A STUDY OF THE CAPACITY OF THE STOCHASTIC HILL CLIMBING TO SOLVE MULTI-OBJECTIVE PROBLEMS

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ABSTRACT

This paper presents a study of the ability of the Stochastic Hill Climbing algorithm to solve multi-objective problems. In these problems exist two or more functions to be optimized and there is not a criterion to select between two solutions unless one of them dominates the other according to the Pareto's concept. In these problems it is desired to obtain the highest quantity of non-dominated solutions. This work shows that Stochastic Hill Climbing is able to have a good performance in this kind of problems, and the advantages of restarting the search when it is stagnated. Successful results are shown in comparison with others obtained via Genetic Algorithms.

1.INTRODUCTION

In many real-world problems, the users desire to obtain solutions that satisfy more than one objective. In some cases, exists a clear relationship between the objectives to be optimized, and that permits their conversion into a single-objective function, which is the combination of all the others by a weighted sum. In other situations this relationships can't be established. That's the case of the multi-objective problems.

In the field of the Evolutionary Algorithms exists a big interest for this kind of problems. Coello [1] has made a recent revision of the state-of-the-art in that field. Although the use of the weighted sums to face this kind of problems is very common, the convenience of other alternatives that permits the user to choose between two possible solutions is undeniable.

In that way, the objective of the multi-objective problems is to find the highest possible quantity of non dominated solutions. One solution dominates another when it is better in at least one criterion to be optimized and it is not worst in any of the others criteria. For example, if solution A is better than solution B by the criteria C1 and is worst in criteria C2 neither of the solutions dominates the other. The set of all non-dominated solutions is named Pareto Set. Summarizing, the objective of a multi-objective optimization is to find the highest quantity of the Pareto Set solutions, giving them to the user or decision-maker to make the final decision.

It is very common to believe that the Evolutionary Algorithms [7] have a special advantage to this kind of problems due to the existence of population of solutions. In this work we show that a simple method (Stochastic Hill Climbing) can be successfully used for this kind of problems. This research, continues other works that have tried to establish the conditions in which an optimization method surpasses another [8][11][4], and led to Wolpert & MacReady's NFL Theorem [12]. This theorem states that is not possible to establish the superiority of a method over any other when they are averaged over all the possible functions. In [6][5][10] can be found many real-world problems in which a simple Stochastic Hill Climbing surpasses other methods as the Genetic Algorithms [2], Taboo Search or Simulated Annealing.

This work follows that research line in the multi-objective optimization field.

2. ADAPTATION OF THE METHODS TO SOLVE MULTI-OBJECTIVE PROBLEMS

In order to solve multi-objective problems, the traditional meta-heuristic methods (e.g. Genetic Algorithm) need to be modified. This section presents some of these modifications that are relevant to this paper.

2.1 Genetic Algorithms

Genetic Algorithms are the most popular Evolutionary Algorithms. They mainly consist of three main processes: Selection, Crossover and Mutation. The work of Coello [1] provides a good source of information about the solution of multi-objective problems via Evolutionary Algorithms. To the objectives of our work, only three modifications of the traditional methods will be explained:

Vector Evaluated Genetic Algorithm (VEGA) [2][3]: The change consists of the split of the population to create fractions or groups which will be evaluated and submitted to selection considering only one objective. Thus, if there are two objectives, one half of the population will be evaluated and selected according one objective function, and the other half according the second function. VEGA's weakness is its tendency to return only the extreme solutions of the Pareto Set, in other words: those, which optimize only one of the many objectives [3][9].

Niched Pareto Genetic Algorithm (NPGA) [3]: NPGA presents a modification of the tournament selection [2]: to choose between two individuals, the dominance between them is considered and compared with a random selected group of other individuals. If the dominance relationship can't be established it is chosen the less common individual in regard of its phenotypic characteristics (its evaluation values). That technique is known as "sharing" in the GAs and its main purpose is to guarantee the individuals diversity.

Multi-Objective Genetic Algorithm (MOGA) [9]: Here all the objectives are integrated in a single-objective function via a weighted sum. The weights are chosen in a random way in each generation. In that manner the direction of the search will vary during the evolution, leading to a greater distribution of the obtained Pareto Set.

2.2 Stochastic Hill Climbing

The Stochastic (SHC) is a very simple method [5][6][10]. It starts from an initial random solution. Later a set of iterations is made, in each one of them an unary mutation operator is applied over the current solution. If the new solution is better than the previous one, then the new solution will be taken as the current solution for the next iteration. Many works have shown the convenience of also accepting a new solution if their evaluation is equal to the one of the current solution [5][6][10].

A simple manner to make the SHC useful in multi-objective problems is the modification of the comparison to set it in function of the dominance. In that way, the modified SHC, accepts a new solution if this one is not dominated by the current solution. As a complement, a list of non-dominated solution is maintained. Thus, each time a new solution that is not dominated by the current solution is found, it is compared with the whole list to delete the new dominated solutions. When the iterations finish, the list will contain all the solutions of the Pareto group that were obtained with no duplicates. We named this method Multi-Objective Stochastic Hill Climbing (MOSHC).

It is also very accepted that SHC improved its performance if the search is restarted whenever the search is stagnated. .We also made an adaptation of the MOSHC algorithm with the inclusion of this "restart" technique, which consists in checking if all the possible mutations to the current solution were made, and if so, it is erased and a new individual is randomly generated to be the new current solution. This modification leads to a method named Multi-Objective Stochastic Hill Climbing with Restart (MOSHC-R).

In the future, another SHC alternatives to face multi-objective problems could be studied. For example, it is possible to randomize the weighted vector as in MOGA [9] or to stimulate diversity with the "sharing" mechanism as in the NPGA [3].

3. EXPERIMENTAL RESULTS

To study the performance of MOSHC and MOSHC-R, three problems where tested:

Units & Pairs in a 12-bits-length string (UP12). Here, the functions to optimize are the quantity of bits with value one (1), and the quantity of pairs of consecutive different bits (01 or 10) in binary strings. For example, in the string 111101111111 the quantity of '1's is 11 and has only 2 pairs. This function was used in [3][9]. The optimal Pareto set is formed by the solutions with the following values (Units, Pairs): (12,0), (11,2), (10,4), (9,6), (8,8), (7,10) y (6,11). In [3] it is said that although this problem seems very artificial, it has a big relationship with structural design problems.

Units & Pairs in a 28-bits-length string (UP28). It is similar to the previous one, but for a bigger string. It was used in [3]. The optimal Pareto group is formed by the solutions with the following values (Units, Pairs): (28,0), (27,2), (26,4), (25,6), (24,8), (23,10), (22,12), (21,14), (20,16), (19,18), (18,20), (17,22), (16,24), (15,26) y (14,27).

Schaffer's F2. The objective functions are X^2 and $(X-2)^2$ where [-6,6]. The X variable is codified in a 14-bits-length string. The optimal Pareto set contains the 2730 solutions in the interval [0,2].

In all cases the mutation used in MOSHC consisted in the random mutation of a single bit.

In [3][9] the frequency of obtaining their results is not shown, but they said that the showed results are typical. For the MOSHC, 20 independent executions were made.

In [3][9], the duplicated solutions are not eliminated. For example, for UP12 only one solution exists where Units=12, that is (12,0). However, it is reported that NPGA obtains 26 solutions of this kind, VEGA obtains 38 and MOGA 3, in the final population of 100 individuals. This is a difficulty to make a most fair comparison with MOSHC which eliminates this duplication. The experimental results obtained by MOSHC and its comparison with MOGA, VEGA and NPGA in each problem are shown below.

UP12: The three studied genetic algorithms had a population of 100 individuals and were executed during 100 generations [3][9]. For MOSHC, were established 10000 iterations as an equivalent. MOGA, regularly found solutions of the seven kinds. NPGA didn't find solutions of (6,11). VEGA didn't encounter solutions with values (10,4) nor (9,6). The results obtained for MOSHC are shown in Table 1.

According the obtained Pareto group size and eliminating the evident duplicates of the (12,0) solution, NPGA obtains 75 "different" solutions, VEGA obtains 40, and MOGA 64. The word different is quoted, because there is no warranty that they are really different, because other type of solutions (different to (12,0)) are possibly presented in the final set. This value is only the maximum of the really different solutions found by each method. MOSHC obtained an average of 58.55 really different Pareto solutions.

Kinds of Solutions (Units, Pairs)	Occurrence %
(6,11)	45
(7,10)	65
(8,8)	100
(9,6)	100
(10,4)	100
(11,2)	100
(12,0)	55

Table 1: Performance of the algorithms in problem UP12

UP28: For this problem we have the results obtained by NPGA after 200 generations, with 400 individuals each [3]. As an equivalent, MOSHC was allowed to do 80000 evaluations. NPGA got solutions of all the kinds but (14,27), and 376 "different" solutions after deleting the evident duplicates of (28,0). However, when the "sharing" mechanism is omitted in the NPGA, no solutions for (28,0), (27,2), (26,21), (25,6), (14,27) are obtained. MOSHC results are shown in Table 2.The average of the found Pareto solutions was of 22428.3, widely overcoming the NPGA results.

Kinds of Solutions (Units,Pairs)	Ocurrence %
(14,27)	5
(15,26)	45
(16,24)	80
(17,22)	95
(18,20)	100
(19,18)	100
(20,16)	100
(21,14)	100
(22,12)	100
(23,10)	95
(24,8)	90
(25,6)	90
(26,4)	90
(27,2)	70
(28,0)	10

Table 2: Performance of the algorithms in problem UP12

Schaffer's F2: Here, only were available the results for NPGA and VEGA [3], after 200 generations and a population of size 30. In correspondence, the MOSHC worked with 60000 evaluations of the objective functions. Both NPGA and VEGA obtained 2 non-optimal (dominated) solutions in the final population of 30 individuals, and consequently, a 28-solutions Pareto optimal group. The difference between them, lays on the Pareto group distribution, NPGA obtains a Pareto group much better distributed in [0,2], while VEGA almost hasn't solutions for the intervals [0,0.2], [0.5,1.2] and [1.5,1.8]. MOSHC found an average of 875.6 in the Pareto set. It also represents the 32.7% of all the possible Pareto solutions. This solutions are well distributed on the interval [0,2].

In summary, MOSHC is able to have a general performance very similar to the GA. In some cases it is much better, although in certain executions was observed a convergence to values that can not generate individuals better than themselves via mutation, and that, provokes a little quantity of solutions, which could even not belong to the Pareto group.

To solve this situation, we modified MOSHC by using the "restart" technique producing the Multi-Objective Stochastic Hill Climbing with Restart (MOSHC-R). It was tested in the three previous problems with the same number of iterations and executions than the original MOSHC. The results for each case were the following.

UP12: It was found an average of 100.25 Pareto solutions. Those of (6,11) kind appeared the 55% of the times, (7,10) the 95% and (12,0) the 90%. All the other valid types occurred always.

UP28: The Pareto solutions average was of 33156.2, occurring all the other kinds the 100% of the times, but (14,27) and (28,0) that appeared the 15% and (15,26) the 75%.

Schaffer's F2: An average of 1869.35 solutions of the Pareto group were found, representing the 68.47% of the possible valid solutions.

According the results exposed above, MOSHC-R widely overcomes the original method and the other GA variants, in both quantity and quality of the final results due to its ability to escape from the influence of weak individuals which can't generate further better solutions.

4.CONCLUSIONS

In this work were proposed two modifications of Stochastic Hill Climbing (SHC) in order to solve multi-objective problems: Multi-Objective Stochastic Hill Climbing (MOSHC), and Multi-Objective Stochastic Hill Climbing with Restart (MOSHC-R)

We have proved that the original MOSHC has a similar or better performance than the GA, even when in certain cases the MOSHC performance was affected due to a convergence to values that provoke the algorithm to get stucked. On the other hand, MOSHC-R has much better behavior than the Genetic Algorithm versions in these multi-objective problems. The adaptation made to MOSHC to enhance its performance, the MOSHC-R, demonstrated being very superior in all cases with respect to their ancestor and the GAs adaptations studied in [3][9] showing an increase in quantity and quality of the obtained results.

Although the proposed algorithms are very simple, the results obtained were successful.

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