Using Artificial Immune Systems to Solve Optimization Problems

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General objective

This project is focused on the analysis and development of new algorithms based on artificial immune systems to solve optimization problems. Two types:

- How to incorporate constraints of any type into a GA for single-objective optimization.
- How to generate the Pareto optimal set of a multiobjective optimization problem.

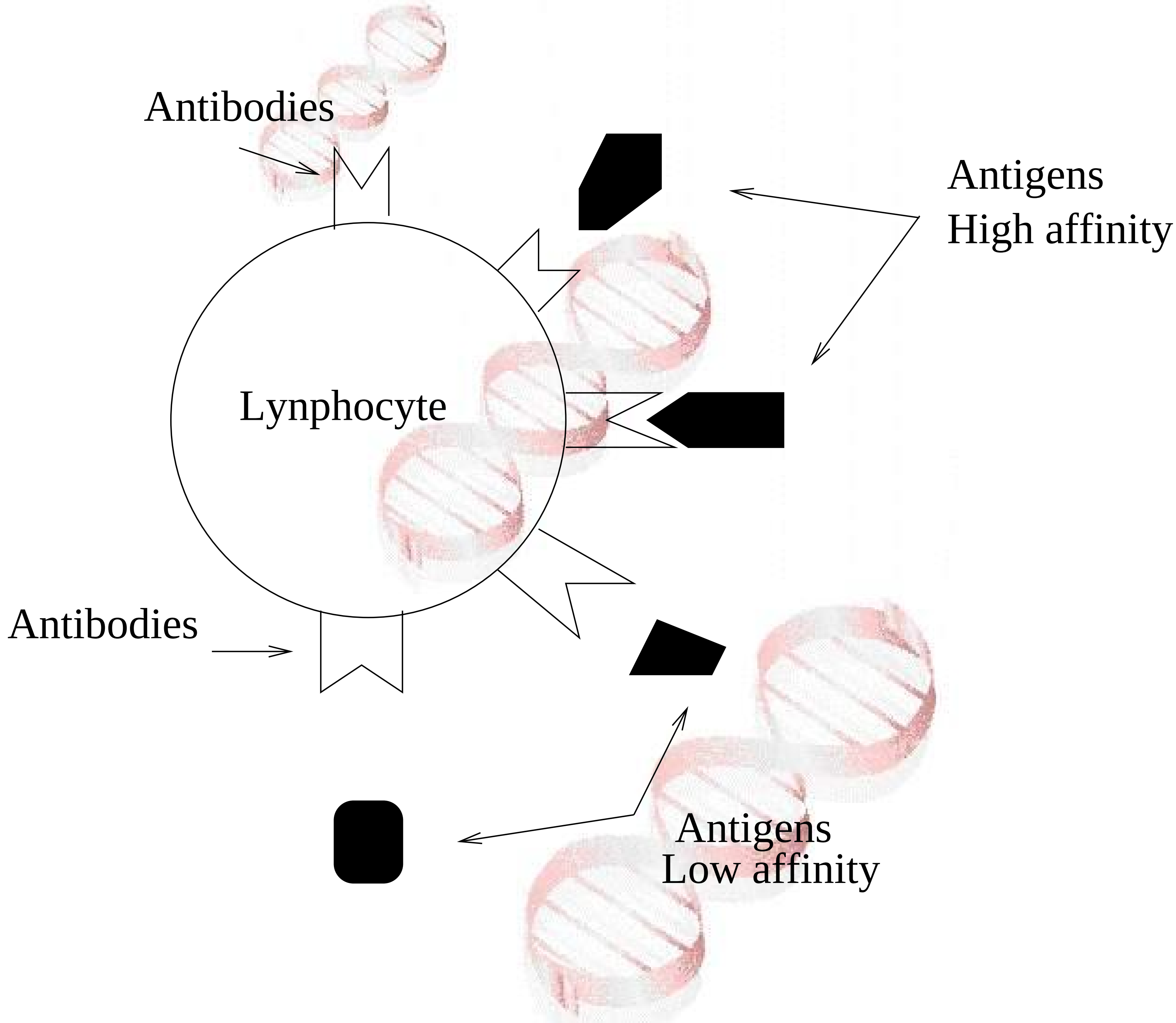
Artificial immune system

"Artificial immune systems are adaptive systems, inspired by theoretical immunology and observed immune functions, principles and models, which are applied to problem solving"

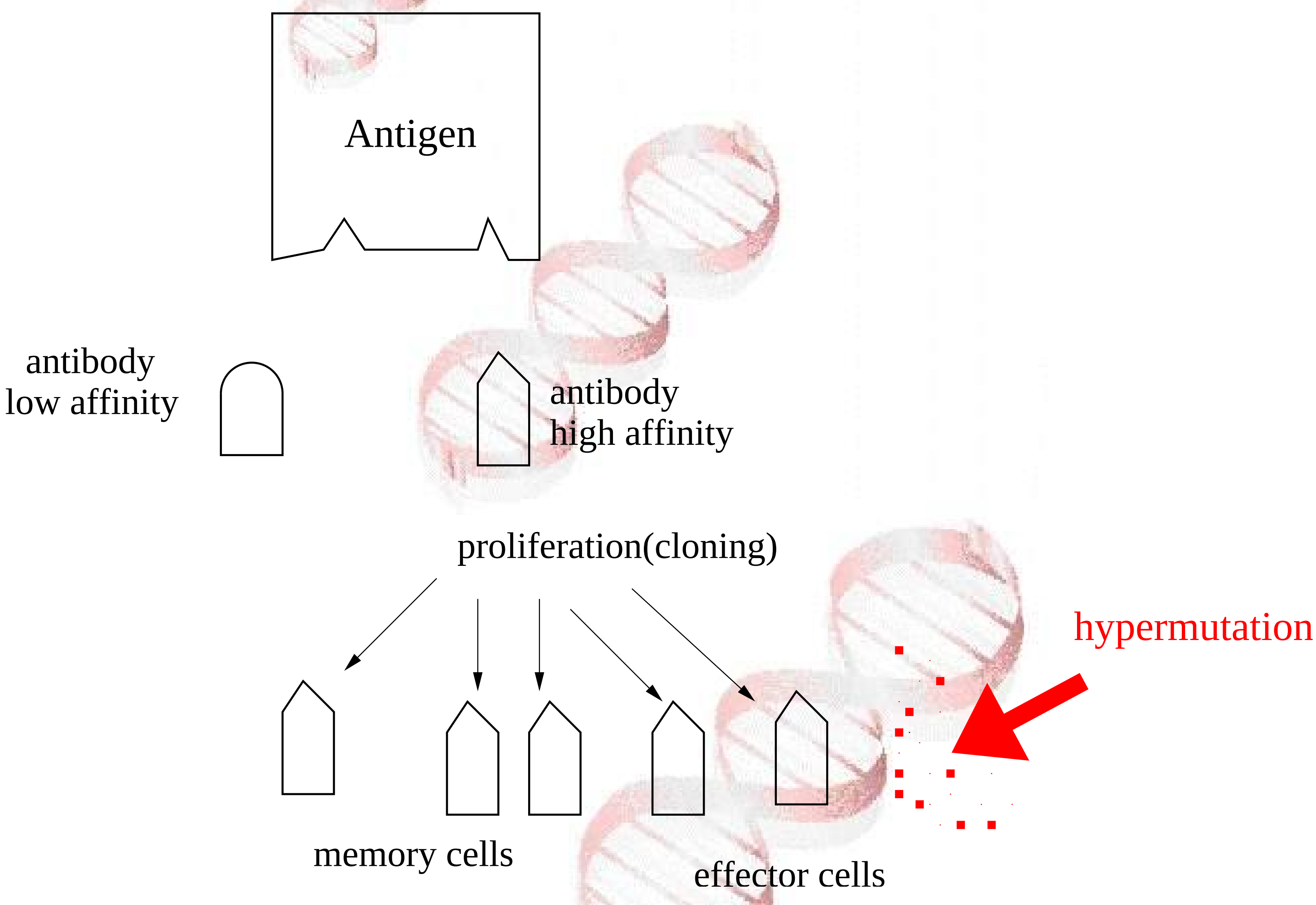
De Castro & Timmis (2002)

Why immune system?

- Learning
- Memory
- Self identity
- Pattern recognition
- Diversity
- Distributed
- Fault tolerance
- Robustness


Antibodies
Antigens
High affinity
Lynphocyte
Antibodies
Antigens
Low affinity

The clonal selection principle



General optimization problem

$$\text{Find} \quad \vec{x} \text{ which optimizes } f(\vec{x}) \tag{1}$$

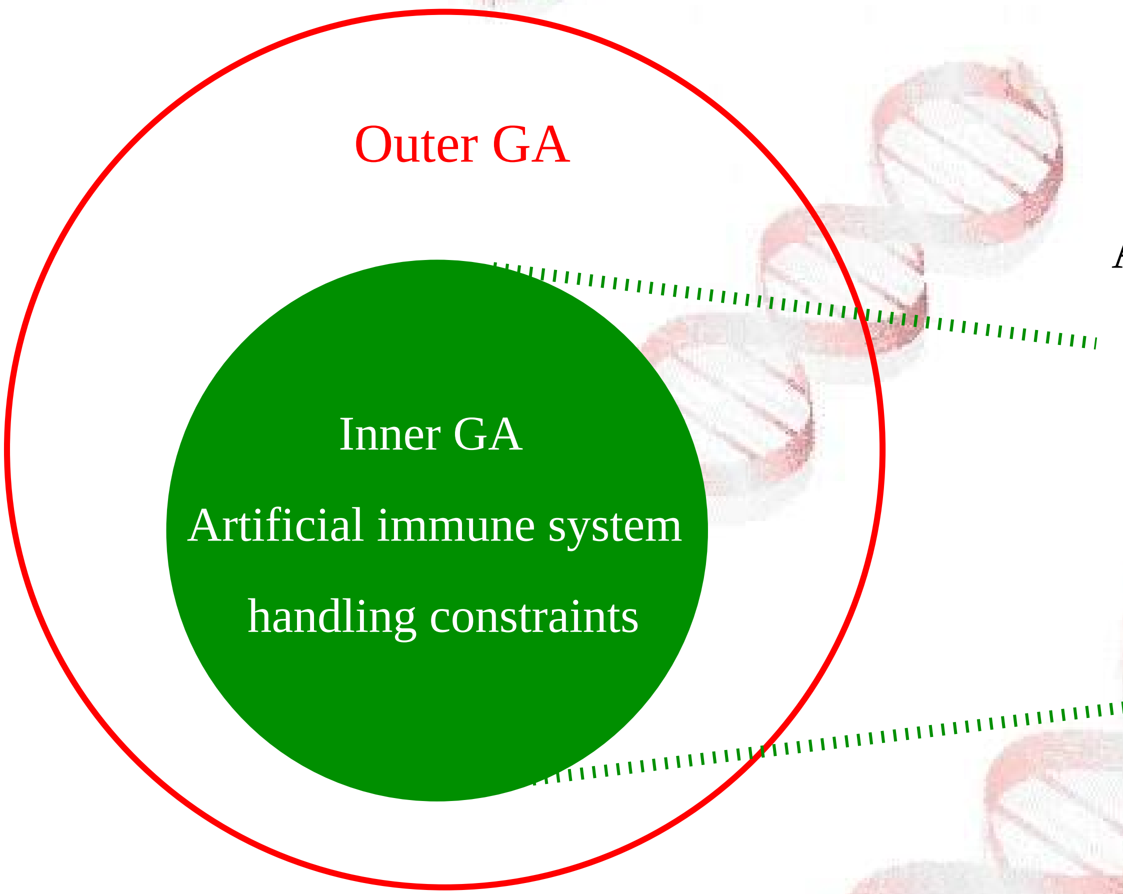
subject to:

$$g_i(\vec{x}) \leq 0, \quad i = 1, \ldots, n \tag{2}$$

$$h_j(\vec{x}) = 0, \quad j = 1, \ldots, p \tag{3}$$

where $\vec{x}$ is the vector of solutions $\vec{x} = [x_1, x_2, \ldots, x_r]^T$, n is the number of inequality constraints and p is the number of equality constraints.

Hajela and Lee's proposal (1996)



Antigens = feasible individuals

0 0 1 1 1 1 1 0 1 0 0 0 1
0 1 1 1 1 1 1 0 1 0 1 1 1
0 0 0 0 1 1 1 10 1 1 0 0
1 1 1 0 1 1 1 1 1 0 1 0 0

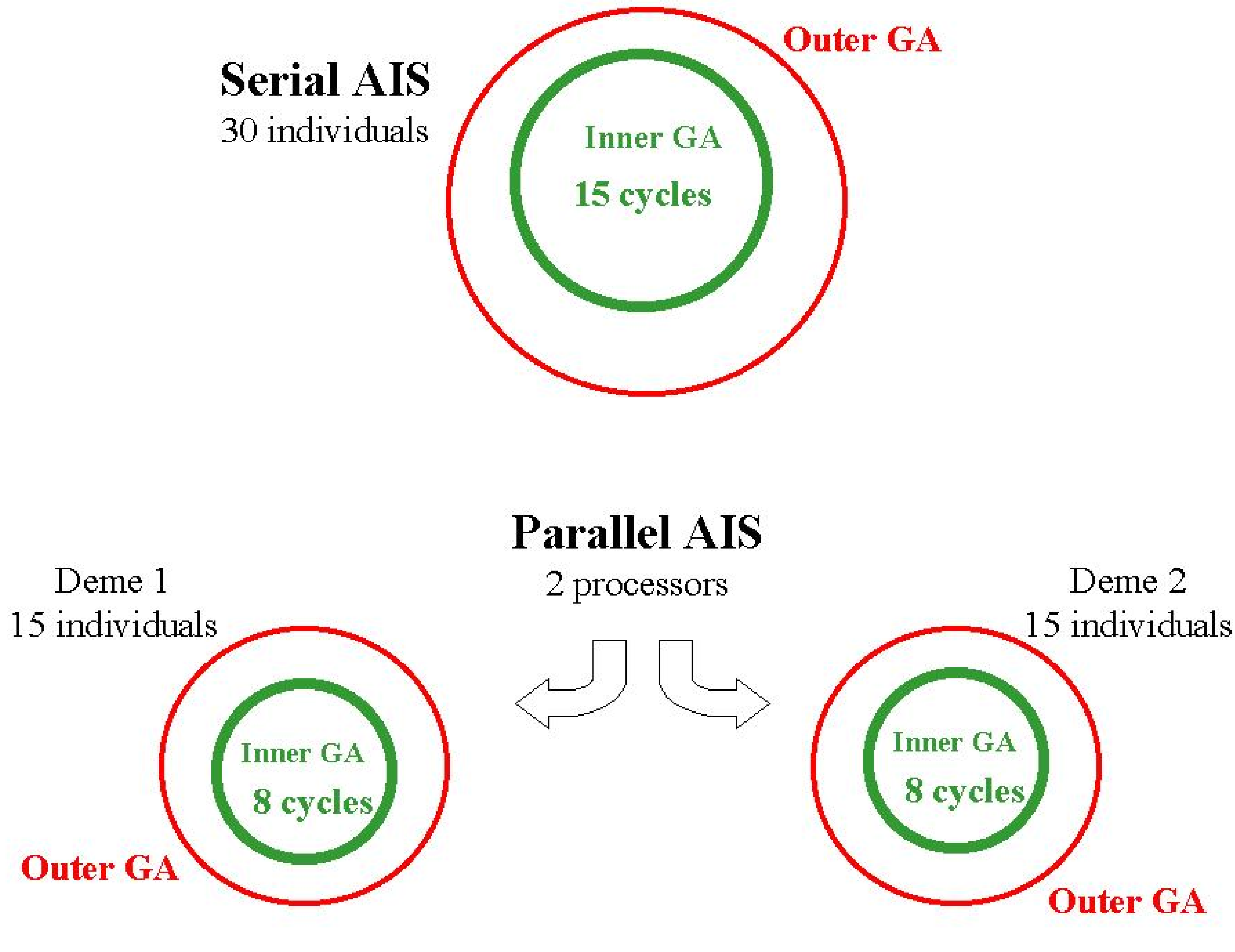
0 0 1 1 1 0 01 0 1 0 0 1
1 0 1 1 0 0 0 0 1 0 1 1 1
0 1 1 1 1 1 1 0 1 0 1 1 1
1 1 1 0 1 1 1 0 1 1 1 0 0

Antibodies = infeasible individuals

Analysis of Variance (ANOVA)

It was done an analysis of variance in order to know the effects of the parameters over the performance of this algorithm. Those parameters with the biggest effects are:

- Mutation rate (the best performance is obtained when using adaptive rates)
- Type of selection (binary tournament)
- Type of crossover (2 points and uniform)


Serial AIS
30 individuals
Outer GA
Inner GA
15 cycles
Parallel AIS
2 processors
Deme 1
15 individuals
Inner GA
8 cycles
Outer GA
Deme 2
15 individuals
Inner GA
8 cycles
Outer GA

Example 1

Minimize:

$$f(\vec{x}) = 5.3578547x_3^2 + 0.8356891x_1x_5 + 37.293239x_1 - 40792.141$$

subject to:

$$g_1(\vec{x}) = 85.334407 + 0.0056858x_2x_5 + 0.0006262x_1x_4 - 0.0022053x_3x_5 - 92 \leq$$
$$g_2(\vec{x}) = -85.334407 - 0.0056858x_2x_5 - 0.0006262x_1x_4 + 0.0022053x_3x_5 \leq$$
$$g_3(\vec{x}) = 80.51249 + 0.0071317x_2x_5 + 0.0029955x_1x_2 + 0.0021813x_3^2 - 110 \leq$$
$$g_4(\vec{x}) = -80.51249 - 0.0071317x_2x_5 - 0.0029955x_1x_2 - 0.0021813x_3^2 + 90 \leq$$
$$g_5(\vec{x}) = 9.300961 + 0.0047026x_3x_5 + 0.0012547x_1x_3 + 0.0019085x_3x_4 - 25 \leq$$
$$g_6(\vec{x}) = -9.300961 - 0.0047026x_3x_5 - 0.0012547x_1x_3 - 0.0019085x_3x_4 + 20 \leq$$

where: $78 \leq x_1 \leq 102$, $33 \leq x_2 \leq 45$, $27 \leq x_i \leq 45$ $(i = 3, 4, 5)$. The global optimum of this problem is $f(\vec{x}^*) = -30665.539$. Two constraints are active at the optimum: g_1 and g_6.

Results example 1

optimal=-30665.536

	KM	Serial	2 P SP=1.1049	3 P SP=2.4799	4 P SP=4.9239
	$f(x)$	$f(x)$	$f(x)$	$f(x)$	$f(x)$
Best	-30664.5	-30663.3	-30659.98	-30663.35	-30664.74
Mean	-30655.3	-30651.1	-30638.01	-30638.17	-30646.71
Worst	-30645.9	-30626.15	-30591.15	-30580.6	-30605.97
Std.Dev.	N/A	11.74	20.29	20.85	16.85

Table 1: Results for the first example. P indicates the number of processors used and SP refers to the speedup achieved. KM are Koziel & Michalewicz results.

Example 2

Minimize:

$$
\begin{aligned}
f(\vec{x}) = (x_1 - 10)^2 + 5(x_2 - 12)^2 + x_3^4 + \\
3(x_4 - 11)^2 + 10x_5^6 + 7x_6^2 + x_7^4 \\
-4x_6x_7 - 10x_6 - 8x_7
\end{aligned}
$$

subject to:

$$
\begin{aligned}
g_1(\vec{x}) = -127 - 2x_1^2 + 3x_2^4 + x_3 + 4x_4^2 + 5x_5 \leq 0 \\
g_2(\vec{x}) = -282 + 7x_1 + 3x_2 + 10x_3^2 + x_4 - x_5 \leq 0 \\
g_3(\vec{x}) = -196 + 23x_1 + x_2^2 + 6x_6^2 - 8x_7 \leq 0 \\
g_4(\vec{x}) = 4x_1^2 + x_2^2 - 3x_1x_2 + 2x_3^2 + 5x_6 - 11x_7 \leq 0
\end{aligned}
$$

where $-10 \leq x_i \leq 10$ for $(i = 1, \ldots, 7)$. The optimum solution is $\vec{x}^*$ = (2.330499, 1.951372, -0.4775414, 4.365726, -0.6244870, 1.038131, 1.594227) where $f(\vec{x}^*)$=680.6300573. Two constraints are active

Results example 2

Optimal=680.630

	KM	Serial	2 P SP=1.84	3 P SP=4.60	4 P SP=6.53
	$f(x)$	$f(x)$	$f(x)$	$f(x)$	$f(x)$
Best	680.91	680.8196	680.9747	681.2443	680.7950
Mean	681.16	681.3433	682.3126	682.1855	681.3369
Worst	683.18	682.3872	686.6091	686.8962	682.3619
Std.Dev.	N/A	0.3516	1.4787	1.2683	0.4204

Table 2: Results for the second example. P indicates the number of processors used and SP refers to the speedup achieved. KM are Koziel & Michalewicz results.

General Multiobjetive Optimization Problem

Find the vector $\vec{x}^* = [x_1^*, x_2^*, \ldots, x_n^*]^T$ which will satisfy the m inequality constraints:

$$g_i(\vec{x}) \geq 0 \quad i = 1, 2, \ldots, m \tag{4}$$

the p equality constraints

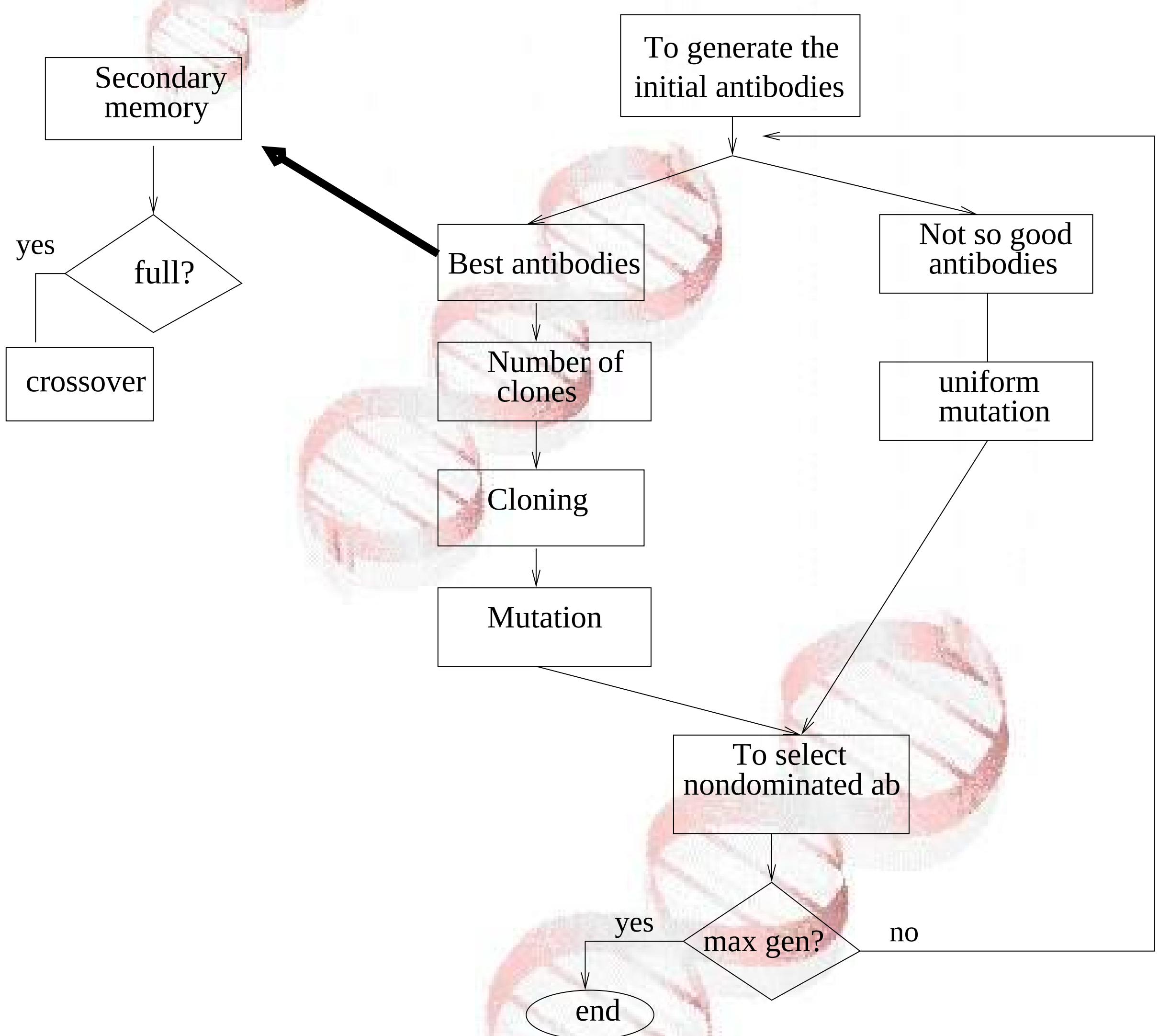
$$h_i(\vec{x}) = 0 \quad i = 1, 2, \ldots, p \tag{5}$$

and will optimize the vector function

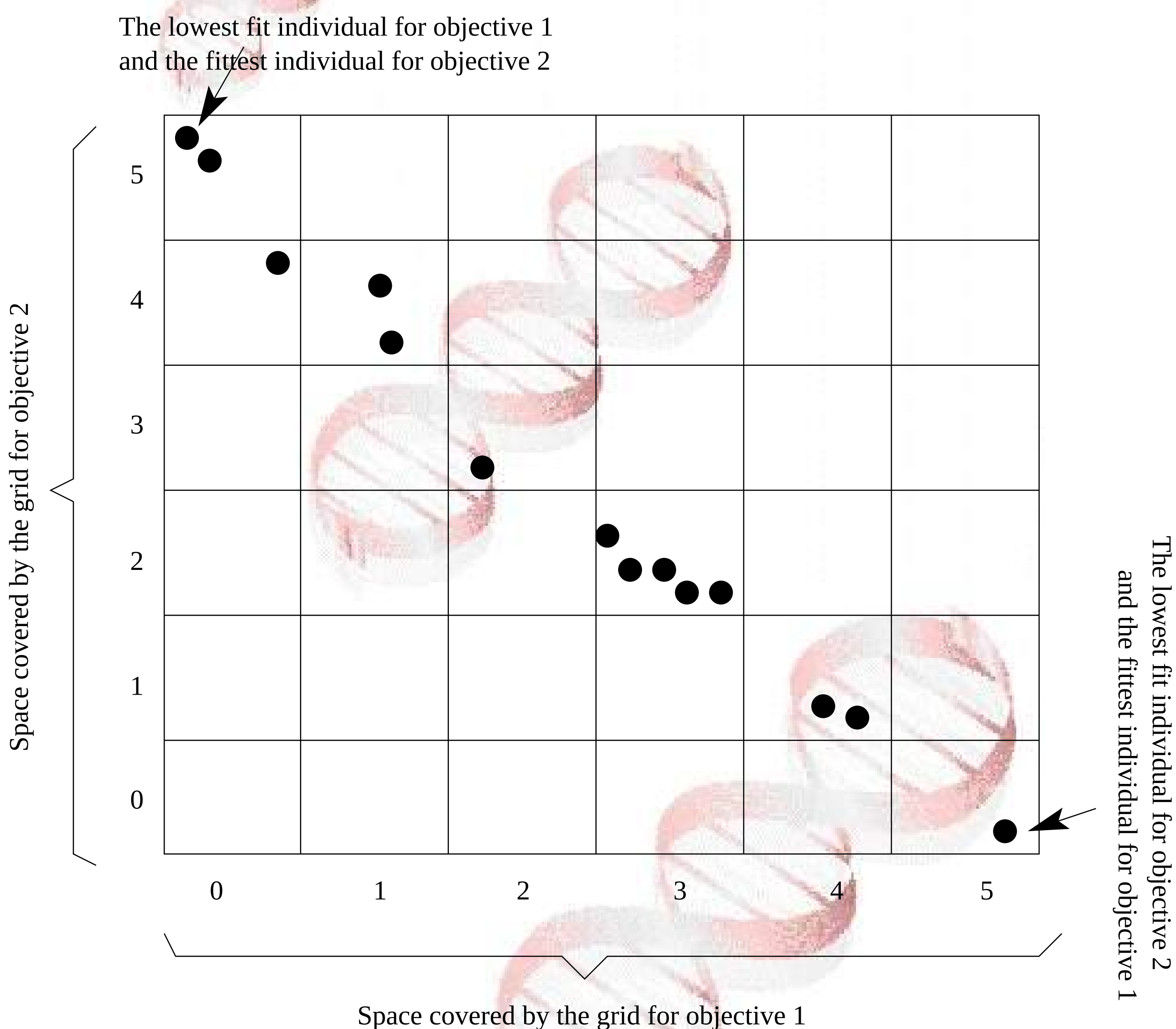
$$\vec{f}(\vec{x}) = [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})]^T \tag{6}$$

where $\vec{x} = [x_1, x_2, \ldots, x_n]^T$ is the vector of decision variables.

The proposed algorithm



The adaptive grid



Comparison of results

We compare our algorithm against two algorithms which are representative of the state-of-the-art:

- Nondominated Sorting Genetic Algorithm (NSGA-II)
- Pareto Archived Evolution Strategy (PAES)

Metrics

- Error ratio (Van Veldhuizen, 1999)
- Spacing (Schott, 1995)
- Generational Distance (Van Veldhuizen & Lamont, 2000)

Example 1

Proposed by Kita (1996):

Maximize

$$F = (f_1(x,y), f_2(x,y))$$

where: $f_1(x,y) = -x^2 + y$, $f_2(x,y) = \frac{1}{2}x + y + 1$, $x, y \geq 0$,
$0 \geq \frac{1}{6}x + y - \frac{13}{2}$, $0 \geq \frac{1}{2}x + y - \frac{15}{2}$, $0 \geq 5x + y - 30$.

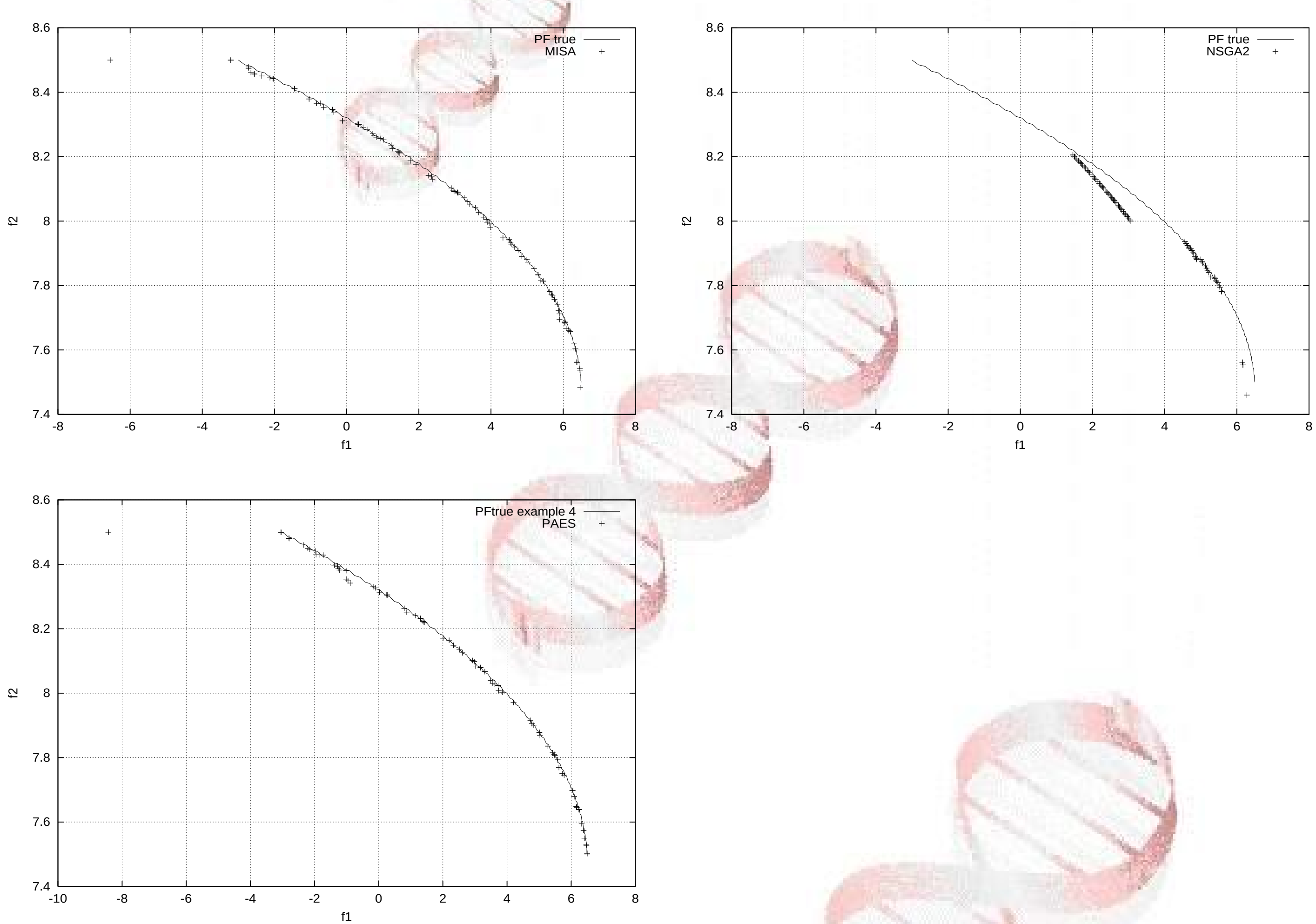


Figure 1: Pareto front obtained by MISA, NSGA-II and PAES, example 1

Example 2

Proposed by Kursawe (1991):

$$\text{Minimize } f_1(\vec{x}) = \sum_{i=1}^{n-1}\left(-10\exp\left(-0.2\sqrt{x_i^2 + x_{i+1}^2}\right)\right) \tag{7}$$

$$\text{Minimize } f_2(\vec{x}) = \sum_{i=1}^{n}\left(|x_i|^{0.8} + 5\sin(x_i)^3\right) \tag{8}$$

where: $-5 \leq x_1, x_2, x_3 \leq 5$

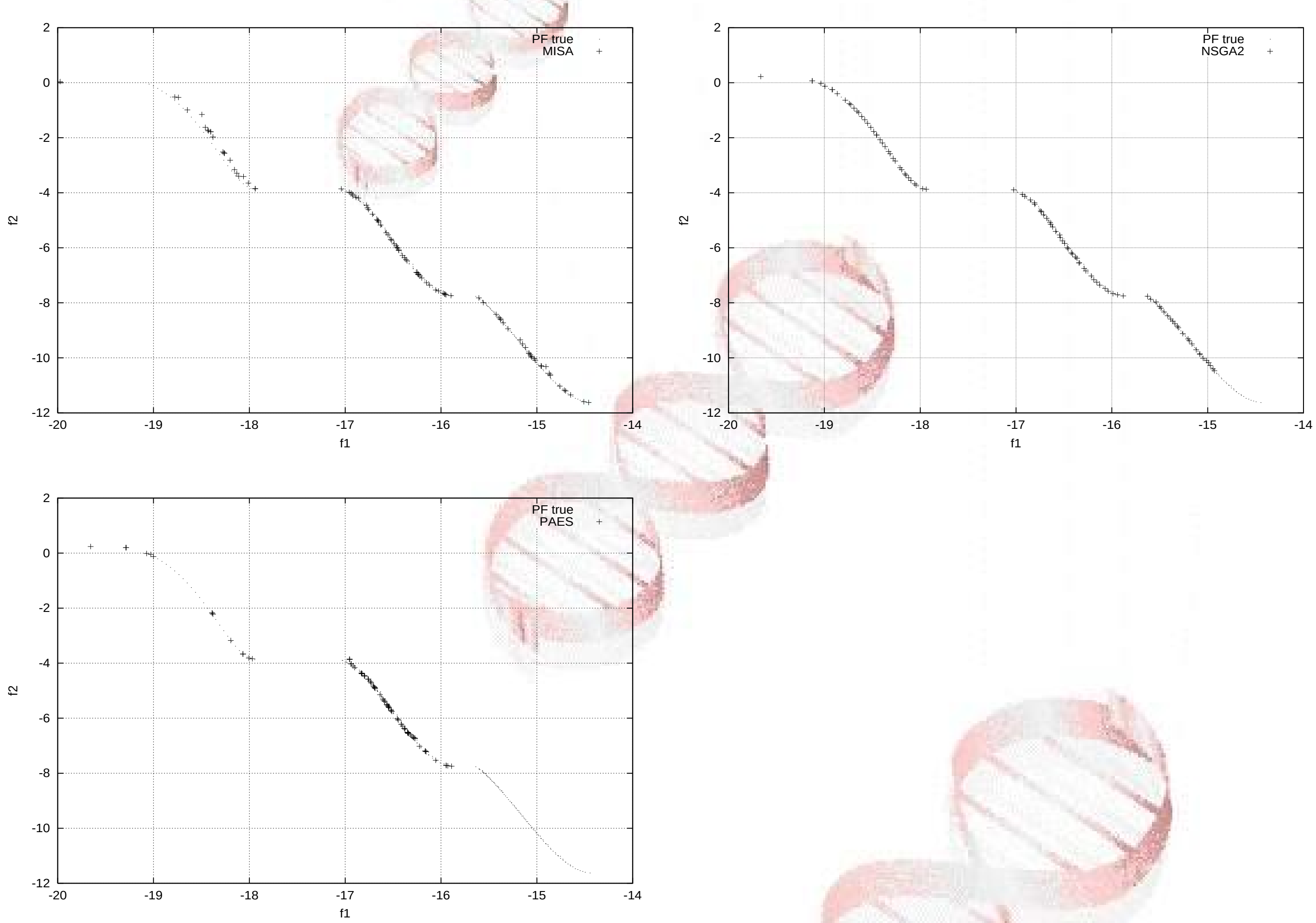


Figure 2: Pareto front obtained by MISA, NSGA-II and PAES, example 2

Publications

- Carlos Coello-Coello and Nareli Cruz-Cortés, "Use of emulations of the immune system to handle constraints in evolutionary algorithms", Intelligent Engineering Systems through Artificial Neural Networks (ANNIE'2001), St. Louis Missouri, USA, November 2001. Best paper award.

- Carlos Coello-Coello and Nareli Cruz-Cortés, "A Parallel Implementation of an Artificial Immune System to Handle Constraints in Genetic Algorithms: Preliminary Results", Congress on Evolutionary Computation (CEC'2002), IEEE, Honolulu, Hawaii, USA, May 2002.

Publications

- Carlos Coello-Coello and Nareli Cruz-Cortés, ”An Approach to Solve Multiobjective Optimization Problems Based on an Artificial Immune System”, First International Conference on Artificial Immune Systems (ICARIS-2002), University of Kent at Catenbury, UK, September 9th-11th, 2002.
- Nareli Cruz-Cortés, and Carlos Coello-Coello, ”Multiobjective Optimization using the Clonal Selection Principle”, in Genetic and Evolutionary Computation Conference (GECCO'2003), Chicago, Illinois, USA, July 2003.

Conclusions

We have presented the design and analysis of algorithms using ideas from the immune system combining them with genetic algorithms (GA) to solve numerical optimization problems:

- An artificial immune system to handle constraints in genetics algorithms to solve single objective optimization problems, as well as a parallel version of that. The parallel version shows remarkable speedups.

conclusions

- An algorithm taking ideas from the clonal selection principle of the immune system to solve multiobjective optimization problems, with or without constraints.

The algorithms were compared againts other ones which are representative of the state-of-the-art. The results are competitive even the proposed approach is (conceptually) very simple.

Future work

- To design an algorithm based on the clonal selection principle to handle constraints in genetic algorithms, and compare it against the previous algorithm.
- To improve the distribution of the solutions along the Pareto front found by our multiobjective optimization algorithm
- To design a mathematical model of one of the artificial immune systems proposed