best solutions for *Problem 3-SER.*

***Problem 3-SER.*: 6 materials, 7 angles**	**Solution**	**Obj. Funct. (Nm)**
Without restrictions	$[(C3, \pm45^o)_3; (G3, \pm45^o)]_s$	14.3
Cost <= 40	$[(C3, \pm45^o)_3; (G3, \pm45^o)]_s$	14.3
Cost <= 30	$[(C3, \pm45^o)_2; (G2, \pm45^o)_2]_s$	15.1
Cost <= 20	$[(C2, \pm45^o); (C1, \pm45^o); (G1, \pm45^o)_2]_s$	17.3
Cost <= 10	$[(G2, \pm45^o)_3; (G1, \pm45^o)]_s$	30.1

Table 11: Best solutions for *Problem 3-LAG.*

***Problem 3-LAG.*: 6 materials, 7 angles**	**Solution**	**Obj. Funct. (Nm)**
Without restrictions	$[(C3, \pm45^o)_4]_s$	16.6
Cost <= 40	$[(C3, \pm45^o)_3; (G3, \pm45^o)]_s$	16.8
Cost <= 30	$[(C2, \pm45^o)_2; (C1, \pm45^o); (G1, \pm45^o)]_s$	18.4
Cost <= 20	$[(C2, \pm45^o); (C1, \pm45^o); (G1, \pm45^o)_2]_s$	20.2
Cost <= 10	$[(G2, \pm45^o)_3; (G1, \pm45^o)]_s$	42.0

The number of biquadratic elements used for objective function evaluation was varied in order to investigate how it affects the results. In this experiment no relative cost constraints were considered. The number of serendipity and lagrangean elements was fixed by 4, 16 and 64.

Table 12 presents the results, in terms of the average number of generations and average number of function evaluations, using the same GA parameters of the previous problem, but with a population of 100 chromosomes. Table 13 lists the best solutions for different number of biquadratic elements.

Table 12: Results for *Problem 3*

Number of elements	Type of elements	Number of Generations	Number of Function Evaluations
4	Serendipity	31	1387
	Lagrangean	33	1400
16	Serendipity	31	1375
	Lagrangean	27	1339
64	Serendipity	34	1419
	Lagrangean	34	1504

Table 13: Best solutions for *Problem 3*

Number of elements	Type of elements	Solution	Obj. Funct. (Nm)
4	Serendipity	$[(C3, \pm 45°)_3; (G3, \pm 45°)]_s$	14.28
	Lagrangean	$[(C3, \pm 45°)_4]_s$	16.64
16	Serendipity	$[(C3, \pm 45°)_4]_s$	16.63
	Lagrangean	$[(C3, \pm 45°)_4]_s$	16.67
64	Serendipity	$[(C3, \pm 45°)_4]_s$	16.67
	Lagrangean	$[(C3, \pm 45°)_4]_s$	16.67

Figure 3 shows how the GA execution time varies with the number of elements considered on the objective function evaluation. Figure 4 shows the variation of the objective function value (the compliance value) versus the number of elements used on the objective function evaluation.

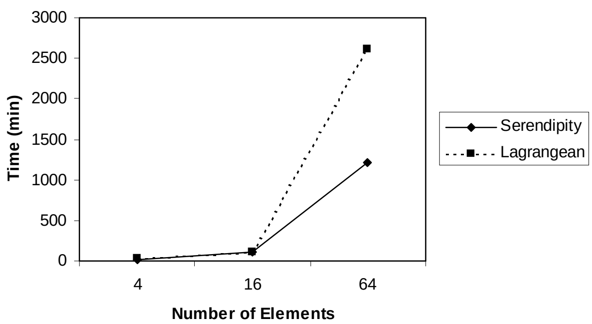

Figure 3: Time versus Number of elements

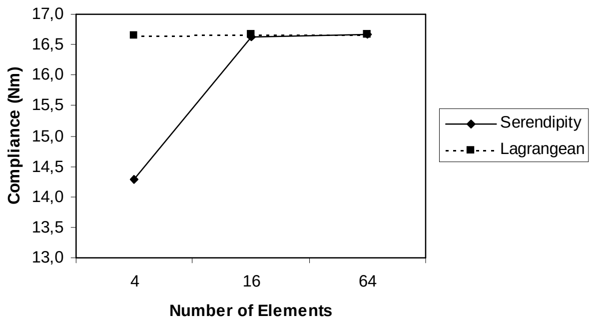

Figure 4: Compliance versus Number of elements

The computation time grows exponentially with the number of elements considered. However, the accuracy of the solution is, in practical terms, approximately the same for 16 and 64 elements.

5 CONCLUSION

The results presented, for four sides supported and clamped plates, show that Genetic Algorithms can be used in the optimization of the design of composite laminated plates, producing solutions with physical meaning, while considering different cost restrictions. Moreover, as expected, the computational cost grows exponentially with the number of elements considered, that is to say, with the accuracy imposed on the finite element code (the evaluation function); however, it has been shown that a solution with 16 elements is accurate enough, with reasonable computational times. It seems contradictory that the number of function evaluations does not increase monotonically as the cost restriction is minimized. It should be pointed out that this is a combinatorial problem, and the admissible set for different instances of cost, can be quite different. Thus, the number of observed function evaluations (an average value of 10 executions) reflects this difference. Future work will aim at the reduction of the computational times, while considering a finite element methodology, based on third order theory, and the study of one side clamped plates.

Acknowledgments

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