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A NEW EVOLUTIONARY APPROACH FOR MULTIOBJECTIVE OPTIMIZATION

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ABSTRACT. Several evolutionary algorithms for solving multiobjective optimization problems have been proposed ([2, 5, 6, 7, 8, 9, 10, 12, 13], see also the reviews [1, 11, 14]). All algorithms aim to give a discrete picture of the Pareto optimal set (and of the corresponding Pareto frontier). But Pareto optimal set is usually a continuous region in the search space. It follows that a continuous region is represented by a discrete picture. When continuos decision regions are represented by discrete solutions there is an information loss. In this paper we propose a new evolutionary approach combing a new solution representation, new variation operators and a multimodal optimization technique. In the proposed approach continuous decision regions may be detected. A solution is either a closed interval or a point. The solutions in the final population will give a realistic representation of Pareto optimal set. Each solution in this population corresponds to a decision region of Pareto set. Proposed technique does not use a secondary population of non-dominated already founded.

Keywords: evolutionary algorithms, multiobjective optimization, Pareto optimal set, Pareto frontier, Genetic chromodynamics.

Let $f_1, f_2, \ldots, f_N$ be N objective functions.

$$f_i : \Omega \to \mathbb{R}, \Omega \subset \mathbb{R}.$$

Consider the multiobjective optimization problem:

$$\begin{cases} \text{optimize } f(x) = (f_1(x), \ldots, f_N(x)) \\ \text{subject to } x \in \Omega \end{cases}$$

The key concept in determining solutions of multiobjective problems is that of Pareto optimality.

Definition. (*Pareto dominance*) Consider a maximization problem. Let x, y be two decision vectors (solutions) from Ω. Solution x is said to *dominate* y (also written as $x \phi y$) if and only if the following conditions are fulfilled:

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(i) $f_i(x) \geq f_i(y)$, $\forall i = 1, 2, \ldots, n$.
(ii) $\exists j \in \{1, 2, \ldots, n\} : f_j(x) > f_j(y)$.

Definition. Let $S \subseteq \Omega$. All solutions which are not dominated by any vector of S are called *nondominated* with respect to S.

Definition. Solutions that are nondominated with respect to the entire search space Ω are called *Pareto optimal* solutions.

Pareto optimal set may consist from decision regions represented as:

(i) a set of points;
(ii) a set of disjoint intervals;
(iii) a set of disjoint intervals and a set of points.

Usual multiobjective optimization algorithms may deal with the first case. The second case is solved in a quite artificial manner. Obtained solutions represent points in a set of non-disjoint intervals. It is problematic to obtain a realistic representation of a union of continuous Pareto optimal regions using such a discrete picture.

When continuous decision regions are modeled by discrete solutions there is an information loss due to fidelity loss between continuous and discrete representations. Any multiobjective optimization problem being computationally solved suffers this fate. Methods for finding Pareto optimal set and Pareto optimal front using discrete solutions are computationally very difficult. Moreover the resulting sets are still only a discrete representation of their continuous counterparts. However the results may be accepted as the 'best possible' at a given computational resolution.

In this paper we propose a new evolutionary approach combing a non-standard solution representation and a multimodal optimization technique. In the proposed approach a solution is either a closed interval or a point. The solutions in the final population will give a more adequate representation of Pareto optimal set.

To evolve population we use a multi-modal optimization metaheuristic called Genetic Chromodynamics ([4]). Each individual from the population is selected for recombination or mutation. A mate for an interval (individual) is another interval that intersects it. If an individual has a mate then they are combined. Otherwise it is mutated. Mutation consists from normal perturbation of interval extremities.

A new variation operator called *splitting operator* is considered. By splitting an interval-solution containing a dominated point is splitted. In this way several Pareto regions existing in the same solution are separated. Performing this operation population size is increased.

Two population decreasing mechanisms are used: *merging* (if an interval is wholly contained in other interval, the first one is remove from the population) and *vanishing* (very bad intervals are removed from the population). The algorithm stops when the optimal number of solutions is achieved. The evolutionary multiobjective procedure proposed in this paper is called *Continuous Pareto Optimal Set* (CPOS).

1. Solution representation and domination

In this paper we consider solutions are represented as intervals in the search space Ω.

Each interval-solution k is encoded by an interval $[x_k, y_k] \subset \mathbb{R}$. Degenerated intervals are allowed. Within degenerate case $y_k = x_k$ the solution is a point. To deal with this representation a new domination concept needed.

Definition. An interval-solution $[x, y]$ is said to be *interval-nondominated* if and only if all points of that interval $[x, y]$ are nondominated.

Remark. If $x = y$ this concept reduced to the ordinary non-domination notion.

Definition. An interval-solution $[x, y]$ is said to be *total dominated* if and only if each point within $[x, y]$ is dominated (by a point inside or outside the interval).

Remarks.

(i) If no ambiguity arise we will use nondominated (dominated) instead of interval-nondominated (interval-dominated).
(ii) An interval-solution may contain dominated as well as nondominates points.

A common approach of multiobjective optimization is to use a Pareto-ranking mechanism for fitness assignment (see for instance). In our interval-representation this approach is difficult to be used directly due to the infinite member of points to be tested in each interval. For this reason we propose a new approach. The idea is to approximate the concept of *total domination*. In this respect we introduce the notion of *non-domination degree*.

A non-domination concept may be introduced by considering some random points in the solution interval. The number Kxy of random points is proportional to the interval size $|x - y|$. We may define K_{xy} as

$$K_{xy} = F(|x - y|),$$

where F is a linear function.

Let S_{xy} be a set of random numbers within the solution-interval $[x, y]$. The size of the sampling set S_{xy} is equal to K_{xy}:

$$\text{card } S_{xy} = K_{xy}.$$

Definition. Non-domination degree of the interval-solution $[x, y]$ is the number N_{xy} defined as follows:

(1) $x \neq y$ then

$$N_{xy} = \frac{N_1 - N_2}{K_{xk}}.$$

where N_1 (N_2) is the number of non-dominated (dominated) points in the set S_{xy} and $K_{xy} \geq 1$.

(2) $x = y$ then

$$N_{xy} = \begin{cases} 1 & \text{if } x \text{ is non-dominated} \\ 0 & \text{otherwise} \end{cases}$$

Definition. Solution $[x, y]$ is said to be *t-nondominated* if the inequality

$$N_{xy} \geq t$$

holds. In this inequality t is a threshold, $0 \leq t \leq 1$.

2. Fitness assignment

Within our evolutionary multiobjective optimization procedure fitness assignment is realized using non- domination degree.

Let $[x, y]$ be a solution. Fitness of the solution $[x, y]$ is denoted eval($[x, y]$) and

$$\text{eval}([x, y]) = N_{xy}.$$

Remark. Proposed fitness assignment scheme may supply different fitness values for several sampling sets S_{xy}. This is not a major drawback. As a matter of fact, we may consider the statistical character of fitness assignment process as an advantage. It may results in an increasing flexibility of the corresponding search procedure.

3. Population model and search operators within CPOS procedure

For preserving all useful solutions in the population CPOS procedure use a multi?modal optimization technique. Our experiments emphasize that Genetic chromodynamics meta?heuristic proposed in [4] outperforms other standard methods like niching, restricted mating or island models. Genetic chromodynamics uses a variable?sized population and a local mating scheme.

The method allows a natural termination condition. Each solution in the last population supplies a Pareto optimal region contributing to the picture of Pareto optimal set.

Most of the multiobjective optimization techniques based on Pareto ranking use a second population that stores nondominated individuals. Members of second population P_{second} may be used to guide the search process. As dimension of secondary population may dramatically increase several mechanisms to reduce

P_{second} size have been proposed. In [13] and [14] a population decreasing technique based on a clustering procedure is considered. We may observe that preserving only one individual from each cluster implies a loss of information. Present approach does not use a secondary population. This makes CPOS procedure more robust and less costly. It does not imply a loss of information about Pareto optimal set during the search process.

3.1. **Selection for recombination.** Only most fit individuals are allowed to recombine. According to our elitist scheme only 1?nondominated individuals are recombined. Each 1-nondominated solution is considered for recombination. For each parent a restricted mating scheme is used to find the other parent. Let $[x, y]$ be an 1?nondominated solution. If the solution is the degenerate interval $x = y$ then its mate is selected from the closed ball $V(x, R)$, where R is the mating range. R represents a parameter of the procedure.

The mate of the one non-degenerate interval $[x, y]$ is selected from all one non-degenerate solutions $[u, v]$ such that they are not disjoint and do not include each other. This means that the following conditions are have to be fulfilled for recombining interval solutions $[x, y]$ and $[u, v]$:

(i) $[x, y] \cap [u, v] \neq \emptyset$;
(ii) $[x, y] \cap [u, v] \neq [x, y]$;
(iii) $[x, y] \cap [u, v] \neq [u, v]$.

The individuals that can be selected as mates of $[x, y]$ represent the breeder set of $[x, y]$. From the breeder set the second parent is selected using a certain procedure like a tournament or proportional selection schemes.

3.2. **Recombination operator.** Recombining the individual $[x, y]$ and its selected mate it results a unique offspring. The first parent $[x, y]$ will be replaced by this offspring.

If the parents are nondegenerated solutions the offspring is the union of the parent intervals.

For degenerated case the offspring may be, for instance, the convex combination of its parents.

According to the proposed recombination operator the mate of a (non) degenerated solution has to be (non) degenerated too.

3.3. **Mutation operator.** An individual $[x, y]$ is mutated if and only if no mate can be selected for it. This happens when the breeder set of $[x, y]$ is empty.

3.3.1. *Mutating an interval.* There are several ways of realizing mutation. These possibilities are:

a) mutate the left extremity of the interval;

b) mutate the right extremity of the interval;
c) mutate the both extremities of the interval.

Remarks.

(i) For extremities perturbation we use an additive normal perturbation with standard deviation σ, where σ is a parameter of the method.
(ii) Degenerated?solutions mutation is included in the general scheme.
(iii) Mutation type (a, b or c) is randomly chosen.

3.3.2. *Degenerated interval-solutions.* By mutation an interval can be reduced to a point. This may happen in the following situations:

(i) mutation of the right (left) interval extremity is less (greater) than the left (right) interval extremity;
(ii) if by mutation the interval extremities coincide (with respect to a given computational resolution).

3.4. **Splitting operator.** For segregation two disjoint Pareto regions that are represented by the same interval-solution we introduce a new type of variation operator called *splitting operator*.

Splitting operator is applied to an interval-solution and produces two offspring. This operator increases population size.

Applying recombination or mutation to all individuals in the current population $P(t)$ a new intermediary population $P^1(t)$ is obtained. Splitting operator is applied to the intermediate population $P^1(t)$.

To apply splitting operator an interval-solution $[x, y]$ is randomly chosen from $P^1(t)$. A cut-point p, $x < p < y$, is randomly chosen. If p is a dominated point then $[x, y]$ may include disjoint Pareto regions. For separating these regions we apply the splitting operator.

The offspring resulted by splitting the solution $[x, y]$ are $[x, p]$ and $[p, y]$. We may thus write

$$\text{split}[x, y] = \{[x, p], [p, y]\}.$$

Splitting operator is not applied if the randomly generated point p is nondominated.

4. Population dynamics within CPOS algorithm

To detect the correct number of Pareto optimal regions it is necessary to have only one solution per Pareto optimal region. Using Genetic Chromodynamics technique population size decreases during the search process such that eventually equals the number of optimal solutions.

Several population decreasing mechanisms may be used. In our implementation we consider two complementary schemes. Two new operators implement the considered population decreasing mechanisms. The proposed operators are called merging and vanishing. They act only on nondegenerated solutions.

(i) **Merging operator.** If an 1-nondominated solution T_1 is completely included in another 1- nondominated solution T_2, then the solution are merged. The solution T_1 is discarded.

(ii) **Vanishing operator.** If a solution is (-1) nondominated then the solution is discarded. This operation is very useful because performing split mutation the number of bad solutions may grow considerably.

These verifications needed by the operators are done when a new solution is included in the population.

5. Stop condition

Genetic chromodynamics deals with a very natural termination condition. According to this stop condition the chromosome population remains unchanged for a fixed number of generations (given by the parameter `MaxIteration` in our algorithm) then the search process stops.

6. CPOS algorithm

Continuos Pareto optimal set (CPOS) algorithm proposed in this paper may be outlined as below:

CPOS Algorithm
begin
    Population initialization:
    generate randomly a interval population $(P(0))$;
    $t = 0$;
    Evolving intervals:
    repeat
        for each individual c **in** $P(t)$
            if `Has_Mate`(c) $\{c$ has a possible mate$\}$
            then
                select b – a mate for c; {select mate using proportional selection}
                Perform recombination:
                $z =$`Recombination`(b, c);
            else ***Perform mutation of individual*** c**:**
                $z =$`Mutate`(c);
            endif
            add z to intermediate population $P'(t)$;
        endfor
Apply merging operator on individuals in intermediate population $P'(t)$**:**

$P''(t)$ =merge($P'(t)$);
Apply vanishing operator on individual on $P''(t)$***:***
$P'''(t)$ =vanish($P''(t)$);
$P(t+1) = P'''(t)$; {next generation}
$t = t + 1$;
until `MaxIterations` is reached
end.

Remark. Algorithm stops if there is no population modification for a number of `MaxIterations` successive iterations.

7. Numerical experiments

Several numerical experiments using CPOS algorithm have been performed. For all examples the detected solutions gave correct representations of Pareto set with an acceptable accuracy degree. Some particular examples are given below.

Example 1. Consider the functions $f_1, f_2 : [-4, 6] \to \mathbb{R}$ defined as

$$f_1(x) = x^2,$$

$$f_2(x) = (x-2)^2.$$

Consider the multiobjective optimization problem:

$$\begin{cases} \text{minimize } f_1(x), f_2(x) \\ \text{subject to} x \in [-4, 6] \end{cases}$$

Pareto optimal set for this multiobjective problem is the interval [0, 2].

The initial population is depicted in Figure 1. For a better view the chromosomes are drawn one above another.

For the value

$$\sigma = 0.1$$

of the standard deviation parameter solutions obtained after 10 generations are depicted in Figure 2.

The population obtained after 24 generations is depicted in Figure 3.

The final population, obtained after 40 generations, is depicted in Figure 4.

Final population obtained after 40 generations contains only one individual. This individual is:

$$s = [0.01, 1.98],$$

and represent a continuous Pareto optimal solution.

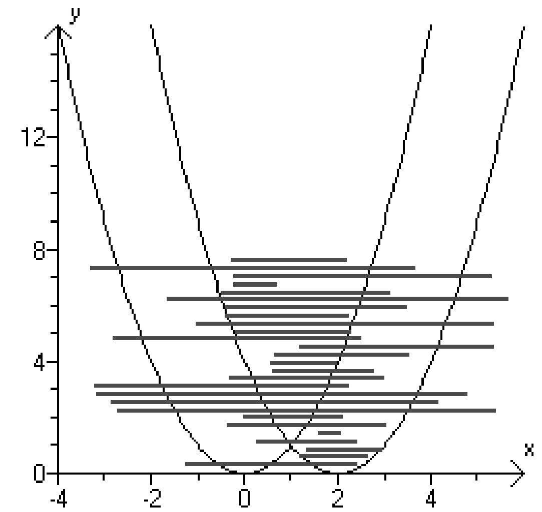

FIGURE 1. Initial population

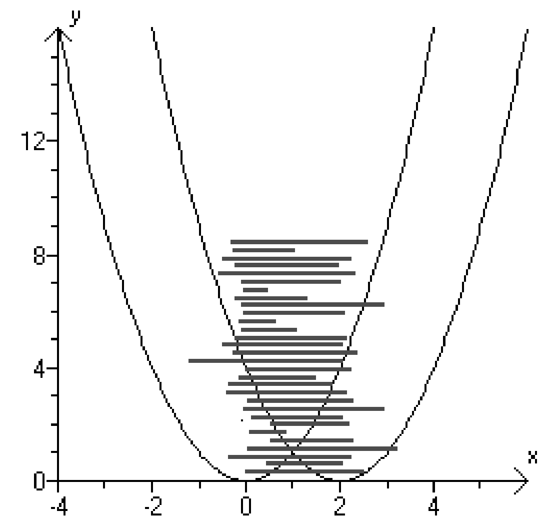

FIGURE 2. Population after 10 generations

The obtained solution accuracy may be increased, if necessary, by decreasing the parameter standard deviation of normal perturbation. Of course the number of iterations needed for convergence increases this case.

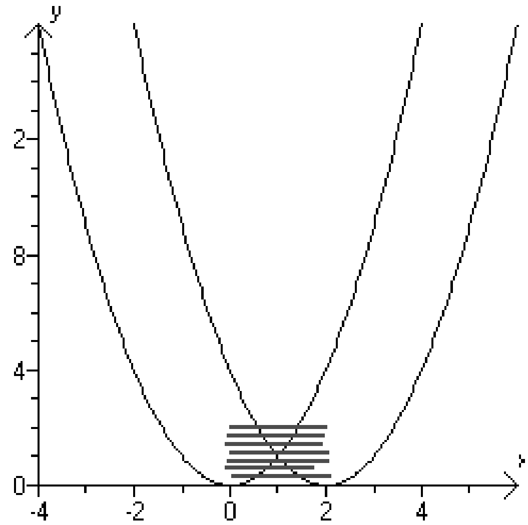

FIGURE 3. The population obtained after 24 generations

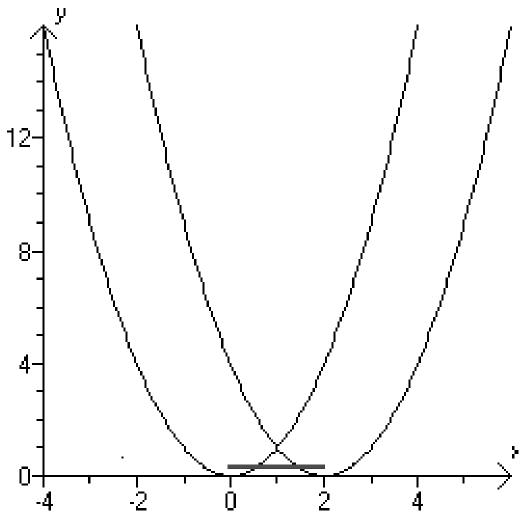

FIGURE 4. Final population obtained after 40 generations

For example, if we consider the value

$$\sigma = 0.01,$$

the solution

$$s = [0.004, 1.997],$$

is obtained after 60 iterations.

Example 2. Consider the functions $f_1, f_2 : [-10, 13] \to \mathbb{R}$ defined as

$$f_1(x) = \sin(x),$$

$$f_2(x) = \sin(x + 0.7).$$

and the multiobjective optimization problem:

$$\begin{cases} \text{minimize } f_1(x), f_2(x) \\ \text{subject to} x \in [-10, 13] \end{cases}$$

The initial population is depicted in Figure 5.

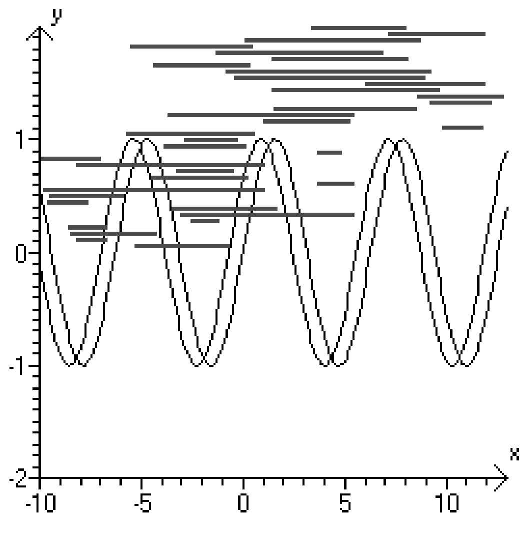

FIGURE 5. Initial population

For the value

$$\sigma = 0.1$$

solutions obtained after 5 generations are depicted in Figure 6.

We may observe four distinct, well-separated, subpopulations are already segregated after 5 generations. Therefore useful subpopulations are stabilized very early. Let us remark that, for the sake of clarity, segments in the same class are separately represented. In reality they partially overlap.

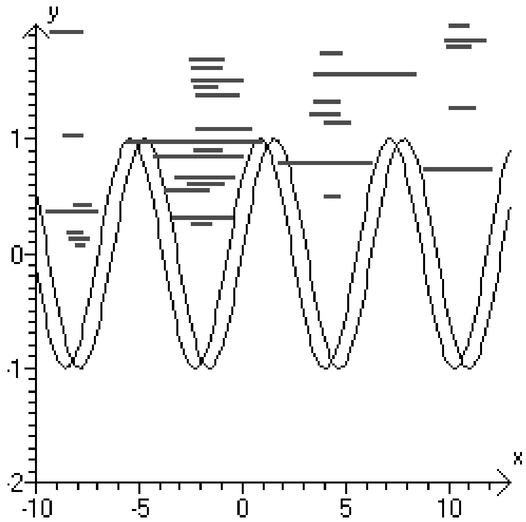

FIGURE 6. The population after 5 generations

The population after 10 generations is depicted in Figure 7. Subpopulations are well individualized and nested.

The final population, obtained after 120 generations, is depicted in Figure 8.

Solutions in the final population are:

$$s_1 = [-8.47, -7.86],$$
$$s_2 = [-2.26, -1.56],$$
$$s_3 = [4.01, 4.69],$$
$$s_4 = [10.29, 10.99].$$

Example 3. Consider the functions $f_1, f_2 : [-9, 9] \to \mathbb{R}$ defined as

$$f_1(x) = x^2,$$
$$f_2(x) = 9 - \sqrt{81 - x^2}.$$

and the multiobjective optimization problem:

$$\begin{cases} \text{minimize } f_1(x), f_2(x) \\ \text{subject to} x \in [-9, 9] \end{cases}$$

The initial population is depicted in Figure 9.

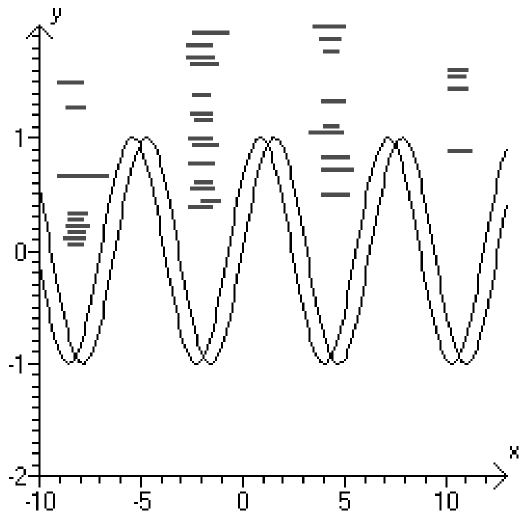

FIGURE 7. Population after 10 generations

Consider the standard deviation parameter value

$$\sigma = 0.1,$$

In this case population obtained after 3 generations is depicted in Figure 10.

It is very interesting to observe that very early population stabilizes to a single individual. This individual will be improved at subsequent iterations.

The population after 7 generations is depicted in Figure 11.

The final population, obtained after 120 generations, is depicted in Figure 12.

Final population obtained at convergence after 120 generations contains only one individual represented as degenerated interval (i.e. a point)

$$s = -0.001.$$

Therefore detected Pareto optimal set consists from a single point:

$$P_{\text{detect}} = \{-0.001\}.$$

We may remark that detected Pareto set represents a good estimation of the correct Pareto optimal set

$$P_c = \{0\}.$$

Accuracy of this estimation can be easy improved by using smaller values of the parameter ? (standard deviation). In this case a larger number of generations are needed for convergence.

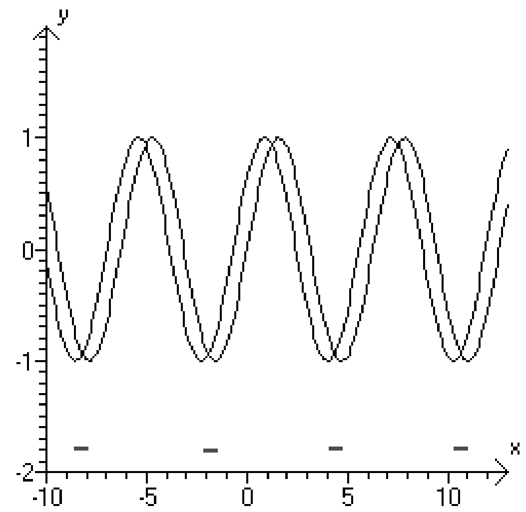

FIGURE 8. Four solutions within the final population (obtained after 120 generations)

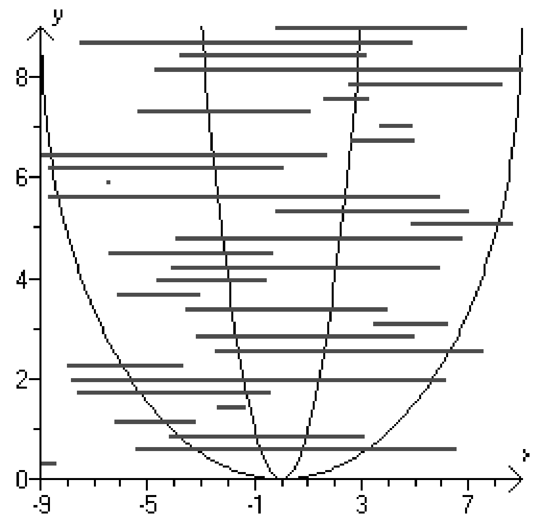

FIGURE 9. Initial population

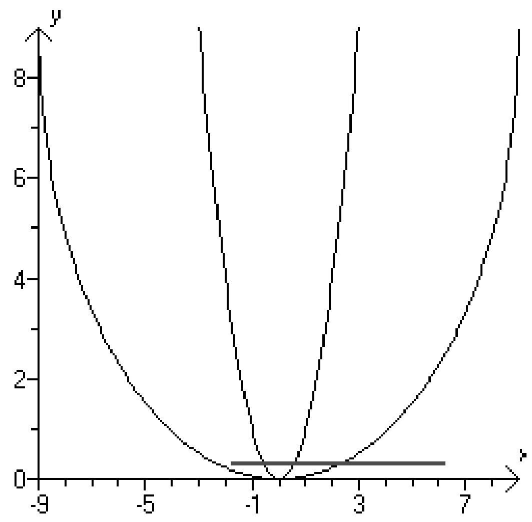

FIGURE 10. Population after 3 generations

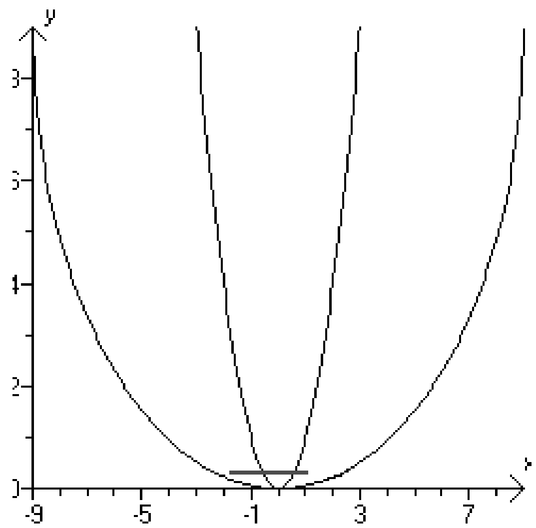

FIGURE 11. Population after 7 generations

For instance, if we put

$$\sigma = 0.01,$$

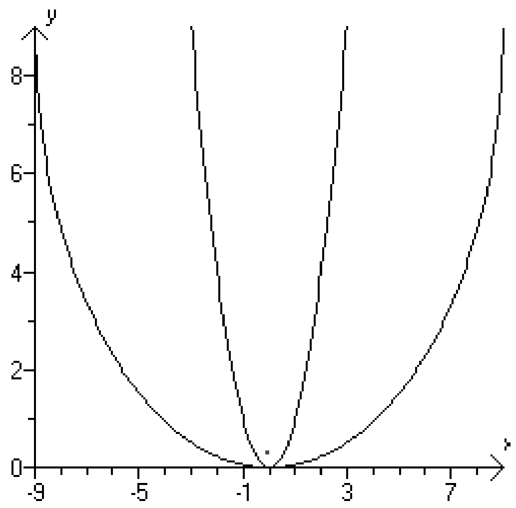

FIGURE 12. Final population obtained after 120 generations

the obtained solution is

$$s = 0.0008.$$

8. Concluding remarks and further researches

A new evolutionary technique for solving multiobjective optimization problems involving one variable functions is proposed. A new solution representation is used. Standard search (variation) operators are modified accordingly. Three new search operators are introduced. The proposed evolutionary multiobjective optimization technique does not use a secondary population of non-dominated solutions.

Proposed multiobjective optimization method uses a new evolutionary metaheuristic called Genetic chromodynamics for maintaining multiple optimal solutions on the calculated Pareto set during the search process.

All known multiobjective optimization techniques supply a discrete picture of Pareto optimal solutions and of Pareto frontier. But Pareto optimal set is usually non-discrete. Finding Pareto optimal set and Pareto optimal frontiers using a discrete representation is not a very easy computationally task (see [11]).

CPOS technique supplies directly a continuous picture of Pareto optimal set and of Pareto frontier. This makes our approach very appealing for solving problems where very accurate solutions detection is needed.

Another advantage is that CPOS technique has a natural termination condition derived from the nature of evolutionary method used for preserving population diversity.

Experimental results suggest that CPOS algorithm supplies correct solutions in a very few iterations.

Further research will concentrate on the possibilities to extend the proposed technique to deal with multidimensional domains.

Another direction is to exploit the solution representation as intervals for solving inequality systems and other problems for which this representation is natural.

References

[1] Coello, C. A. C, A comprehensive survey of evolutionary- based multiobjective optimization techniques, *Knowledge and Information Systems*, 1(3), 1999, 269-308.

[2] Deb, K., Multiobjective evolutionary algorithms: problem difficulties and construction of test problems, *Evolutionary Computation*, 7, 1999, 205-230.

[3] Dumitrescu, D, Lazzerini, B, Jain, L., C., Dumitrescu, A., *Evolutionary Computation*, CRC Press, Boca Raton, 2000.

[4] Dumitrescu, D., Evolutionary Chromodynamic, Studia Univ. Babeş-Bolyai, Ser. Informatica, 2000.

[5] Fonseca, C.M., Fleming, P.J., An overview of evolutionary algorithms in multiobjective optimization, *Evolutionary Computation*, 3, 1995, 1-16.

[6] Goldberg, D.E., *Evolutionary Algorithms in Search, Optimization and Machine Learning*, Addison Wesley, Reading, 1989.

[7] Horn, J., Nafpliotis, N., Multiobjective optimization using niched Pareto evolutionary algorithms, *IlliGAL Report 93005*, Illionois Evolutionary Algorithms Laboratory, University of Illinois, Urbana Champaingn.

[8] Horn, J., Nafpliotis, N., Goldberg D. E. A niche Pareto evolutionary algorithm for multiobjective optimization, *Proc. 1st IEEE Conf. Evolutionary Computation*, Piscataway, vol 1, 1994, 82-87.

[9] Schaffer, J.D., Multiple objective optimization with vector evaluated evolutionary algorithms, *Evolutionary Algorithms and Their Applications*, J.J. Grefenstette (Ed.), Erlbaum, Hillsdale, NJ, 1985, 93-100.

[10] Srinivas, N., Deb, K., Multiobjective function optimization using nondominated sorting evolutionary algorithms, *Evolutionary Computing*, 2, 1994, 221-248.

[11] Veldhuizen, D.A.V., Multiobjective Evolutionary Algorithms: Classification, Analyses and New Innovations, Ph.D Thesis, 1999, Graduated School of Engineering of the Air Force Institute of Technology, Air University.

[12] Veldhuizen, D.A.V., Lamont, G.B., Multiobjective evolutionary algorithms: analyzing the state-of-the-art, Evolutionary Computation, 8, 2000, 125-147.

[13] Zitzler, E., Thiele, L, Multiobjective evolutionary algorithms: A comparative study and the strength Pareto approach, IEEE Trans on Evolutionary Computation, 3 (1999), 257-271.

[14] Zitzler, E., Evolutionary Algorithms for Multiobjective Optimization: Methods and Applications, Doctoral Dissertation, 1999, Swiss Federal Institut of Technology Zurich,Tik-Schriftenreihe nr. 30.

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