STUDIA UNIV. BABEŞ–BOLYAI, INFORMATICA, Volume **XLVI**, Number 1, 2001

GENETIC CHROMODYNAMICS FOR OBTAINING CONTINUOUS REPRESENTATION OF PARETO REGIONS

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Abstract. In [5] an evolutionary algorithm for detecting continuous Pareto optimal sets has been proposed. In this paper we propose a new evolutionary elitist approach combing a non-standard solution representation and an evolutionary optimization technique. The proposed method permits detection of continuous decision regions. In our approach an individual (a solution) is either a closed interval or a point. The individuals in the final population give a realistic representation of Pareto optimal set. Each solution in this population corresponds to a decision region of Pareto optimal set. Proposed technique is an elitist one. It uses a unique population. Current population contains non- dominated solutions already founded.

Keywords: evolutionary multiobjective optimization, Pareto set, Pareto frontier, Pareto interval

1. Introduction

Several evolutionary algorithms for solving multiobjective optimization problems have been proposed ([2], [5]–[10], [12], [13]; see also the reviews [1], [11] and [14]).

Usually Pareto evolutionary algorithms aim to give a discrete picture of the Pareto optimal set (and of the corresponding Pareto frontier). But generally Pareto optimal set is a continuous region in the search space. Therefore a continuous region is represented by a discrete set. When continuous decision regions are represented by discrete solutions there is loss of information. Moreover reconstructing continuous Pareto set from a discrete picture is not an easy task [11].

In [5] an evolutionary algorithm for detecting continuous Pareto optimal sets has been proposed. The method proposed in [5] uses Genetic chromodynamics evolutionary technique [4] to maintain population diversity.

2000 *Mathematics Subject Classification.* 68T05.

1998 *CR Categories and Descriptors.* I.2.8 [**Computing Methodologies**]: Artificial Intelligence – *Problem Solving, Control Methods, and Search.*

In this paper a new evolutionary approach, combing a non-standard solution representation and a Genetic Chromodynamics optimization technique, is considered. Within the proposed approach continuous decision regions may be detected. A solution (individual) is either a closed interval or a point (considered as a degenerated interval). Mutation is the unique search operator considered.

The mutation operator idea is to expand each individual toward left and toward right. In this respect both interval extremities are mutated. The left extremity is mutated towards left and the right extremity is mutated towards right.

To reduce population size and to obtain the correct number of solutions within the final population the *merging* operator introduced in context of Genetic Chromodynamics is used.

The solutions in the final population supply a realistic representation of Pareto optimal set. Each solution in this population corresponds to a decision region (a subset of Pareto set). A decision region will also be called a *Pareto region*.

The solutions are detected in two stages. In the first stage a Genetic Chromodynamics technique is used to detect all (local and global) Pareto solutions. In the second stage the solutions are refined. During the fine tunning the sub-optimal regions are removed.

The evolutionary multiobjective technique proposed in this paper is called *Continuous Pareto Set* (CPS) algorithm.

2. Problem statement

Let Ω be the search space. Consider n objective functions $f_1, f_2, \ldots, f_n$,

$$f_i : \Omega \to \mathcal{R},$$

where $\Omega \subset \mathcal{R}$.

Consider the multiobjective optimization problem:

$$\begin{cases} \text{optimize } f(x) = (f_1(x), \ldots, f_n(x)) \\ \text{subject to } x \in \Omega \end{cases}$$

The key concept in determining solutions of multiobjective optimization problems is that of Pareto optimality. In what follows we recall some basic definitions.

Definition1. *(Pareto dominance)* Consider a maximization problem. Let x, y be two decision vectors (solutions) from Ω. Solution x *dominate* y (also written as $x \succ y$) if and only if the following conditions are fulfilled:

(i) $f_i(x) \geq f_i(y), \forall i = 1, 2, \ldots, n,$

(ii) $\exists j \in \{1, 2, \ldots, n\} : f_j(x) > f_j(y).$

Definition 2. Let $S \subseteq \Omega$. All solutions, which are not dominated by any vector of S, are called *nondominated* with respect to S.

Definition 3. Solutions that are nondominated with respect to S, $S \subset \Omega$, are called *local Pareto* solutions or *local Pareto regions*.

Definition 4. Solutions that are nondominated with respect to the entire search space Ω are called *Pareto optimal* solutions.

Let us note that when the search space is a subset of $\mathcal{R}$, then Pareto optimal set may be represented as:

(i) a set of points;
(ii) a set of disjoint intervals;
(iii) a set of disjoint intervals and a set of points.

Remark. In each of the cases (i), (ii) and (iii) a point or an interval represents a Pareto region.

Evolutionary multiobjective optimization algorithms are intended for supplying a discrete picture of Pareto optimal set and of Pareto frontier. But Pareto set is usually a union of continuous set. When continuous decision regions are represented by discrete solutions there is some information loss. The resulting sets are but a discrete representation of their continuous counterparts.

Methods for finding Pareto optimal set and Pareto optimal front using discrete solutions are computationally very difficult. However the results may be accepted as the 'best possible? at a given computational resolution. Methods for obtaining the continuous representation using discrete outputs of evolutionary techniques are considered in Veldhuizen [11].

The evolutionary method proposed in this paper directly supply the true (i.e. possibly continuous) Pareto optimal set.

3. Solution representation and domination

In this paper we consider solutions are represented as intervals in the search space Ω.

Each interval-solution k is encoded by an interval $[x_k, y_k] \in \mathcal{R}$. Degenerated intervals are allowed. Within degenerate case $y_k = x_k$ the solution is a point.

In order to deal with proposed representation a new domination concept is needed. This domination concept is given by the next definition.

Definition 5. An interval-solution $[x, y]$ is said to be *interval-nondominated* if and only if all points of that interval $[x, y]$ are nondominated point-wise solutions. An interval-nondominated solution will be called a *Pareto-interval*. **Remarks.**

(i) If $x = y$ this concept reduced towards ordinary non-domination notion.
(ii) If no ambiguity arise we will use *nondominated* instead of *interval-nondominated*.

4. Mutation

Problem solutions are detected in two stages. In the first stage a Genetic Chromodynamics technique is used to detect all (global and local) solutions. This represent the *evolution stage*.

In the second stage (*fine tuning* or *refinement stage*) solutions that have been detected in the first stage are refined. By using the refinement procedure sub-optimal Pareto regions are removed from the final population.

Most of the multiobjective optimization techniques based on Pareto ranking use a secondary population (an archive) denoted P_{second} for storing nondominated individuals. Archive members may be used to guide the search process. As dimension of secondary population may dramatically increase several mechanisms for reducing archive size have been proposed. In [13] and [14] a population decreasing technique based on a clustering procedure is considered. We may observe that preserving only one individual from each cluster implies a loss of information.

In our approach the population size does not increase during the search process even if the number of Pareto optimal points increase. The population size is kept low due to the special representation we consider.

When a new nondominated point is found it replaces another point solution in the population or it is used for building a new interval solution with another point in the population. This does not cause any information loss concerning Pareto optimal set during the search process.

The algorithm starts with a population of degenerated intervals (i.e. a population of points). The unique variation operator is mutation. It consists of normal perturbation of interval extremities. Mutation can also be applied to point-solutions (considered as degenerated intervals). Each interval extremity is mutated. The left extremity of an interval is always mutated towards left and the right interval extremity is mutated only towards right.

For mutation two cases are to be considered.

a) Degenerated interval: The individual is mutated towards left or right with equal probability. The obtained point represents the offspring. Parent and offspring compete for survival. The best, in the sense of domination, enter the new population.

If parent and offspring are not comparable with respect to domination relation then the two points defines an interval solution. The new interval solution is included in the new generation. The point solution representing the parent is discarded.

b) Nondegenerated interval: (i) Firstly the left extremity of the interval $[u, v]$ is mutated towards left. A point-offspring u' is obtained. Consider the case when the offspring u' and the parent u do not dominate each other. In this situation a new interval solution $[u', v]$ is generated. The new solution has the offspring u' as its left extremity and v as its right extremity. If the offspring u' dominates the parent u, then the interval solution $[u, v]$ enters the new population.

(ii) A similar mutation procedure is applied to the right interval extremity of the previously obtained solution ($[u, v]$ or $[u', v]$).

5. Population model

For each generation every individual in the current population is mutated. Parents and offspring directly compete for survival. The domination relation guides this competition.

For detecting the correct number of Pareto optimal regions it is necessary to have, in the final population, only one solution per Pareto optimal region.

In this paper we consider the merging operator of Genetic Chromodynamics for implementing the population decreasing mechanism. Very close solutions are fused and population size decreases accordingly.

In the framework of this paper the merging operator is described as bellow:

(i) if two interval solutions overlap the shortest interval solution is discarded. Degenerated interval- solution included into non-degenerated interval-solutions are removed too;

(ii) if two degenerated solutions are closer than a fixed threshold, then the worst solution is discarded.

The merging operator is applied each time a new individual enters the population.

The method allows a natural termination condition. The algorithm stops when there is no improvement of the solutions for a fixed number of generations. Each solution in the last population supplies a Pareto optimal region contributing to the picture of Pareto optimal set.

6. Fine tuning

During the fine tuning stage sub-optimal solutions (regions) are removed. For this aim each continuous Pareto region is transformed into a discrete set. Discretized version is obtained considering points within each interval solution at a fixed step size.

Let us denote by ss the step size. Consider an interval solution $[x, y]$. From this solution consider the points xj fulfilling the conditions:

- $x_j = x + j \cdot ss, \; j = 0, 1, \ldots;$
- $x_j \leq y.$

These points represent the discretized version (denoted D) of the interval solution $[x, y]$.

Each point x_j within the interval solutions is checked. If a point from the discretized set D dominates the point x_j then x_j is removed from the Pareto interval $[x, y]$ together with a small neighboring region. The size of the removed region is equal with ss.

The intervals obtained after this stage are considered as the true Pareto sets.

7. Algorithm Complexity

The complexity of the proposed algorithm is low. Let m be the number of objectives and N the population size. The first stage requires

$$K_1 = O(m \cdot N \cdot IterationNumber)$$

operations.

Denote by I_{max} is the longest interval solution in the population. Consider a function

$$F : \mathcal{R} \times \mathcal{R} \to N.$$

Admit that F fulfills the following conditions:

(i) F is a linear function,
(ii) $F([a, a]) = 1$, for each $a \in (?\infty, \infty)$.

Second stage (fine tuning) requires

$$K_2 = O(N^2 \cdot F(I_{max})^2)$$

operations.

8. CPS Algorithm

Using the previous considerations we are ready to design a new multiobjective optimization algorithm.

The evolution stage of the CPS algorithm is outlined as bellow:

Algorithm **CPS**:
generates an initial population P(0) {all intervals are degenerated i.e. $x_k = y_k$}
$t = 0$;
while not (Stop_Condition) **do**

```
begin
    for each individual k in P(t) do
    begin
        p = random {generate a random number between 0 and 1}
        if p < 0.5
        then
            Left_offspr = MutateTowardsLeft(x_k);
            if dominate(Left_offspr, x_k)
            then
                if x_k = y_k
                then x_k = y_k = Left_offspr
            else
                if nondominated(Left_offspr, x_k)
                then x_k = Left_offspr;
        else
            Right_offspr = MutateTowardsRight(y_k);
            if dominate(Right_offspr, y_k)
            then
                if x_k = y_k
                then x_k = y_k = Right_offspr
            else
                if nondominated(Right_offspr, y_k)
                then y_k = Right_offspr;
        endif
    endfor
    t = t + 1
endwhile
```

Fine tuning part of CPS algorithm is obvious.

9. Numerical experiments

Several numerical experiments using CPOS algorithm have been performed. For all examples the detected solutions gave correct representations of Pareto set with an acceptable accuracy degree. Some particular examples are given below.

Example 1. Consider the functions $f_1, f_2 : [-10, 13] \to \mathcal{R}$ defined as

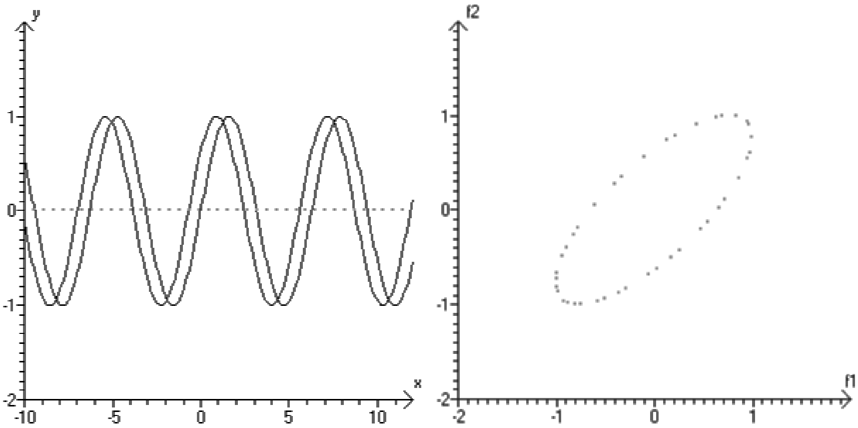
$$f_1(x) = sin(x),$$

$$f_2(x) = sin(x + 0.7).$$

Consider the multiobjective optimization problem:

$$\begin{cases} \text{optimize } f(x) = (f_1(x), f_2(x)) \\ \text{subject to } x \in [-10, 13] \end{cases}$$

The initial population is depicted in Figures 1(a) and 1(b).



(a) (b)

FIGURE 1. (a) Initial population represented within solution space; (b) Initial population represented within objective space

Consider the value

$$\sigma = 0.1$$

for the standard deviation of mutation operator. Solutions obtained after 3 generations are depicted in Figures 2(a) and 2(b).

The final population, obtained after 42 generations, is depicted in Figures 3(a) and 3(b).

The final population after the refinement stage is depicted in Figures 4(a) and 4(b).

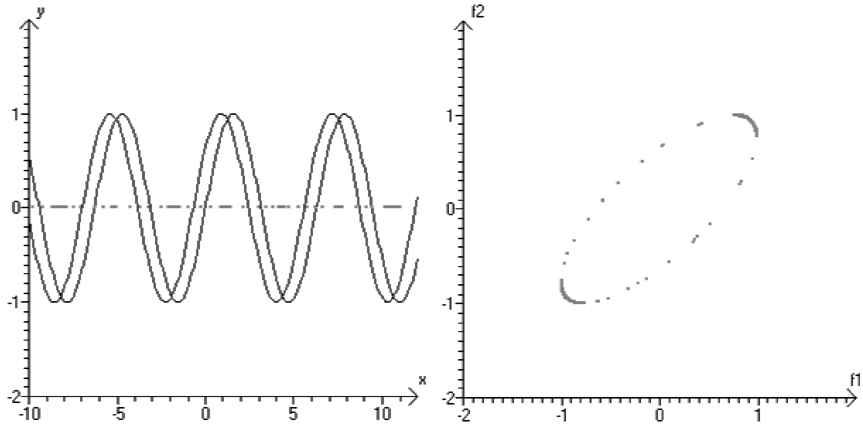
Solutions in the final population are:

$$s_1 = [-8.47, -7.86],$$

$$s_2 = [-2.26, -1.56],$$

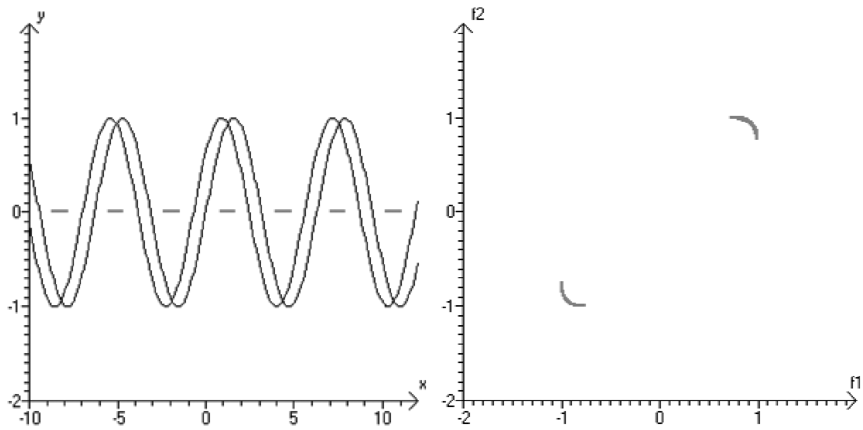
$$s_3 = [4.01, 4.69],$$

$$s_4 = [10.29, 10.99].$$



(a) (b)

FIGURE 2. (a) Population obtained after 3 generations represented within solution space; (b) Population obtained after 3 generations represented within objective space



(a) (b)

FIGURE 3. (a) Population obtained at convergence (after 42 generations) represented within solution space; (b) Population obtained at convergence (after 42 generations) represented within objective space

Example 2. Consider the functions $f_1, f_2 : [-10, 20] \to \mathcal{R}$ defined as

$$f_1(x) = sin(x),$$

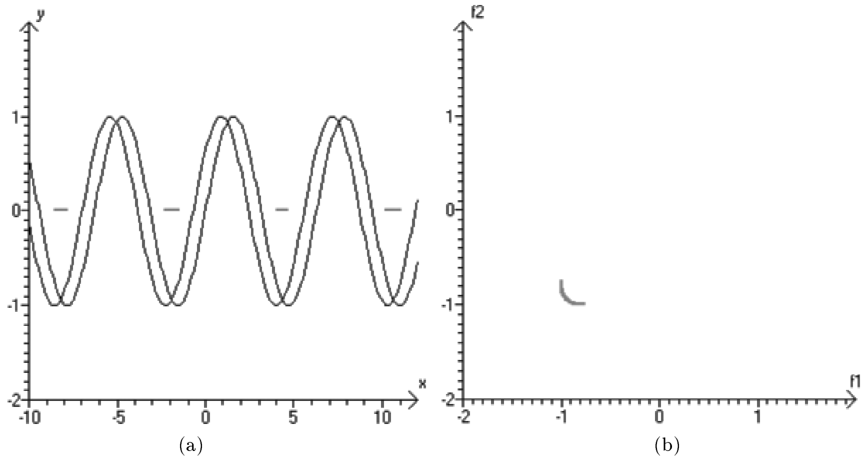


FIGURE 4. (a) Final population obtained after fine tuning stage represented within solution space; (b) Final population obtained after fine tuning stage represented within objective space

$$f_2(x) = 2 \cdot sin(x) + 1.$$

Consider the multiobjective optimization problem:

$$\begin{cases} \text{optimize } f(x) = (f_1(x), f_2(x)) \\ \text{subject to } x \in [-10, 20] \end{cases}$$

The initial population is depicted in Figures 5(a) and 5(b).

For the value

$$\sigma = 0.1$$

solutions obtained after 14 generations are depicted in Figures 6(a) and (b).

The final population, obtained after 70 generations, is depicted in Figures 7(a) and 7(b).

Example 3. Consider the functions $f_1, f_2 : [0, 24] \to \mathcal{R}$ defined as

$$f_1(x) = sin(x),$$

$$f_2(x) = \begin{cases} \frac{-4 \cdot x}{\pi} + 8 \cdot k, & 2 \cdot k \cdot \pi \leq x < 2 \cdot k \cdot \pi + \frac{\pi}{2}, \\ \frac{4 \cdot x}{\pi} - 4 \cdot (2 \cdot k + 1), & 2 \cdot k \cdot \pi + \frac{\pi}{2} \leq x < (2 \cdot k + 1) \cdot \pi, \\ \frac{-2 \cdot x}{\pi} + 2 \cdot (2 \cdot k + 1), & (2 \cdot k + 1) \cdot \pi \leq x < (2 \cdot k + 1) \cdot \pi + \frac{3 \cdot \pi}{2}, \\ \frac{2 \cdot x}{\pi} - 4 \cdot (k + 1), & 2 \cdot k \cdot \pi + \frac{3 \cdot \pi}{2} \leq x < 2 \cdot (k + 1) \cdot \pi + \frac{\pi}{2}. \end{cases}$$

where $k \in \mathbb{Z}^+$.

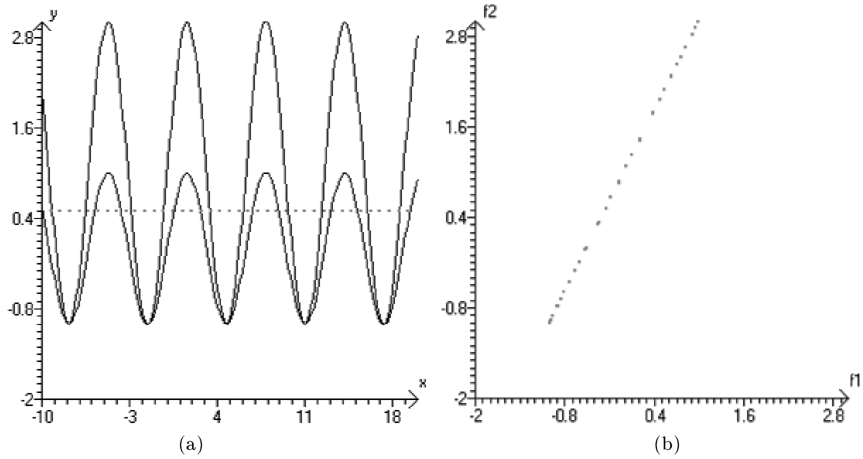


FIGURE 5. (a) Initial population represented within solution space; (b) Initial population represented within objective space

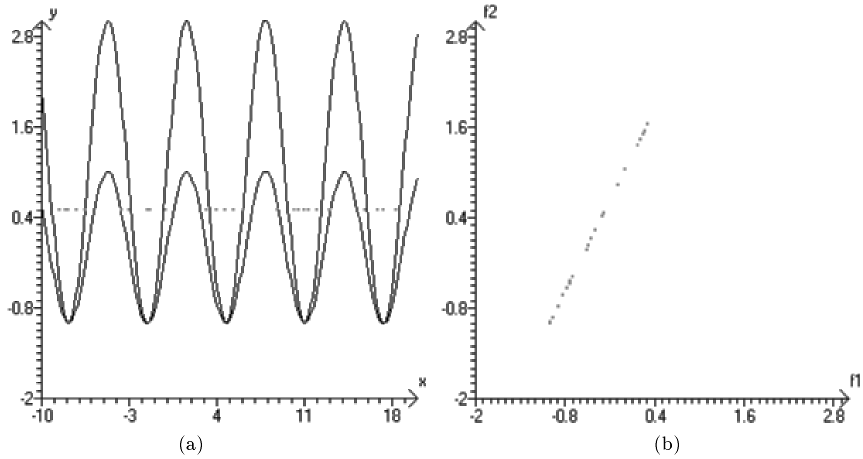
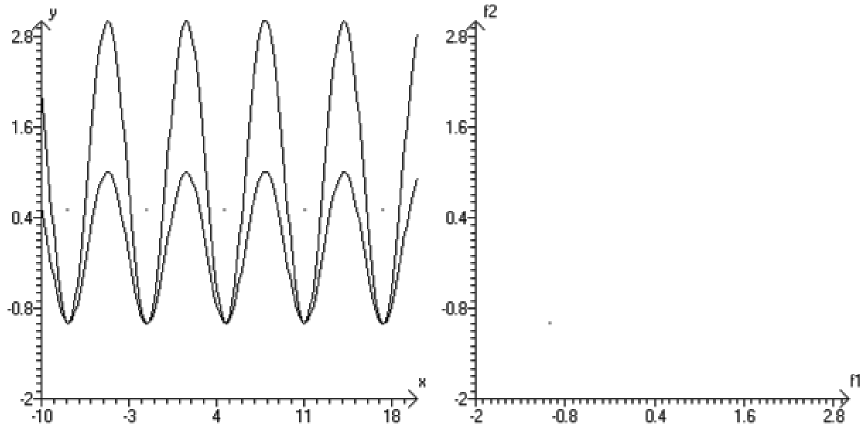


FIGURE 6. (a) Population obtained after 14 generations represented within solution space; (b) Population obtained after 14 generations represented within objective space

Consider the multiobjective optimization problem:

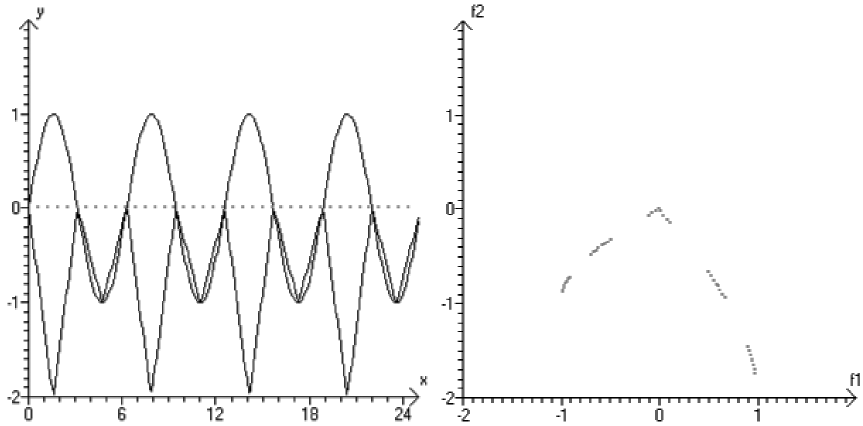
$$\begin{cases} \text{optimize } f(x) = (f_1(x), f_2(x)) \\ \text{subject to } x \in [0, 24] \end{cases}$$



(a) (b)

FIGURE 7. (a) Final population obtained after 70 generations represented within solution space; (b) Final population obtained after 70 generations represented within objective space

The initial population is depicted in Figure 8(a) and 8(b).



(a) (b)

FIGURE 8. (a) Initial population represented within solution space; (b) Initial population represented within objective space

For the value

$$\sigma = 0.1$$

solutions obtained after 4 generations are depicted in Figures 9(a) and 9(b).

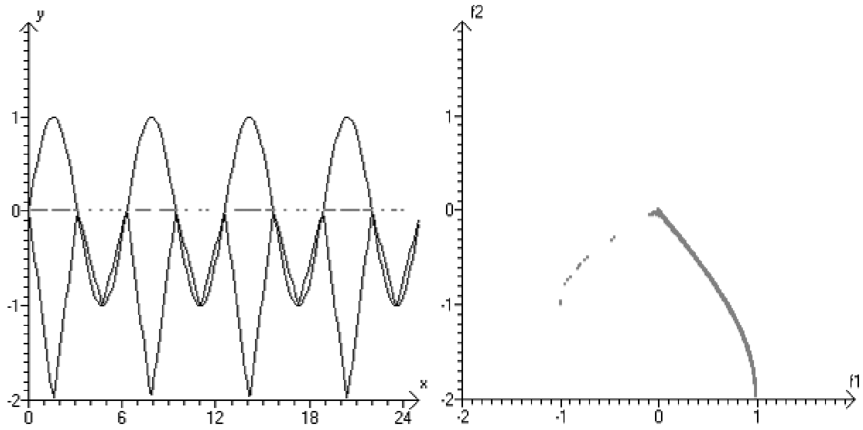


FIGURE 9. (a) Population obtained after 4 generations represented within solution space; (b) Population obtained after 4 generations represented within objective space

The final population, obtained after 60 generations, is depicted in Figures 10(a) and 10(b).

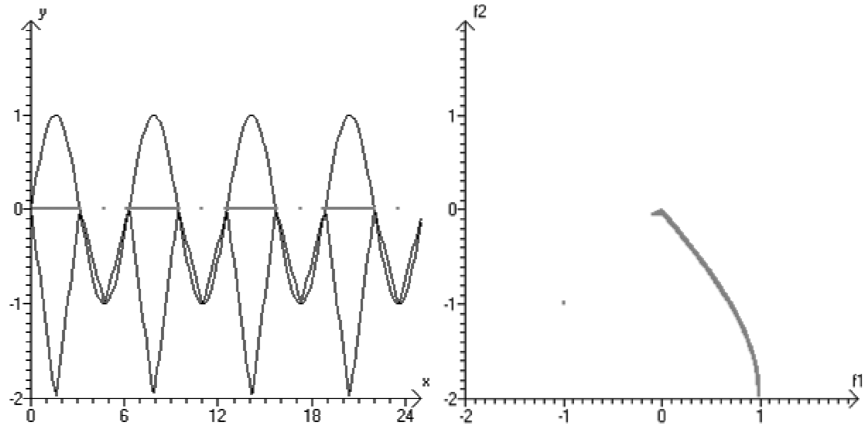
The final population after the refinement stage is depicted in Figure 11(a) and 11(b).

10. Concluding remarks and further research

A new evolutionary technique for solving multiobjective optimization problems involving one variable functions is proposed. A new solution representation is used. Standard search (variation) operators are modified accordingly. The proposed evolutionary multiobjective optimization technique uses only one population. This population consists of nondominated solutions already founded.

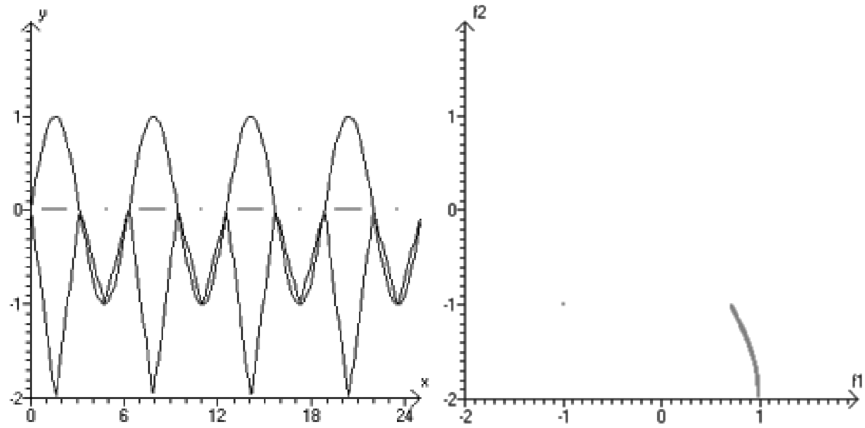
All known standard or recent multiobjective optimization techniques supply a discrete picture of Pareto optimal solutions and of Pareto frontier. But Pareto optimal set is usually non-discrete. Finding Pareto optimal set and Pareto optimal frontiers using a discrete representation is not a very easy computationally task (see [11]).

Evolutionary technique proposed in this paper supplies directly a continuous picture of Pareto optimal set and of Pareto frontier. This makes our approach



(a) (b)

FIGURE 10. (a) Population obtained at convergence (after 60 generations) represented within solution space; (b) Population obtained at convergence (after 60 generations) represented within objective space



(a) (b)

FIGURE 11. (a) Final population obtained after fine tuning stage represented within solution space; (b) Final population obtained after fine tuning stage represented within objective space

very appealing for solving problems where very accurate solutions detection is needed.

Another advantage is that CPS technique has a natural termination condition derived from the nature of evolutionary method used for preserving population diversity.

Experimental results suggest that CPS algorithm supplies correct solutions after few generations.

Further research will focus on the possibilities to extend the proposed technique to deal with multidimensional domains.

Another research direction is to exploit the solution representation as intervals for solving inequality systems and other problems for which this representation seems to be natural.

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