Mechanical Component Design for Multiple Objectives Using Elitist Non-Dominated Sorting GA

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Abstract. In this paper, we apply an elitist multi-objective genetic algorithm for solving mechanical component design problems with multiple objectives. Although there exists a number of classical techniques, evolutionary algorithms (EAs) have an edge over the classical methods in that they can find multiple Pareto-optimal solutions in one single simulation run. The proposed algorithm (we call NSGA-II) is a much improved version of the originally proposed non-dominated sorting GA (NSGA) in that it is computationally faster, uses an elitist strategy, and it does not require fixing any niching parameter. On four mechanical component design problems borrowed from the literature, we show that the NSGA-II can find a much wider spread of solutions than classical methods and the NSGA. The results are encouraging and suggests immediate application of the proposed method to other more complex engineering design problems.

1 Introduction

Most engineering design problems involve multiple and often conflicting objectives. In principle, the presence of conflicting objectives results in a number of optimal solutions, commonly known as Pareto-optimal solutions. Since no one Pareto-optimal solution can be said to be better than another without further considerations, it is desired to find as many such Pareto-optimal solutions as possible. For the last decade or so, a number of multi-objective evolutionary algorithms (MOEAs) have been suggested, mainly because of the their ability to find multiple Pareto-optimal solutions in one single simulation run. The non-dominated sorting genetic algorithm (NSGA) was one such algorithm suggested by Srinivas and Deb in the year 1994 [10]. NSGA and a few other algorithms were mainly generational in approach and did not use any elitism. Realizing the need of elitism for faster convergence, researchers have recently introduced elitism in the paradigm of MOEAs [12, 6]. Recently, we have proposed a fast elitist NSGA, we called NSGA-II [4], to alleviate three major difficulties of NSGA: (i) large computational effort for non-dominated sorting, (ii) no preservation of elites, and (iii) need to fix a niche parameter. NSGA-II has also possess the lowest possible computational complexity achievable with any non-dominated sorting approach. NSGA-II has been shown to outperform PAES—another elitist EA which is explicitly designed for maintaining spread among non-dominated solutions—in terms of spread of trade-off solutions on a number of difficult test problems.

In this paper, we investigate the efficiency of NSGA-II in finding diverse Pareto-optimal front in a number of engineering design problems. In a two-bar truss design problem, NSGA-II is compared with a classical ϵ-constraint method. In a gear train design, a spring design, and a welded beam design problem, NSGA-II is compared with the original NSGA and with the best reported single-objective optimizers.

2 Non-dominated Sorting GA (NSGA)

Srinivas and Deb [10] proposed NSGA in 1994 for multi-objective optimization. NSGA is different from a single-objective GA in the way the fitness is assigned to individuals. In order to assign fitness, first the population is sorted according to their non-domination level. Thereafter, all solutions of the first (best) non-dominated front is assigned a large dummy fitness value. In order to preserve diversity among solutions of this front, a niche-preservation technique (sharing function method) is used to find a shared fitness value for each solution in the front. The dummy fitness value is degraded according to the density of solutions in the neighborhood of each solution. Thereafter, the solutions in the second front are assigned a dummy fitness value smaller than the smallest shared fitness value of the previous front. Once again, the shared fitness values of the second front is found by using the sharing strategy. This process continues till all solutions are assigned a shared fitness. A proportionate selection method is used with the shared fitness values. Search operators are used as usual. On a number of test problems [10] and on a number of engineering design problems [7, 11], NSGA is reported to find a number of non-dominated solutions.

However, NSGAs have been criticized for the following three reasons: (i) the non-dominated sorting approach is $O(N^3)$, where N is the population size, (ii) no elitism approach is used, and (iii) a sharing parameter σ_{share} needs to be fixed. Recently, we have developed an improved NSGA (we called NSGA-II), which eliminates all these difficulties. In the following subsection, we briefly describe that algorithm.

3 The Elitist Non-dominated Sorting GA: NSGA-II

Initially, a random parent population P_0 is created. The population is sorted based on the non-domination. A special book-keeping procedure is used in order to reduce the computational complexity down to $O(N^2)$ [4]. Each solution is assigned a fitness equal to its non-domination level (1 is the best level). Thus, minimization of fitness is assumed. Binary tournament selection, recombination, and mutation operators are used to create a child population Q_0 of size N. Thereafter, we use the following algorithm in every generation. First, a combined population $R_t = P_t \cup Q_t$ is formed. This allows parent solutions to be compared with the child population, thereby ensuring elitism. The population R_t is of size $2N$. Then, the population R_t is sorted according to non-domination. The new parent population P_{t+1} is formed by adding solutions from the first front and continuing to other fronts successively till the size exceeds N. Thereafter, the solutions of the last accepted front are sorted according to a *crowded comparison criterion* and the first N points are picked. Since the diversity among the solutions is important, we use a partial order relation $\geq_n$ as follows:

$i \geq_n j$ if ($i_{rank} < j_{rank}$) or (($i_{rank} = j_{rank}$) and ($i_{distance} > j_{distance}$))

That is, between two solutions with differing nondomination ranks we prefer the point with the lower rank. Otherwise, if both the points belong to the same front then we prefer the point which is located in a region with lesser number of points (or with larger crowded distance). This way solutions from less dense regions in the search space are given importance in deciding which solutions to choose from R_t. This constructs the population P_{t+1}. This population of size N is now used for selection, crossover and mutation to create a new population Q_{t+1} of size N. We use a binary tournament selection operator but the selection criterion is now based on the crowded comparison operator $\geq_n$. The above procedure is continued for a specified number of generations.

It is clear from the above description that NSGA-II uses (i) a faster non-dominated sorting approach, (ii) an elitist strategy, and (ii) no niching parameter. Diversity is preserved by the use of crowded comparison criterion in the tournament selection and in the phase of population reduction. NSGA-II has been shown to outperform other current elitist multi-objective EAs on a number of difficult test problems [4].

4 Mechanical Component Design Problems

In the following, we discuss four mechanical component design problems which we have studied. Although different problems have been tried, we have kept the GA parameters same in all problems: Population size of 100, crossover probability of 0.8, mutation probability of $1/n$ (where n is the number of variables) are used. The SBX operator [3] with a spread factor of 20 and the real-parameter mutation operator [2] with a spread factor of 500 are used. In all simulations, we run for a maximum of 100 generations. All constraints are normalized and a sum of all constraint violations is added to all objective functions.

4.1 Two-bar truss design

This problem was originally studied using the ϵ-constraint method [8]. The truss (Figure 1) has to carry a certain load without elastic failure. Thus, in addition to the objective of designing the truss for minimum volume (which is equivalent to designing for minimum cost of fabrication), there are additional objectives of minimizing stresses in each of the two members AC and BC. We construct the following two-objective optimization problem for three variables y (vertical distance between B and C in m), x_1 (length of AC in m) and x_2 (length of BC in m):

$$\begin{array}{ll} \text{Minimize } f_1(\boldsymbol{x}) = x_1\sqrt{16+y^2} + x_2\sqrt{1+y^2} \\ \text{Minimize } f_2(\boldsymbol{x}) = \max(\sigma_{\text{AC}}, \sigma_{\text{BC}}) \\ \text{subject to } \max(\sigma_{\text{AC}}, \sigma_{\text{BC}}) \leq 1(10^5) \\ \qquad\qquad 1 \leq y \leq 3 \quad \text{and} \quad \boldsymbol{x} \geq \boldsymbol{0} \end{array} \tag{1}$$

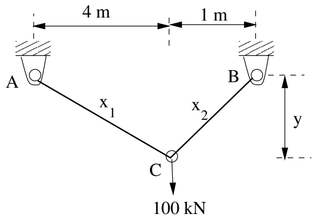

Fig. 1. The two-bar truss is shown.

The stresses are calculated as follows:

$$\sigma_{\mathrm{AC}} = \frac{20\sqrt{16+y^2}}{yx_1} \qquad \sigma_{\mathrm{BC}} = \frac{80\sqrt{1+y^2}}{yx_2}$$

The original study reported only five solutions with the following spread: (0.004445 m^3, 89983 kPa) and (0.004833 m^3, 83268 kPa). In order to restrict solutions with stress in the above range, we have added an additional constraint of maximum stress being smaller than $1(10^5)$. A penalty parameter of 10^3 is used to handle this constraint. We apply the proposed method with $0 \leq x_i \leq 0.01$. Figure 2 shows the optimized front found using the proposed method. The solutions are spread in the following range: (0.00407 m^3, 99755 kPa) and (0.05304 m^3, 8439 kPa), which indicates the power of NSGA-II compared to the ϵ-constraint method. The ϵ-constraint method could not find wide variety of solutions in terms of the second objective. If minimization of stress is important, NSGA-II finds a solution with stress as low as 8439 kPa, whereas the ϵ-constraint method has found a solution with minimum stress of 83268 kPa, an order of magnitude higher than that found in NSGA-II. What is also important that all these solutions have been found in just one simulation run of NSGA-II. NSGA-II solutions are better than NSGA solutions, both in terms of closeness to the optimum front and in their spread.

4.2 Gear train design

A compound gear train is to be designed to achieve a specific gear ratio between the driver and driven shafts (Figure 3). The objective of the gear train design is to find the number of teeth in each of the four gears so as to minimize (i) the error between the obtained gear ratio and a required gear ratio of 1/6.931 [5] and (ii) the maximum size of any of the four gears. Since the number of teeth must be integers, all four variables are strictly integers. By denoting the variable vector $x = (x_1, x_2, x_3, x_4) = (T_d, T_b, T_a, T_f)$, we write the two-objective optimization problem:

$$\begin{array}{l} \text{Minimize } f_1(\boldsymbol{x}) = \left[\frac{1}{6.931} - \frac{x_1 x_2}{x_3 x_4}\right]^2 \\ \text{Minimize } f_2(\boldsymbol{x} = \max(x_1, x_2, x_3, x_4) \\ \text{Subject to } 12 \leq x_1, x_2, x_3, x_4 \leq 60, \\ \qquad\qquad \text{all } x_i\text{'s are integers.} \end{array}$$

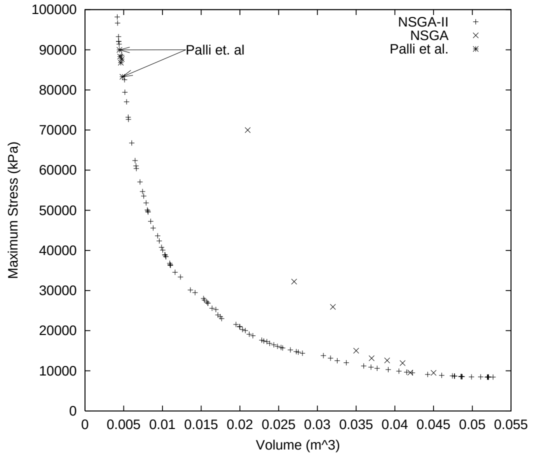

Fig. 2. Optimized solutions obtained using the NSGA-II and NSGA for the two-bar truss problem. Five solutions found in [8] are also shown for comparison.

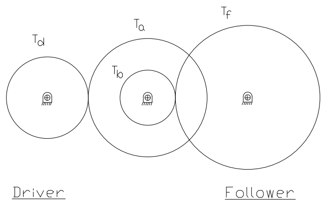

Fig. 3. A compound gear train is shown.

A discrete version of SBX operator is used to make sure that only integer children solutions are created from two integer parents. Rigid bounds are used for each variable in order to handle the constraints. Figure 4 shows the obtained optimized solutions. The solutions obtained by the single objective GAs (GeneAS-I and GeneAS-II) [2], by the augmented Lagrangian (AL), and the branch-and-bound (BB) methods for the error minimization are also shown. The figure shows that although the proposed method could not find the best single-objective solutions (GeneAS-I and II), the solution E is close to them and is better than all other single-objective optimizers. Both GeneAS solutions are very sensitive to the variables and using a multi-objective optimization algorithms, it may be difficult to find the individual optimum solutions in this problem. Nevertheless, the plot shows the spread in solutions obtained by the proposed method. The solutions marked as ‘E’ and ‘D’ are shown in the following table:

Solution	x_1	x_2	x_3	x_4	Error	Max. Diameter
E	12	12	27	37	$1.83(10^{-8})$	37
D	12	12	30	30	$2.47(10^{-4})$	30

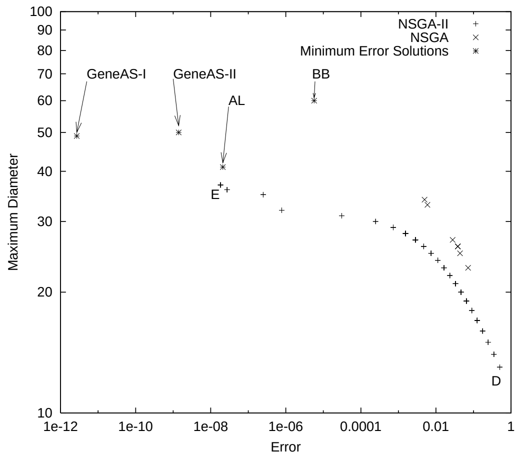

Fig. 4. Optimized solutions obtained using NSGA-II and NSGA for the gear train design problem. Previously found solutions with GeneAS-I and II, and augmented Lagrangian (AL) and branch-and-bound (BB) methods are also shown.

The table shows that a wide variety of optimal solutions have been obtained. NSGA solutions are not as good as NSGA-II solutions.

4.3 Spring design

A helical compression spring needs to be designed for minimum volume and for minimum stress. Three variables are identified: The number of spring coils N, the wire diameter d, and the mean coil diameter D. Of these variables, N is an integer variable, d is a discrete variable having 42 non-equispaced values as given in [5], and D is a real-parameter variable. Denoting the variable vector $x = (x_1, x_2, x_3) = (N, d, D)$, we write the two-objective optimization problem:

$$
\begin{aligned}
\text{Minimize } & f_1(\boldsymbol{x}) = 0.25\pi^2 x_2^2 x_3 (x_1 + 2) \\
\text{Minimize } & f_2(\boldsymbol{x}) = \frac{8KP_{\max} x_3}{\pi x_2^3} \\
\text{Subject to } & g_1(x) = \ell_{\max} - \frac{P_{\max}}{k} - 1.05(x_1 + 2)x_2 \geq 0, \\
& g_2(x) = x_2 - d_{\min} \geq 0, \\
& g_3(x) = D_{\max} - (x_2 + x_3) \geq 0, \\
& g_4(x) = C - 3 \geq 0, \\
& g_5(x) = \delta_{pm} - \delta_p \geq 0, \\
& g_6(x) = \frac{P_{\max} - P}{k} - \delta_w \geq 0, \\
& g_7(x) = S - \frac{8KP_{\max} x_3}{\pi x_2^3} \geq 0 \\
& g_8(x) = V_{\max} - 0.25\pi^2 x_2^2 x_3 (x_1 + 2) \geq 0 \\
& x_1 \text{ is integer, } x_2 \text{ is discrete, } x_3 \text{ is continuous.}
\end{aligned}
$$

The parameters used above are as follows:

$$
\begin{array}{lll}
K = \frac{4C-1}{4C-4} + \frac{0.615x_2}{x_3} & P = 300\text{lb} & D_{\max} = 3\text{in} \\
k = \frac{Gx_2^4}{8x_1x_3^3} & P_{\max} = 1000\text{lb} & \delta_w = 1.25\text{in} \\
\delta_p = \frac{P}{k} & \ell_{\max} = 14\text{in} & \delta_{pm} = 6\text{in} \\
S = 1,89,000\text{kpsi} & d_{\min} = 0.2\text{in} & C = D/d
\end{array}
$$

We add the last two constraints to restrict the stress to be within allowable strength and the volume to be within a pre-specified volume of $V_{\max} = 30$ in^3. Discrete version of SBX is used to handle the first two variables and the continuous version of SBX is used to handle the third variable. A penalty parameter of 10^3 is used for each normalized constraint.

Figure 5 shows the non-dominated front obtained using the proposed algorithm and the best solutions obtained using two single-objective optimizers—GeneAS [2] and the branch-and-bound (BB) method [5]. The proposed method is able to find solutions close to these single-objective (volume) optimum and, most importantly, is able to maintain a wide spread of different solutions. The extreme solutions are presented in the following table:

Solution	x_1	x_2	x_3	f_1	f_2
Min. volume	5	0.307	1.619	2.690	1,87,053
Min. stress	24	0.500	1.865	24.189	61,949

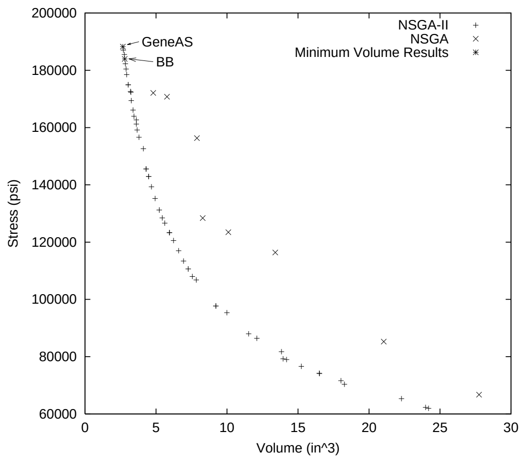

Fig. 5. Optimized solutions obtained using NSGA-II and NSGA for the spring design problem. Previously found solutions with GeneAS and branch-and-bound (BB) methods are also shown.

Once again, NSGA-II solutions are better than NSGA solutions.

4.4 Welded beam design

A beam needs to be welded on another beam and must carry a certain load F (Figure 6). The overhang portion of the beam has a length of 14 inch and $F = 6,000$ lb force is

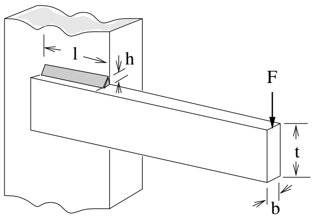

Fig. 6. The welded beam design problem. Minimizations of cost and end deflection are two objectives.

applied at the end of the beam. The objective of the design is to minimize the cost of fabrication and minimize the end deflection. In the following, we formulate the two-objective optimization problem:

$$\begin{array}{rl} \text{Minimize} & f_1(\boldsymbol{x}) = 1.10471h^2\ell + 0.04811tb(14.0+\ell), \\ \text{Minimize} & f_2(\boldsymbol{x}) = \delta(\boldsymbol{x}), \\ \text{Subject to} & g_1(\boldsymbol{x}) \equiv 13,600 - \tau(\boldsymbol{x}) \geq 0, \\ & g_2(\boldsymbol{x}) \equiv 30,000 - \sigma(\boldsymbol{x}) \geq 0, \\ & g_3(\boldsymbol{x}) \equiv b - h \geq 0, \\ & g_4(\boldsymbol{x}) \equiv P_c(\boldsymbol{x}) - 6,000 \geq 0. \end{array} \tag{2}$$

The deflection term $\delta(\boldsymbol{x})$ is given as follows:

$$\delta(\boldsymbol{x}) = \frac{2.1952}{t^3 b}.$$

The first constraint makes sure that the shear stress developed at the support location of the beam is smaller than the allowable shear strength of the material (13,600 psi). The second constraint makes sure that normal stress developed at the support location of the beam is smaller than the allowable yield strength of the material (30,000 psi). The third constraint makes sure that thickness of the beam is not smaller than the weld thickness from a practical standpoint. The fourth constraint makes sure that the allowable buckling load (along t direction) of the beam is more than the applied load F. The stress and buckling terms are given as follows [9]:

$$\begin{aligned} \tau(\boldsymbol{x}) &= \sqrt{(\tau')^2 + (\tau'')^2 + (\ell\tau'\tau'')/\sqrt{0.25(\ell^2+(h+t)^2)}}, \\ \tau' &= \frac{6,000}{\sqrt{2}h\ell}, \end{aligned}$$

$$\tau'' = \frac{6,000(14 + 0.5\ell)\sqrt{0.25(\ell^2 + (h+t)^2)}}{2\left\{0.707h\ell(\ell^2/12 + 0.25(h+t)^2)\right\}},$$
$$\sigma(\boldsymbol{x}) = \frac{504,000}{t^2 b},$$
$$P_c(\boldsymbol{x}) = 64,746.022(1 - 0.0282346t)tb^3.$$

The variables are initialized in the following range: $0.125 \leq h, b \leq 5.0$ and $0.1 \leq \ell, t \leq 10.0$. Penalty parameters of 100 and 0.1 are used for the first and second objective functions, respectively.

Figure 7 shows the non-dominated solutions obtained using NSGA and NSGA-II. It is clear that NSGA-II is able to find a wider distribution of solutions than NSGA. NSGA-II found the best cost solution with a cost of 2.79 units, which is close to the best solution (with a cost of 2.38 units) found using a single-objective GA [1].

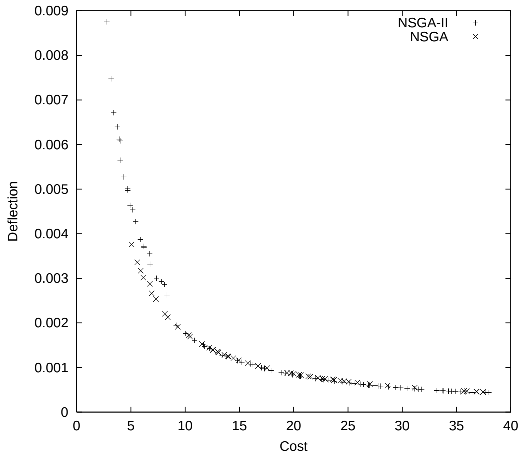

Fig. 7. Non-dominated solutions obtained using NSGA-II and NSGA for the welded beam design problem.

5 Conclusions

In this paper, we have used a modified version of the non-dominated sorting GA (or NSGA) for finding multiple Pareto-optimal solutions in a number of engineering design problems. The NSGA-II is different from its predecessor NSGA in a number of ways: (i) it uses a computationally faster non-dominated sorting approach, (ii) it uses an elitist strategy, thereby not allowing good solutions to be deleted by genetic operators, and (iii) it eliminates the need of any niching parameter. The results on four engineering design problems show that a wide spread of solutions have been obtained.

In a two-member truss design problem, NSGA-II has found many trade-off solutions compared to only 5 solutions reported in the literature using the ϵ-constraint method. In all problems, NSGA-II finds a front better and wider than that found by NSGA. The results of this study are encouraging. The study offers a computationally fast, an elitist, and a parameter-less multi-objective optimizer which should ease the way to solving complex engineering multi-objective optimization problems.

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