MULTI-SEXUAL GENETIC ALGORITHM FOR MULTIOBJECTIVE OPTIMIZATION.

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Abstract

In this paper a new method for solving multicriteria optimization problems by Genetic Algorithms is proposed. Standard Genetic Algorithms use a population, where each individual has the same sex (or has no sex) and any two individuals can be crossed over. In the proposed Multisexual Genetic Algorithm (MSGA), individuals have an additional feature, their sex or gender and one individual from each sex is used in the recombination process. In our multicriteria optimization application there are as many sexes as optimization criteria and each individual is evaluated according to the optimization criterion related to its sex. Furthermore, a multi-parent crossover is applied to generate offspring of parents belonging to all different sexes, so the offspring represents intermediate solutions not totally optimal with respect to any single criterion. During the execution of the algorithm the set of nondominated solutions is updated and this set is presented as the output of MSGA at the end.

I. Introduction

Many real-word design or decision making problems have multiple objectives, in the sense that several aims need to be achieved simultaneously. All these aims (objectives) can be described quantitatively as a set of design objective functions which is associated with a number of inequality and equality constraints. In most cases, the objective functions are in conflict with one another, so that it is not possible to improve on any of the objective functions without deteriorating at least one of the other objective functions. This is known as the concept of Pareto optimality (Rao, 1991). In multiobjective optimization, as opposed to single-objective optimization, there may not exist an unambigous optimal solution (global minimum or global maximum). Characteristic of the multiobjective optimization problems is a very large set of acceptable solutions that are superior to the rest of the solutions in the search space when all objectives are considered. At the same time they are not optimal from the point of any single objective. These solutions are known as Pareto-optimal solutions or nondominated solutions. The rest of the solutions are referred to as dominated solutions. Since none of the solutions in a Pareto-optimal set is absolutely better than any other, any one of them is an acceptable solution. The choice of one particular solution depends on the features of the problem and a number of problem-related factors.

Even in the simplest case of two objective functions without any constraints it is very difficult to get the set of Pareto-optimal solutions. In trying to solve multiobjective optimization problems many classical methods scalarize the objective vector into a single objective using some knowledge about the problem being solved. In these cases, the obtained solution is highly sensitive to the weight vector used in the scalarization process. Moreover, the optimization of this single objective may guarantee a Pareto-optimal solution but it results in a single-point solution. The designer, however, may prefer one (Pareto optimal) point over the others depending on the situation. Consequently, it it may be useful to have (all) other possible Pareto-optimal solutions. The traditional methods cannot find multiple Pareto-optimal solutions simultaneously (Srinivas and Deb, 1995).

Because GAs deal with a population of points it seems natural to use GAs in multiobjective optimization problems to capture a number of solution simultaneously. Several GA-based methods for solving multiobjective optimization problems have been developed recently. The main approaches are: Vector Evaluated Genetic Algorithm (Schaffer, 1985), Niched Pareto Algorithm (Horn et. al, 1993), Pareto Ranking Based Genetic Algorithm (Belegundu et. al., 1994), Nondominated Sorting Genetic Algorithm (Srinivas and Deb, 1995). In this paper a new method for solving multicriteria optimization problems using GAs is presented.

In section II the multiobjective optimization problem is formulated. The MSGA used to solve this problem is described in section III. Numerical experiments and results are presented in section IV, while section V contains the conclusions.

II. Multiobjective optimization

A general multiobjective optimization problem consists of a number of objectives and constraints in the form of inequalities and equalities. Formally, the problem can be described as follows (Rao, 1991):

minimize (or maximize) $\mathbf{F}(\mathbf{x})$, $\mathbf{x} \in \mathbf{R}^n$ under the inequality, resp. equality constraints

$g_k(\mathbf{x}) < 0$, k= 1,..., K and

$h_m(\mathbf{x}) = 0$, m= 1,..., M,

where

$\mathbf{x} = [\, x_1, x_2, \ldots, x_n]$ is n dimensional vector having n design or decision variables and

$\mathbf{F} = [F_1(\mathbf{x}), F_2(\mathbf{x}), \ldots, F_I(\mathbf{x})]$.

Definition 1 $\mathbf{x} \in R^n$ is Pareto-optimal solution if there is no $\mathbf{x}^* \in R^n$ for which $F(\mathbf{x}^*)$ dominates $F(\mathbf{x})$ i.e. there is no $\mathbf{x}^*$ such that $F_i(\mathbf{x}^*) \le F_i(\mathbf{x})$ $\forall$ i = 1,...,I.

Definition 2 A vector **x** and a vector **y** are nondominated by each other if neither **x** dominates **y** nor **y** dominates **x**.

III. Multisexual Genetic Algorithm

The rationale behind the MSGA consists of three ideas:

- provide each individual with an additional feature - the sex or gender;
- map optimization criteria to sexes by a one-to-one mapping and evaluate individuals by the optimization criteria belonging to their sex;
- use multi-parent crossover for recombination, requiring one parent from each sex.

The differences between classical GAs and MSGAs are summarized in Table 1.

Classical (uni-sexual) GA	MSGA
Each individual is evaluated in the same way, i.e. using the same fitness function or the same set of fitness functions.	The individuals with different sex are evaluated using different fitness functions (different optimization criteria)
There is no restriction on which two (or more) parents can be used by the crossover operator	The crossover operator uses the representants of all sexes – exactly one from each sex.

Table 1. The differences between GAs and the MSGA.

The MSGA can be used in natural way to multiobjective optimization problems. By the analogy of specialization and differences between males and females observed in real world, the representatives of different sexes will try to specialize in fulfilling different optimization criteria. Their children are supposed to be intermediate results, probably not the best according to any single criterion, but having a chance to be nondominant according to Pareto definition.

There are additional advantages of the simultaneous crossover of many parents. Eiben *et al.* (Eiben, 1994) and (Eiben, 1995) use multi-parent crossovers in a sexless, or uni-sexual, population. This leads to improved GA performance, the multi-parent crossover operators are more explorative and less sensitive to premature convergence. This mechanism should appear also in multisexual populations - a uni-sexual population is just a special case of a multisexual population, where all of the criteria are the same.

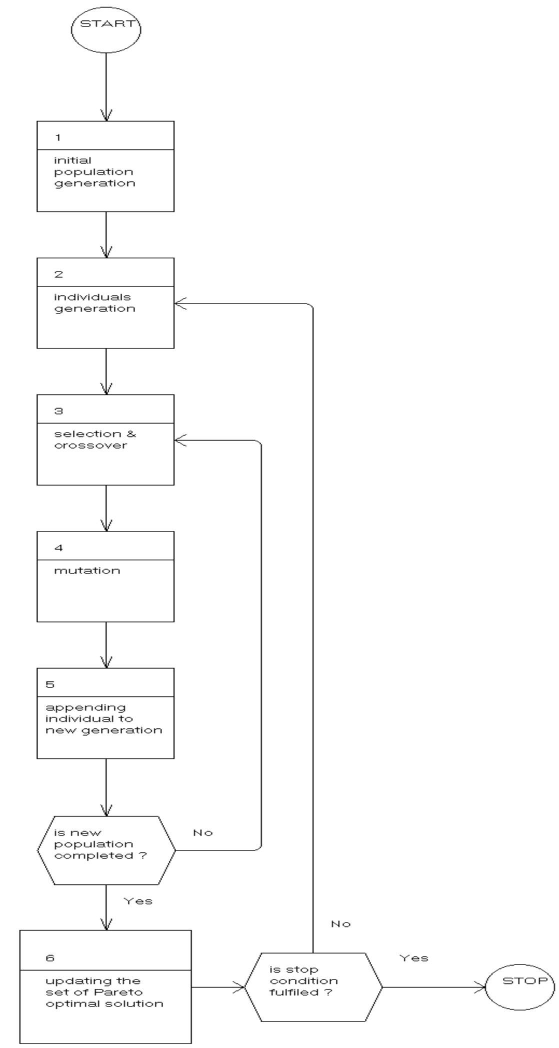

Figure 1. The block scheme of the MSGA.

The most important elements of the MSGA are described below.

Representation of solutions

A solution is represented by a binary string, like in classical GA, and the sex marker - an integer number from the interval [1,...,N] where N is the number of the optimized criteria, see Figure 2.

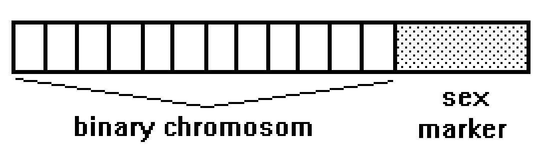

Figure 2. Representation of solutions.

Initial population generation

The usual part of the chromosome, the genetic code, and the sex marker can be chosen randomly or established according to some *a priori* knowledge about the problem being optimized.

Individual evaluation

For each individual the proper fitness function is calculated. The fitness function is chosen according to the sex of individual.

$f(\mathbf{x}) = F_{s(\mathbf{x})}(\mathbf{x})$

where $s(\mathbf{x}) \in \{1, ..., I\}$ is the sex of individual x and F_i ($i \in \{1, ..., I\}$) is the i-th optimization criterion.

Next, for each sex, individuals belonging to that sex are sorted according to their fitness function values. The rank obtained by this sorting is the basis of future selection. The ranks are determined independently for each sex.

Selection and crossover

The steps of selection and crossover are shown in Figure 3 and explained below.

First, according to the predefined crossover rate, the algorithm decides, whether the next member of the new population is simply copied from old population or is generated by crossover.

If crossover is used, then:

For each sex, one individual is chosen from among all the representatives of this sex. The probability of choosing an individual is related to its rank calculated in the evaluation step.

The selected individuals (one representative from each sex) are crossed over by uniform scanning crossover (Eiben, 1994). The uniform scanning crossover operator generates one child from many parents. During this process, each particular bit in the child's binary string is chosen randomly with equal probability from the corresponding bits (bits on the same position) in the parents.

The sex of the offspring is inherited from that parent, which supplied the largest number of genes (bits in binary string). If more then one parent gave to the offspring the same, maximal number of genes, then the sex of offspring is drawn randomly from their sexes.

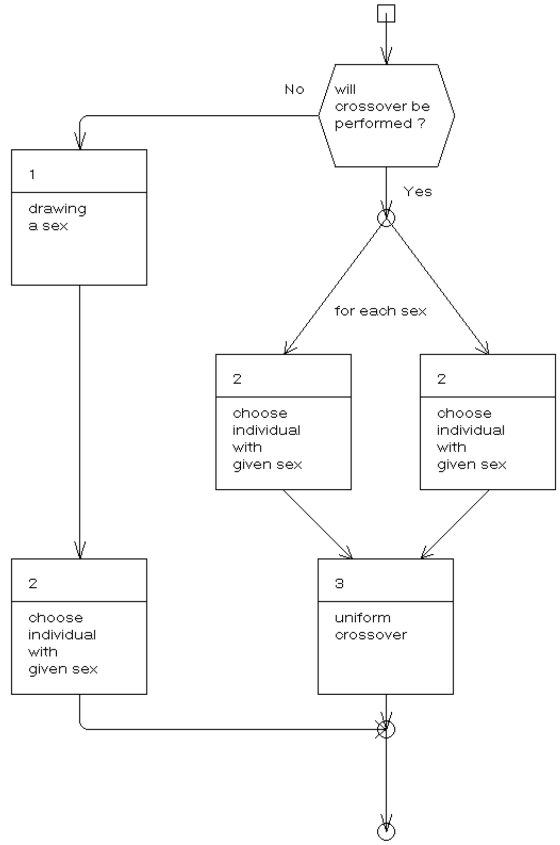

Figure 3. The block scheme of selection and crossover.

If crossover is not applied, one of the existing individuals from the old generation is copied to the new one. Because the fitness function values can be compared only for the same sex, selecting the individual involves two steps.

The sex is selected randomly, with equal probability.

For the given sex, the individual is selected according to its rank.

Mutation

Simple classical bit-flip mutation is applied. Mutation is performed only on the binary part of the chromosom, the sex marker remains unchanged.

Updating the set of nondominated solutions

The group of locally nondominated solutions from the current generation is selected and merged with the existing set of nondominated solutions, collected from all previous generations. The solutions, which become dominated are then removed.

The set of collected nondominated solutions is the actual output of executing the MSGA.

IV. Experiments

The MSGA was applied to solve several test problems. The second of tests presented here was described in (Srinivas and Deb, 1995). The MSGA was implemented in c++.

Test 1

Parameters:
population size: 30
crossover rate: 0.8
mutation rate: 0.01
chromosome length: 32

The goal is to minimize:

$$f_1(x,y) = (x^2+y^2)^{1/8}$$

and $$f_2(x,y) = ((x-0.5)^2+(y-0.5)^2)^{1/4}$$

where $x \in <-5, 10>$

The trade-off surface for this example is concave, what is the potential difficulty (Schaffer, 1985)

After 197 generations 129 nondominated solutions were found. The solutions are shown in Figure 4.

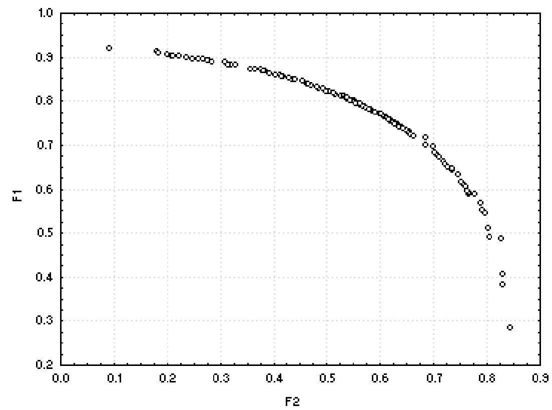

Figure 4. Results of Test 1 in terms of F1 and F2 values.

Test 2

Parameters:
population size: 100
crossover rate: 0.3
mutation rate: 0.001
chromosome length: 16

The goal is to minimize:

$$f_1(x) = \begin{cases} -x & \text{if } x \le 1 \\ -2+x & \text{if } 1 < x \le 3 \\ 4-x & \text{if } 3 < x \le 4 \\ -4+x & \text{if } 4 < x \end{cases}$$

and $$f_2(x) = (x-5)^2$$

After 631 generations 494 nondominated solutions were found. The solutions are shown in Figure 5.

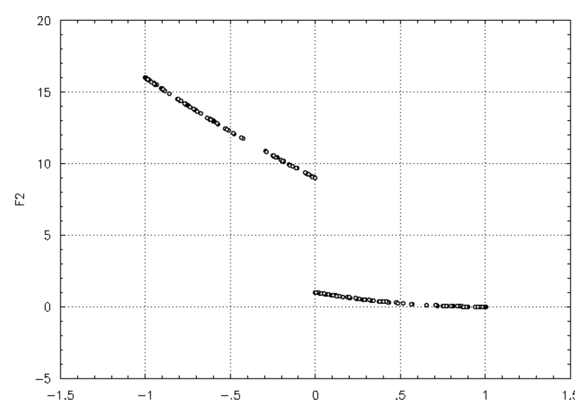

Figure 5. Results of Test 2 in terms of F1 and F2 values.

V. Conclusions and future work

The performed tests confirm the MSGA's ability to find Pareto-optimal solutions. Enforcing that crossover is applied to representatives of different sexes, and the fact that different sexes are evaluated (thus selected) by different optimization criteria prevent the algorithm from convergence to points optimal with respect to only one single optimization criterion.

A particular advantage of the MSGA is that it stores the set of nondominated solutions, which thus may exceed the population size. The set of nondominated solutions, obtained as the output of executing the MSGA, covers the whole analytically computated region of Pareto-optimal solutions. The person choosing one or several solutions from this set, has a clear view where Pareto-optimal solutions really are. This would probably be impossible in other multicriterial GAs, where the size of the set of solutions is limited by the population size, usually not very big.

The results of the preliminary experiments presented in this paper are promising. Nevertheless, there is much to do in order to establish the power and the possible limits of this approach. To this end, a comparison of the MSGA with other multicriterial GA will be performed in the future. There are also numerous possible enhancements and variations of the MSGA, for instance

- using different multi-parent crossovers, e.g. diagonal crossover (Eiben, 1994)
- allowing to recombine less parents then number of sexes
- allowing to recombine more then one parent of a particular sex
- choosing the sex of offspring according to the rank it would obtain if it had a particular sex; or assigning a pre-determined sex to the offspring
- using floating point representation of solutions

VI. Acknowledgments

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VII. References

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