VECTOR SHAPE OPTIMIZATION OF AN ELECTROSTATIC MICROMOTOR USING A GENETIC ALGORITHM

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Abstract. The automated shape optimization of an electrostatic micromotor with radial field is tackled. Two objectives in mutual contrast i.e. static torque and torque ripple, depending on two design variables, are considered. An innovative procedure for vector optimization which aims at obtaining as many optimal solutions as possible, is presented. To this end, a non-dominated sorting genetic algorithm (NSGA) is set up, linking Pareto Optima Theory and Genetic Algorithms. This way, fifty different optimal solutions lying on the Pareto optimal front are obtained. This procedure gives the designer a wide set of optimal solution, each of which corresponds to a different degree of preference with respect to the single objectives.

Keywords: Electrostatic micromotor, shape design, multiobjective optimization, Pareto optima, genetic algorithm.

Introduction

The optimal design of electromagnetic devices and, in particular, of electrostatic micromotors usually is characterized by objectives in mutual contrast, giving rise to a multicriteria optimization problem (Di Barba, 1999). There are two different ways to tackle such a problem. The first one consists of building a single scalar objective function by combining the single objectives in a suitable way. This approach leads to classical multiobjective optimization methods and gives a solution which is supposed to be the optimum. This procedure seems to be highly arbitrary in the choice of both the scalarization criterion and the weighting coefficients implied. The second way to tackle the problem consists of applying the Pareto optima theory (Deb, 1999) linking it to an optimization genetic algorithm (Goldberg, 1989). The result is a family of non-dominated solutions and the procedure does not imply any arbitrary choice.

Design problem

We dealt with a variable-capacitance rotating microactuator with radial field (Serrault, 1993). A simplified scheme of the geometry of the device is shown in Fig. 1. The device, etched on a polySilicon structure, is characterized by 24 or 30 stator electrodes and 16 or 10 rotor teeth, respectively (3/2 geometry or 3/1 geometry). The radial dimensions [μm] of the device are: R_2=22, R_3=40, R_4=60, R_5=63.

Referring to Fig. 1, denoting $x_1=\alpha$ and $x_2=\beta$, we switch on one phase of the 3-phase system of square voltages of amplitude equal to V = 100 V and consider the equivalent capacitance $C_{eq}(\varphi,x_1,x_2)$ where φ is the rotor angle.

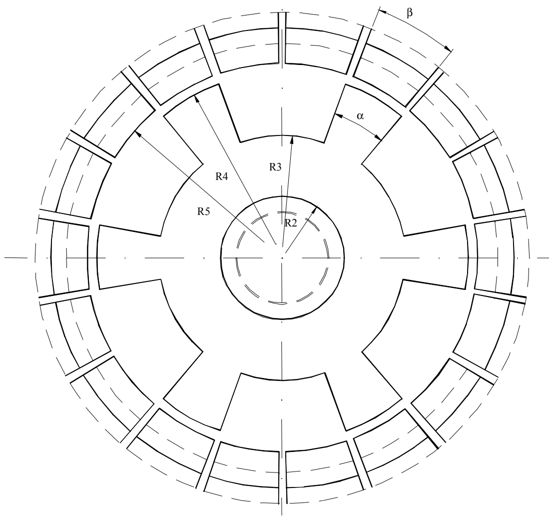

Fig 1 - Cross section of the device.

The basic formula for computing both no-load commutation torque Γ_0 and static torque Γ_S is

$$\Gamma(\varphi, x_1, x_2) = \frac{1}{2} V^2 \frac{\partial C_{eq}(\varphi, x_1, x_2)}{\partial \varphi} \tag{1}$$

When Γ_0 has to be computed, eq. (1) becomes

$$\Gamma_0(x_1, x_2) = \frac{1}{2} V^2 N_S \frac{\left[C_{eqA}(x_1, x_2) - C_{eqB}(x_x, x_2)\right]}{2\pi} \tag{2}$$

where $C_{eqA}(x_1,x_2)$ is the capacitance of the maximum coenergy configuration in which the axis of the supplied electrode is coincident with the axis of the rotor tooth; $C_{eqB}(x_1,x_2)$ is the equivalent capacitance when the rotor position is the same, but the supply has been switched to the next phase and N_S is the number of stator electrodes. When Γ_S has to be computed, eq. (1) becomes:

$$\Gamma_S(\varphi, x_1, x_2) = \frac{1}{4} V^2 N_R \left[C_{\max}(x_1, x_2) - C_{\min}(x_1, x_2)\right] sin(N_R \varphi) \tag{3}$$

where $C_{max}(x_1,x_2)$ and $C_{min}(x_1,x_2)$ are the maximum and minimum capacity with respect to the rotor angle φ while N_R is the number of rotor teeth. We are now able to set up two objective functions we will deal with, plotted in Fig. 2 and Fig. 3, i.e.

maximum static torque

$$F_1(x_1, x_2) = \frac{1}{4} V^2 N_R \left[C_{\max}(x_1, x_2) - C_{\min}(x_1, x_2)\right] \qquad \textit{to be maximized} \tag{4}$$

torque ripple

$$F_2(x_1, x_2) = \frac{F_1(x_1, x_2) - \dfrac{sin(a)}{sin(2a)} \Gamma_0(x_1, x_2)}{\cos(N_R a) - \cos(N_R b)} \qquad \textit{to be minimized} \tag{5}$$

where

$$a = \pi\left(\frac{1}{2N_R} - \frac{1}{N_S}\right) \qquad b = a + \frac{\pi}{N_S}$$

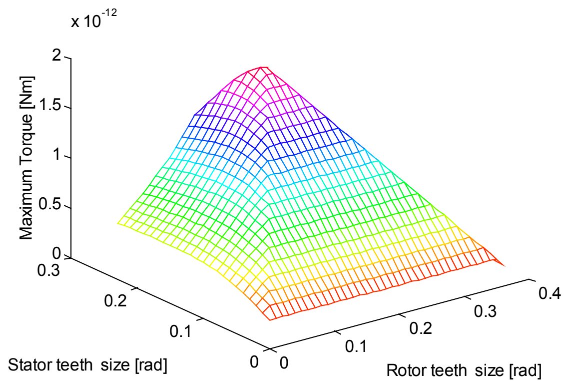

Fig 2 - Maximum static torque objective function.

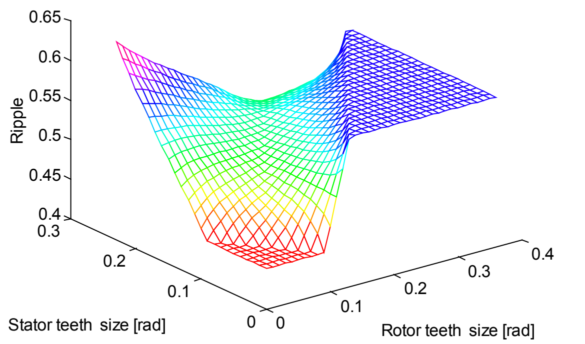

Fig 3 - Torque ripple objective function.

Both the objectives are non-analytical; the second one is potentially ill-conditioned due to a saddle point. Suitable constraints, assuring geometrical congruency and absence of electrostatic discharge, bound the search space.

Optimization strategy

When considering a multicriteria optimization problem, the Pareto optima theory states the existence of a set of non-dominated solutions called Pareto optimal front. The aim of the NSGA algorithm is to obtain as many solutions as possible on the Pareto optimal set preserving diversity among them, which means to obtain solutions equally distributed in the set; a schematic flow chart of the four-steps algorithm that we have implemented is shown in Fig 4.

As can be seen from the flow-chart, in the *first step* of the algorithm we generate, in a random way, an initial population of 100-200 individuals in the search space.

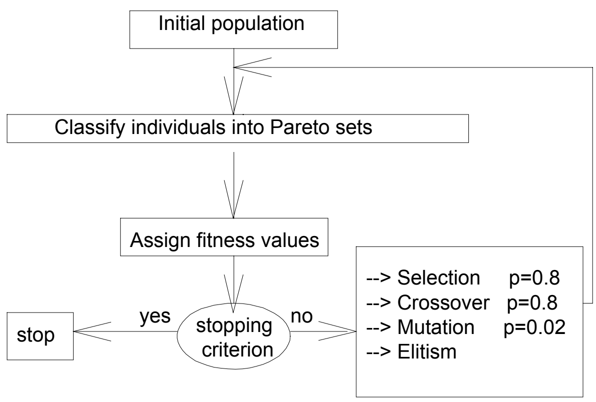

Fig 4 - Flow chart of the NSGA.

In the *second step* we classify individuals into Pareto sets using the following definition. A solution x_1 is said to dominate the other solution x_2 , if both the following conditions are true.

1 The solution x_1 is not worse than x_2 in all objectives.
2 The solution x_1 is strictly better than x_2 in at least one objective.

The *third step* consists of the assignment of a fitness value to each individual; two criteria must be followed: 1) forcing convergence to the Pareto Optimal set, 2) forcing diversity among solutions. In order to do this, the fitness value for each individual depends on the Pareto set which it belongs to and a sharing procedure is implemented in order to favour isolated solution and to avoid clustering of solutions. As *fourth step,* if the stopping criterion is not satisfied, a genetic algorithm produces a new generation of individuals. We have implemented the three classical genetic operators (selection, crossover and mutation) and an elitism procedure that guarantees the survival of the best individual of the previous generation. The probability values for all the operators are listed in Fig 4.

Results

We report results of the optimization of the 3/2 geometry device. In Fig 5 and Fig. 6 the initial 200 individuals population is shown in the objective space and in the variable space, respectively. In Fig. 6 the Pareto front can also be seen.

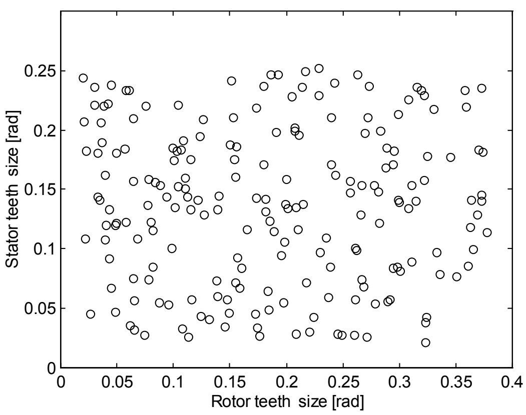

Fig. 5 - Starting population: design space; 200 individuals.

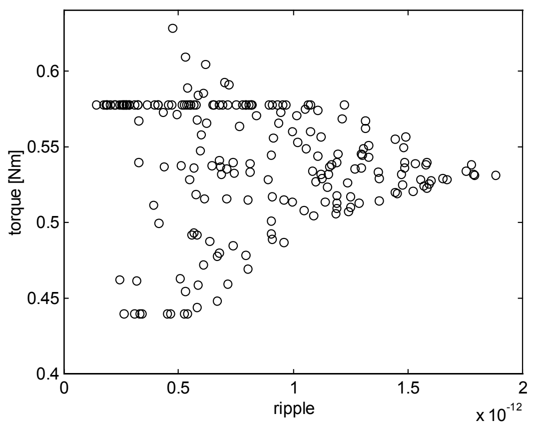

Fig. 6 - Starting population: objective space; 200 individuals.

In Fig. 7 and Fig. 8 the results of the optimization are represented in the objective space and in the design space, respectively.

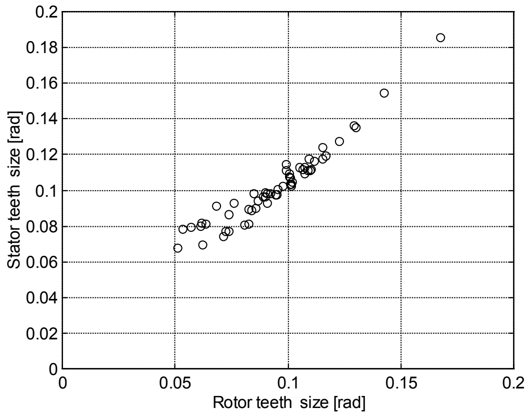

Fig 7 Final population: design space; 50 individuals.

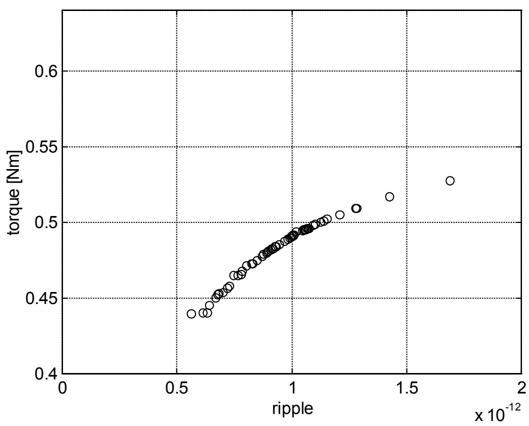

Fig 8 Final population: objective space; 50 individuals.

It can be seen that 50 individuals converge to the Pareto front in a distributed way giving rise to a set of optimal solutions. The latter are distributed both in the objective space and in the variable space, giving rise to different possible geometries. As one can expect, it can be seen that solutions are characterized by almost equal values of x_1 and x_2 in order to maximize the torque and by small values of both x_1 and x_2 in order to minimize the ripple.

Conclusion

An innovative algorithm for the optimization of electromagnetic devices has been applied to the optimal design of an electrostatic micromotor. The maximum static torque and the ripple have been considered as objectives of optimization and a non-dominated sorting genetic algorithm has been set up to tackle the vector design problem. Several optimal solutions have been obtained, all of them distributed on the Pareto optimal front.

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