Multiobjective Design Optimization of Real-life Devices in Electrical Engineering: a Cost-effective Evolutionary Approach

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Abstract. When tackling the multicriteria optimization of a device in electrical engineering, the exhaustive sampling of Pareto optimal front implies the use of complex and time-consuming algorithms that are unpractical from the industrial viewpoint. In several cases, however, the accurate identification of a few non-dominated solutions is often sufficient for the design purposes. An evolutionary methodology of lowest order, dealing with a small number of individuals, is proposed to obtain a cost-effective approximation of non-dominated solutions. In particular, the algorithm assigning the fitness enables the designer to pursue either shape or performance diversity of the device. The optimal shape design of a shielded reactor, based on the optimization of both cost and performance of the device, is presented as a real-life case study.

1 Introduction

Optimal design in electromagnetism has a long history, from Maxwell (1869) on. In other fields of engineering like structural mechanics the history of optimal design is even longer, dating back to Lagrange (1770). In the latter area the modern development has taken place over the past three decades, anticipating the analogue development in electromagnetism and, to some extent, fostering it. In more recent years it has been possible to integrate the analysis of electromagnetic field with optimization techniques, so moving from computer-aided design (CAD) to automated optimal design (AOD).

The essential goal of AOD in electromagnetics is that of identifying, in a completely automatic way, the system or the device that is able to provide some prescribed performance, *e.g.* to minimize weight and materials cost or to maximize some output, taking into account physical constraints and geometrical bounds. This is actually an inverse problem and implies the simultaneous minimization of conflictual objectives.

In real-life engineering the presence of a single criterion or objective is somewhat an exception or a simplification. Therefore, the future of computational electromagnetics seems to be oriented towards, and conditioned by, the development of efficient methodologies and robust algorithms for solving multicriteria design problems.

From a formal viewpoint, a multicriteria problem is cast as follows:

$$\min_x F(x) \qquad (1)$$

$$\text{subject to} \qquad g(x)<0$$
$$h(x)=0$$

where $F(x)=(f_1(x),...,f_m(x))$ is a vector of m criteria or objectives, $x=(x_1,...,x_n)$ is the vector of n design variables defining the device or the system, g(x) and h(x) are inequality and equality constraints, respectively. In general, the utopia solution x*, *i.e.* that minimizing all F_i simultaneously, does not exist and the so-called Pareto solutions are accepted, *i.e.* those for which no decrease in any of the criteria is obtained without a simultaneous increase in at least one of the other criteria.

Traditionally, multicriteria problems are reduced to singlecriterion problems, for instance by means of one of the following procedures:

i) the use of a penalty function composed of the various criteria;

ii) the separate solutions of singlecriterion problems and their trade-off;

iii) the solution of a singlecriterion problem, taking the other criteria as constraints.

This approach leads to classical methods of multiobjective optimization and gives a solution which is supposed to be the optimum.

Often in the design of electromagnetic devices a satisfactory way to tackle the problem of multicriteria optimisation consists of applying the Pareto optima theory in connection with a suitable minimization algorithm. The result is a set of non-dominated solutions: in principle, all of them are optimal; in practice, each of them corresponds to a different degree of minimization of the single objectives.

Moreover, though looking attractive, the non-dominated approach often results to be unaffordable from the computational viewpoint; in fact, the evaluation of each objective may imply heavy non-linear field analyses in three-dimensional geometries. Consequently, the aim of a reliable method of multicriteria optimization should be to approximate the Pareto optimal front by fulfilling three requirements:

- convergence to the front independent on the number, even very low, of non-dominated solutions;
- remarkable diversity among non-dominated solutions;
- moderate computational cost.

An attempt towards this goal is here presented.

2 EMO strategy: methodological aspects

The aim of a stochastic multiobjective optimiser based on non-dominated sorting is to obtain as many solutions as possible lying on the Pareto optimal set while preserving diversity among them.

GA-based strategies [1],[2],[6] typically require some hundreds individuals for ensuring convergence. Moreover, when dealing with real-life optimization problems in electrical engineering, the evaluation of each objective often requires a FEM solution lasting several minutes [7],[10]. This difficulty often makes the use of GA-based strategies computationally unaffordable or highly unpractical from an industrial point of view.

Therefore we have decided to adopt a (1+1) ES algorithm as the optimization engine of the multiobjective strategy shown in Fig. 1 and Fig. 2 because, in our experience, it is robust and gives good convergence even when few individuals are considered. It should be noted that generation, mutation and annealing steps are implemented in parallel; this is possible because in our implementation individuals do not interact each other during the whole process, apart from the steps of Pareto ranking and fitness evaluation. In practice, the general structure of the algorithm is the same as NSGA whereas genetic operators have been replaced with the evolution strategy ones.

Two criteria must be pursued when assigning the fitness value to each individual:
1) forcing global convergence to the Pareto optimal set;
2)forcing diversity among solutions belonging to the same set.

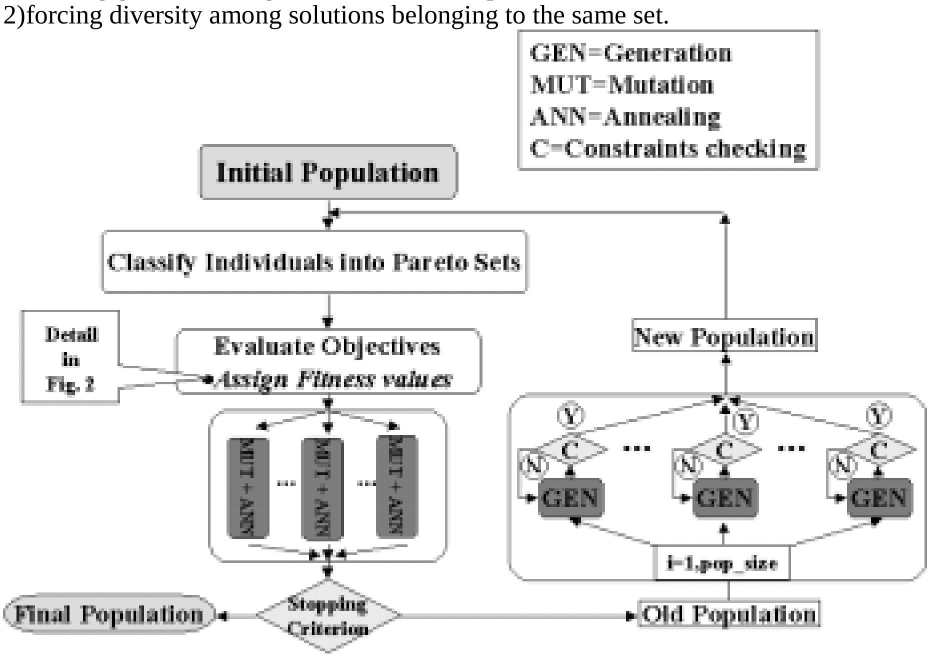

Fig. 1. Non-dominated Sorting Evolution Strategy Algorithm (NSESA): flowchart

To the first purpose, the fitness of each individual is evaluated according to the Pareto set, which it belongs to. To the second purpose, a sharing procedure is implemented within the current set in order to favour isolated solutions and prevent clustering. This step is particularly delicate when using a small number of individuals (say 5 to 10) and some changes with respect to classical sharing procedures [5],[8] are here proposed.

In general, when implementing a fitness sharing procedure, diversity of individuals in either the design space or the objective space can be considered. Moreover solutions with strong diversity in shape can be characterised by weak diversity in objective value (the opposite as well). Both procedures can lead to results useful for the device designer, who is interested in both shape and performance diversity of optimal solutions. This is why a sharing procedure in only one of the two spaces cannot guarantee a satisfactory approximation of the Pareto optimal front in the other space.

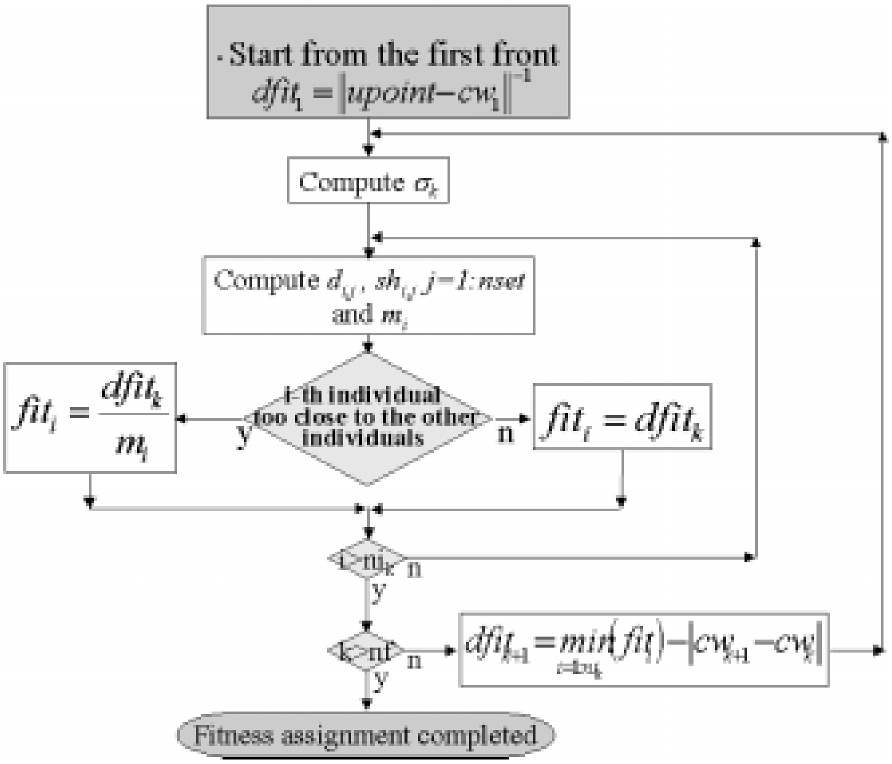

Fig. 2. Fitness assignment algorithm: flowchart

More into details, given a population sorted into Pareto sets, we at first consider the first set and assign a dummy fitness *dfit* to each individual as shown in Fig. 2, where cw_1 is the center of weight of the first front and *upoint* the utopia-point.

In order to set up the sharing procedure we then evaluate the normalized average distances $d_{i,j}$ among elements, in both design and objectives domain. Afterwards we implement the standard sharing formulas [3],[4] for the calculation of the sharing parameter $sh_{i,j}$ and the penalty coefficient $m_{i,}$; we evaluate the niche radius σ in the following way:

$$\sigma_x = \frac{1}{(nset-1)}\sqrt{\sum_{p=1}^{ndof}\left(x\,max_p - x\,min_p\right)^2} \tag{2}$$

$$\sigma_f = \frac{1}{(nset-1)}\sqrt{\sum_{p=1}^{objf}\left(f\,max_q - f\,min_q\right)^2}$$

considering σ_x or σ_f when shape or performances diversity has to be enhanced, respectively; *ndof* and *nobjf* are number of design variables and number of objectives, respectively.

Finally, the fitness value of the i-th individual is evaluated and assigned. Before moving to the *k+1-th* front a new dummy fitness $dfit_{k+1}$ has to be evaluated, as shown in Fig. 2. In order to increase the convergence rate the new value of dummy fitness depends on the center of weights cw_k, cw_{k+1} of current and next set, respectively. The procedure is repeated for all successive sets.

We point out that convergence towards the optimal front is always controlled in the objective space, while sharing procedures can be performed in either design space or objective space.

The following convergence indexes have been defined:

$$C_x(iter) = \frac{1}{npop} \sum_{i=1}^{npop} \sqrt{\sum_{j=1}^{ndof} \left(x_{i\,j}^{\;iter} - x_{i\,j}^{\;iter-1} \right)^2} \tag{3}$$

$$C_f(iter) = \frac{1}{npop} \sum_{i=1}^{npop} \sqrt{\sum_{k=1}^{nobjf} \left(f_{i\,k}^{\;iter} - f_{i\,k}^{\;iter-1} \right)^2}$$

Finally, three stopping criteria have been implemented:

a) maximum number of iterations;

b) minimum value of convergence index in the objective space;

c) maximum number of iterations with no improvement found.

Results of previous investigations [9], [11] on simplified test problems have validated the effectiveness of the strategy proposed.

3 EMO strategy: numerical aspects

Several test cases on real-valued analytical functions have been carried out for validating the code implemented. Here we show results for one of them, namely the Deb's t_3 problem. It is characterized by two variables and two objectives, giving rise to a non-connected Pareto front; the problem can be defined as follows:

$$\min_{(x_1, x_2)} (f_1, f_2) \tag{4}$$

$$\begin{cases} f_1(x_1) = x_1 \\ f_2(x_1, x_2) = 1 + 9x_2 - \sqrt{x_1(1+9x_2)} - x_1 \sin(10\pi x_1) \end{cases}$$

where $(x_1, x_2) \in (0,1) \times (0,1)$

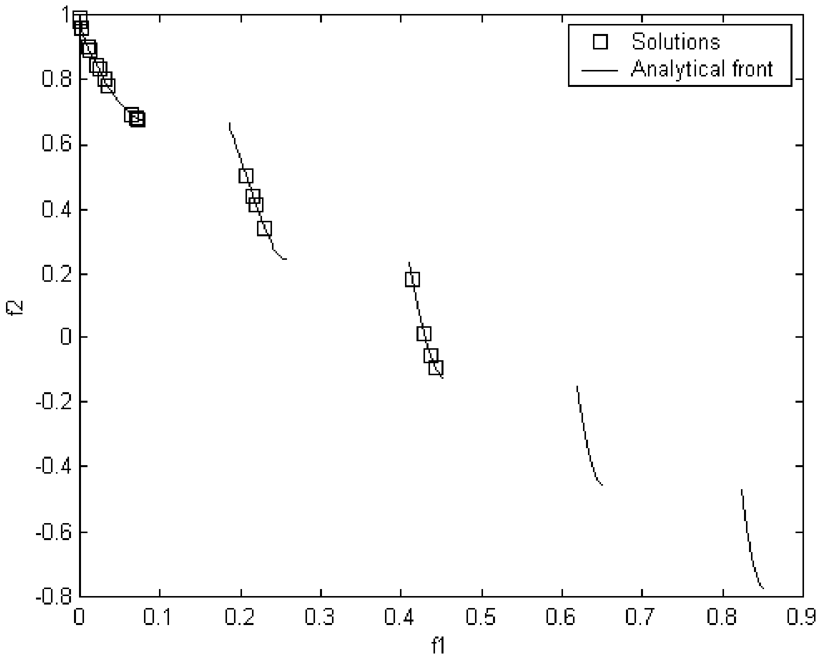

Fig. 3. NSESA: 20 individuals solution for validation test

As can be seen from Fig. 3, a solution composed of twenty individuals has been found; individuals are distributed along three of the five branches the POF is composed of. The starting population was chosen in a random way in the design space. Given the ik-th individual at niter-th iteration, the following two expressions have been used in design space and in objective space, respectively, in order to quantify the POF approximation error all along the evolution:

$$\begin{aligned} errorx(ik, iter) &= x_2(ik, iter) \\ errorf(ik, iter) &= f_2(ik, iter) - 1 + \sqrt{f_1(ik, iter)} + \\ &\quad + f_1(ik, iter)\sin(10\pi\, f_1(ik, iter)) \end{aligned} \tag{5}$$

The log value of both errors is plotted in Fig. 4 with reference to a single individual.

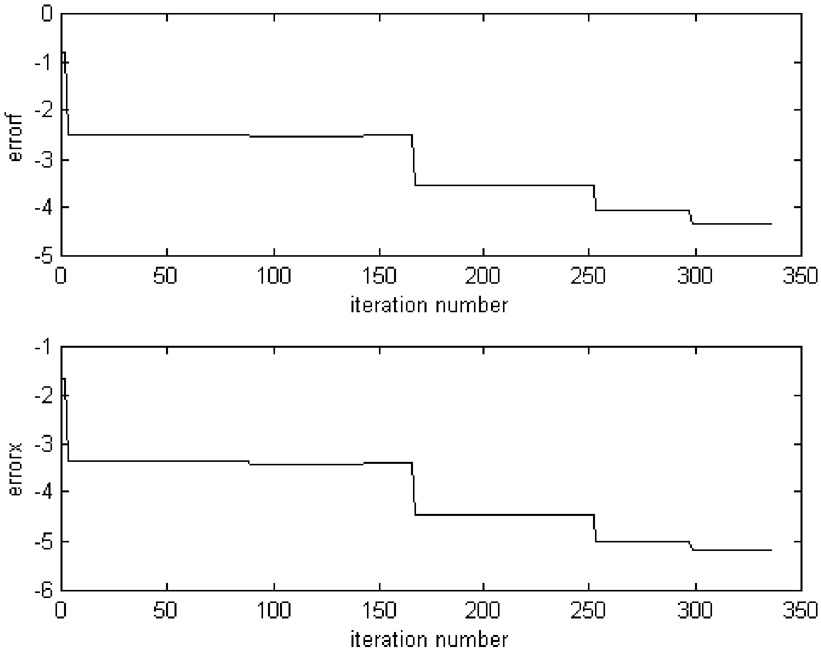

Fig. 4. History of approximation errors for the solution shown in Fig. 3

The log value of both convergence indexes is plotted in Fig. 5.

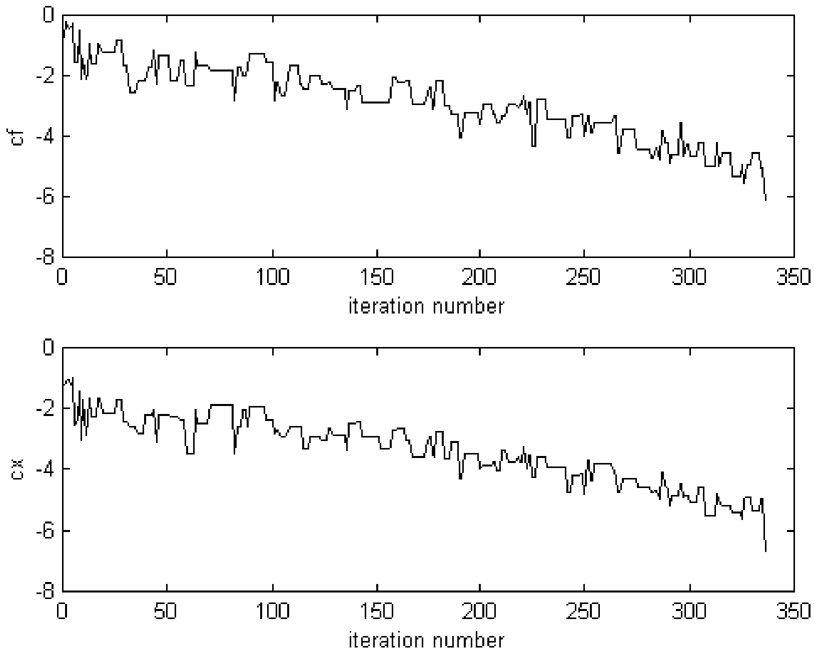

Fig. 5. History of convergence indexes for the solution shown in Fig. 3

4 An industrial case study

4.1 The device

The shape optimization of a single-phase series reactor for power applications is considered [12]; the reactor is employed to reduce the peak value of short-circuit current and so to mitigate its electrodynamical effects.

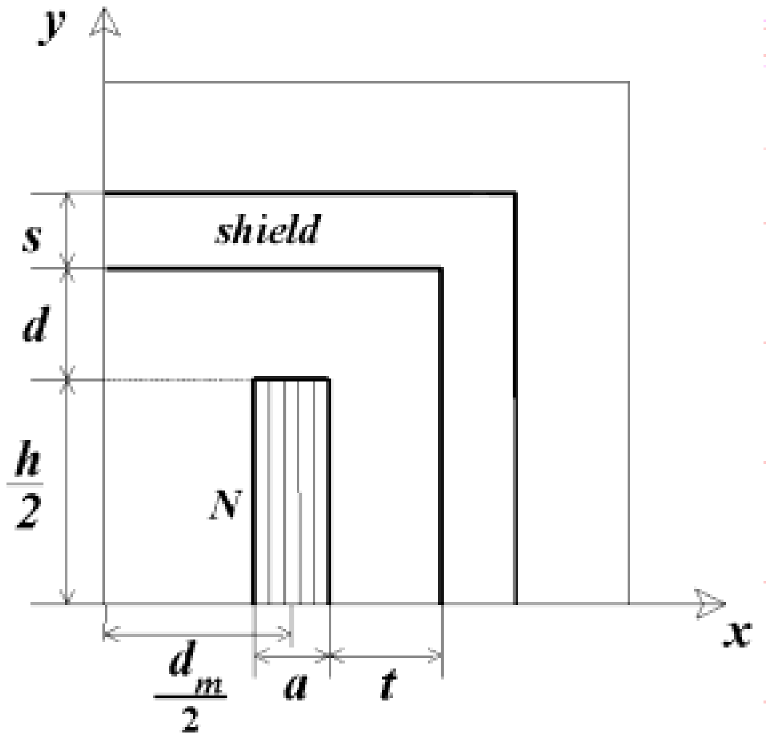

Fig. 6. Cross-section of the reactor (one quarter) and design variables

The reactor, the cross-section of which is shown in Fig. 6, is characterised by a coreless winding with cylindrical shape (foil winding); it is boxed in a laminated magnetic shield with rectangular shape in order to protect the surrounding environment from the strong stray field. The latter, in turn, gives rise to power losses in the winding that limit the operation of the device. The higher the winding, the lesser the stray field; on the other hand, the realization of a higher winding and shield, though reducing the effect of leakage, causes an increase of volume and cost of the reactor so that a conflict of design criteria is originated. For a prototype reactor rating 5.9 MVA at a nominal current of 893 A the following values hold: h= 500 mm, dm=590 mm, a=210 mm, d=80 mm, t=40 mm, N=212, filling factor of the winding k_s= 0.504.

4.2 Analysis

The distribution of magnetostatic field in the reactor, for which the rectangular symmetry is assumed, is governed by the Poisson's equation in terms of vector potential **A**=(0,0,A)

$$-div\left(\frac{1}{\mu}gradA\right)=J \tag{6}$$

subject to boundary conditions A=0 along x=0 and elsewhere; J=3.57 Amm^{-2} is the current density in the winding while $\mu_r=1$ and $\mu_r=10^4$ are the values assumed for relative permeability of non-magnetic materials and iron, respectively. To solve (6) numerically, the two-dimensional field region shown in Fig.6 has been discretized by means of a regular grid of finite elements, namely triangles with quadratic variation of potentials; the total number of elements is *ne*=950 approximately. The evolutionary optimizer calls the MagNet code [13] for performing the field analysis and then updates the finite element grid at each iteration.

4.3 Design

In general, up to seven design variables defining the shape of the device can be considered: geometric height *h*, mean diameter d_m, radial thickness of the winding *a*, number of turns *N*, axial distance *d* between winding and magnetic shield, thickness *s* of the shield, radial distance *t* between winding and shield.

Two conflictual criteria can be defined:

- the material cost f_1 of the reactor, namely the weighted sum of copper and iron weights, to be minimized:

$$f_1=4k_iw_i\left[s\left(\frac{d_m+a}{2}+t\right)l+s\left(\frac{h}{2}+d+s\right)l\right]+k_cw_ck_slah \tag{7}$$

  with $k_i=1$, $k_c=3$ while $w_i=7860$ kgm^{-3} and $w_c=8930$ kgm^{-3} are mass densities of iron and copper, respectively;
- the fringing field f_2 inside the winding, i.e. the mean radial component of magnetic induction in the cross-section of the winding, to be minimized as well:

$$f_2=\frac{1}{NW}\sum_{i=1}^{NW}\left|B_x(i)\right| \tag{8}$$

where NW=64 is the number of points of a grid sampling the radial induction in the winding.

The minimisation of the fringing field has two important benefits: from a global point of view it leads to a strong reduction of additional losses in the winding and thus increases the efficiency of the reactor; on the other hand, the probability of local overheating inside the coil and its consequent failure is reduced.

The following constraints have been prescribed:

- the rated value of inductance L=23.57 mH;
- the induction in the core, not exceeding 0.8 T, when the current is equal to $\sqrt{2}I_{nom}$ with I_{nom}=893 A;
- the current density in the winding;
- the insulation gaps *d* and *t* between winding and core.

Consequently, three independent design variables have been selected, *i.e.* height *h*, mean diameter d_m and number of turns *N* of the winding, respectively. Finally, a set of bounds preserves the geometrical congruency of the model, namely:

$$0.5 \le h \le 1.5 \quad m \qquad 0.1 + 2a \le d_m \le 1.8 \quad m \qquad 162 \le N \le 262 \qquad \textbf{(9)}$$

The sensitivity surfaces of both f_1 and f_2 against (h,d_m) for given number of turns N=200 are reported in Fig. 7 and Fig. 8, respectively.

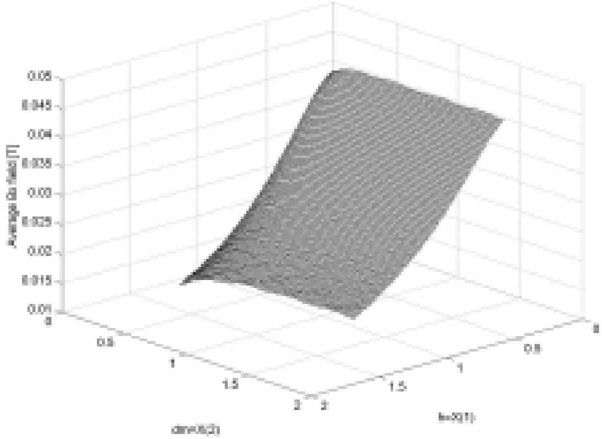

Fig. 7. Average B_x field in the winding as a function of mean diameter d_m and height h of the winding itself

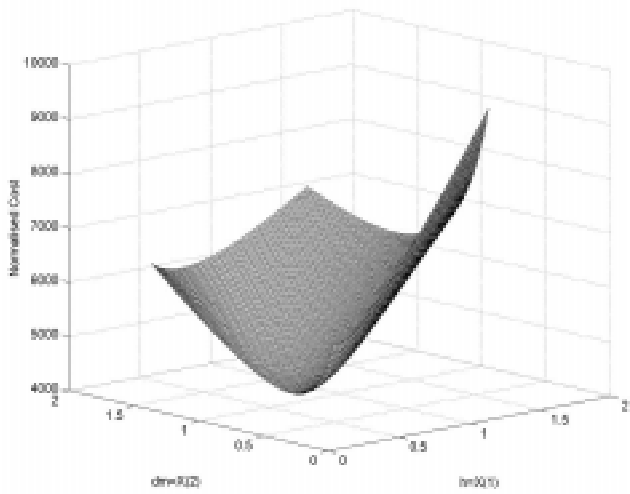

Fig. 8. Normalized cost of the reactor as a function of mean diameter d_m and height h of the winding

The conflict between the two objectives is evident from the comparison of both surfaces.

5 Results and discussion

Aiming at a preliminary investigation, the search space has been randomly sampled by means of 4000 points uniformly distributed; the approximation of the objective space shown in Fig. 9 has then been obtained (in the figure only 1000 samples are represented).

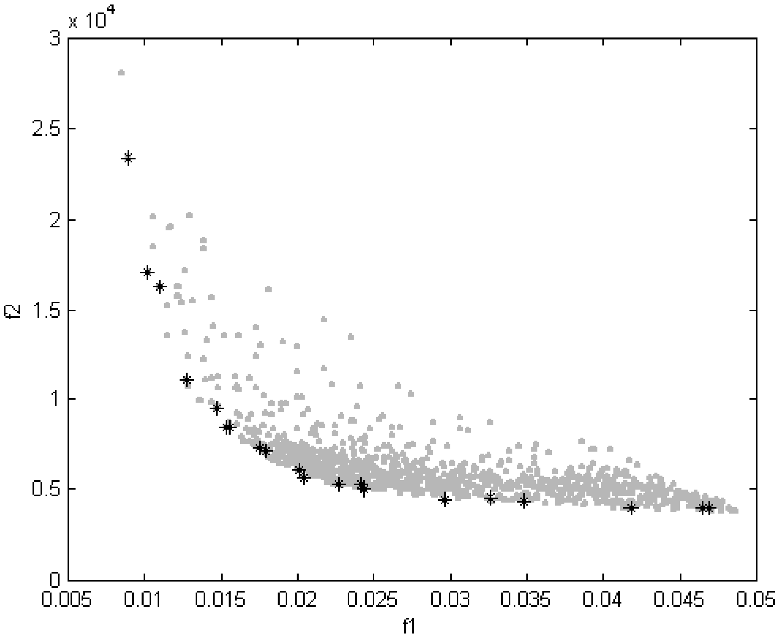

Fig. 9 Objective space: random samples and Pareto solutions (*)

From a practical point of view the optimal front appears to be globally convex, connected and composed of two parts, the one being deceptive and strongly Pareto, the other being non deceptive and weakly Pareto [14]. Moreover, a sub-region concentrating the majority of samples is evident; it corresponds to the weakly Pareto front. As a consequence, individuals during evolution are strongly attracted towards this sub-region.

The EMO strategy has been run in two cases, each of which considering 10 individuals whose initial values have been randomly selected; diversity of individuals in the objective space has been pursued in both cases. After overlapping the two sets of solutions, the approximation of the Pareto optimal front pointed out in Fig. 9 has been obtained.

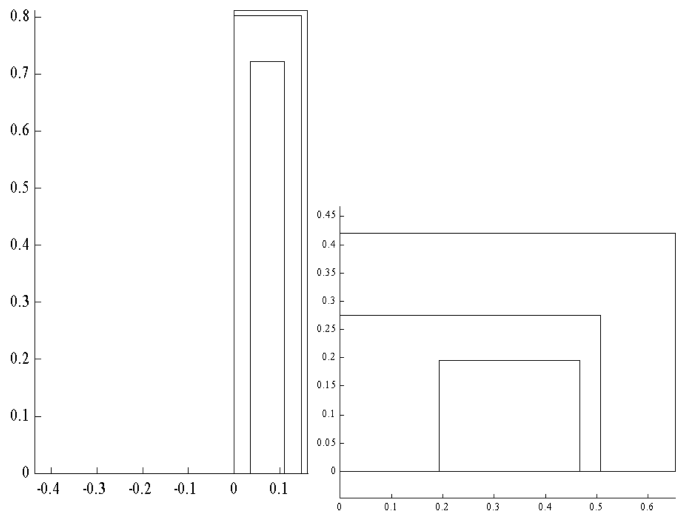

Fig. 10. Shape of extreme solutions: minimum cost (left) and minimum stray configurations (dimensions in m)

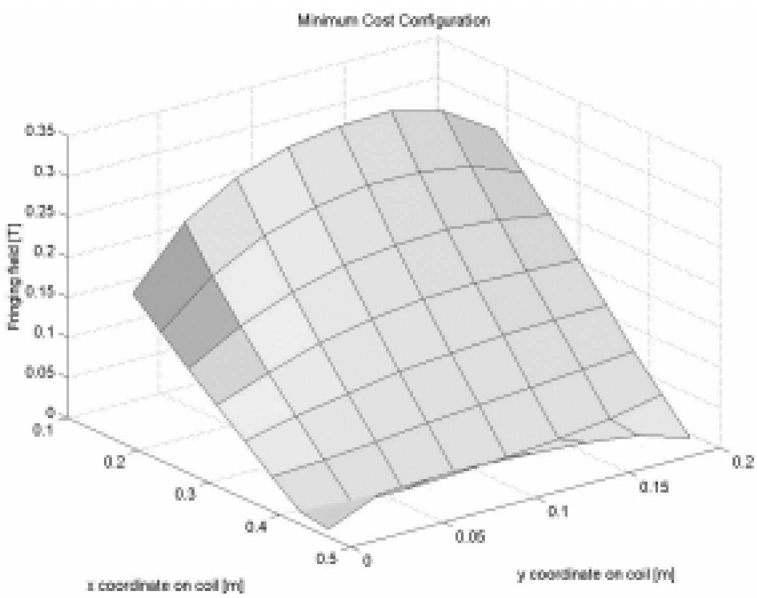

Fig. 11. Fringing field in the winding for the minimum cost configuration

In Fig. 10 the device geometries corresponding to the Pareto-optimal extreme solutions are shown; the corresponding distributions of stray field in the winding are reported in Fig. 11 and Fig. 12 respectively. The variability of both shape and performance is evident.

In order to estimate the maximum cost *totcost* of the EMO strategy implemented, the following formula holds

$$totcost = niter \times npop \times nobj \times femtime \qquad (\mathbf{10})$$

where: number of objective functions *nobj*=2 to 3, maximum number of iterations *niter*=300. In our experience of real-life problems, typical number of individuals is *npop*=5 to 20, while the cost of a single FEM analysis is *femtime*=1 to 5 min.
As for the case study developed, due to the linear magnetostatic analysis and the inexpensive evaluation of f_2 , we had $femtime \times nobj$=0.8 min thus requiring some 48 hours for the stopping criterion to be satisfied.

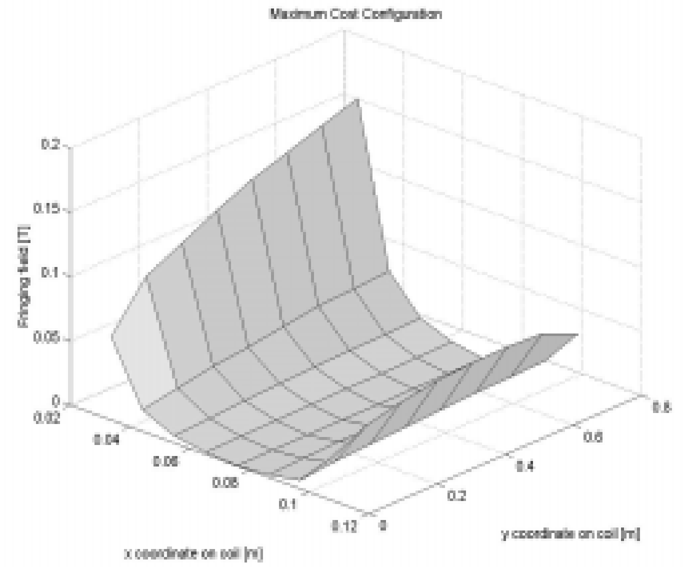

Fig. 12. Fringing field in the winding for the maximum cost configuration

6 Conclusion

In real-life engineering, when adopting an algorithm of multiobjective optimisation based on Pareto optimality, it is of primary importance to reduce the number of calls to the objective function, often requiring a FEM analysis. In the paper, a cost-effective EMO strategy has been developed and applied to the shape design of a realistic electromagnetic device.

From the methodological viewpoint, the results show that a lowest-order evolution strategy algorithm with a small number of individuals (5 to 10) can be conveniently used as the engine of the multiobjective optimization. Nevertheless, the procedure of fitness assignment should be modified with respect to classical formulas. In fact, the latter refer to large number of individuals (50 to 100) and depend on some tuning parameters, usually defined by means of empirical formulas. In the paper a fitness assignment procedure is proposed, making the formulas forcing diversity of individuals univocal and easy-to-implement.

Turning to the case study, the cost and performance optimisation of a shielded reactor has been achieved. A wide number of configurations belonging to the Pareto optimal front have been identified, so offering the designer an effective choice among devices that rank from the best performing one to the less expensive one. From an industrial point of view, having a set of Pareto-optimal solutions makes it easy to fulfil *a posteriori* technology-related constraints that are typical of real-life engineering, whereas in scalar optimization they have to be carefully prescribed *a priori* in order the only solution be feasible.

Finally, the proposed strategy allows the designer to pursue either shape or performance diversity of Pareto-optimal devices.

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