Determining the Color-Efficiency Pareto Optimal Surface for Filtered Light Sources

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Abstract. While there are many factors that determine the commercial potential of an electric light source, color and efficiency are arguably the most important. Tradeoffs between color and efficiency are frequently made in lighting applications, typically by moving between different light source technologies. However, the potential exists to change position in color-efficiency space by filtering a light source. Because color is specified in two dimensions, and efficiency in one, the Pareto-optimal color and efficiency front defines a surface. This paper presents a method for determining color-efficiency Pareto optimal surface for a filtered light source.

1 Introduction

There are many factors that determine the commercial potential of an electric light source, including color, efficiency, color rendering index, color temperature, and the reliability and lifetime of the source. While the success or failure of novel light source technologies in the marketplace is dependent on all of these factors, color and efficiency are arguably the most important factors.

Tradeoffs between color and efficiency are made in different applications. For example, low pressure sodium lamps, which have "bad" color but high efficiency are used in many outdoor applications; meanwhile, incandescent lamps, which have "good" color but low efficiency are used in many indoor applications. These examples switch between light source technology to move around the color-efficiency Pareto optimal surface; however, by filtering a light source, it is also possible to move about on the Pareto optimal surface. In cases where the efficiency of a light source is high, but the color is undesirable, it might be possible to filter the light such that the efficiency is reduced by a small amount, but the color becomes acceptable. The goal of this research is to develop a technique for determining the color-efficiency Pareto optimal surface for filtered light sources.

2 Characterization of Light Sources

Various radiometric, photometric, and colorimetric properties of a light source can be determined from the source's spectral power distribution (SPD) [1], the radiant power per unit wavelength as a function of wavelength. The SPDs of the three light sources used in this paper are plotted in Figure 1. Note that SPDs can be smooth and continuous (e.g., incandescent lamps, sulfur lamps), or more spiky, with power either spread throughout the visible spectrum (e.g., metal halide lamps, fluorescent lamps), or concentrated principally in one portion of the visible spectrum (e.g., high pressure sodium lamps, low pressure sodium lamps).

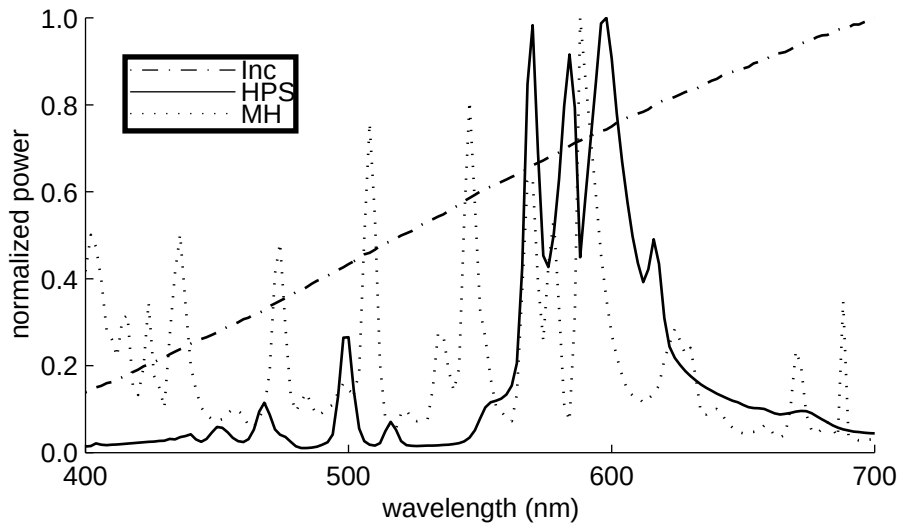

Fig. 1. SPD for metal halide (MH), high pressure sodium (HPS) and incandescent (Inc) lamps.

2. 1 The CIE System

The International Commission on Illumination (CIE[1]) is the ISO recognized body for all matters regarding the science and art of lighting. The CIE has developed [2] a method to represent the color and brightness of a light source from its SPD. The SPD

[1] After the French, 'Commission Internationale de l'Eclairage'.

is weighted by the color matching functions and integrated over the visible spectrum to provide the tristimulus values, X, Y and Z:

$$X = k \int_{\lambda} P_{\lambda} \bar{x}(\lambda) d\lambda \tag{1}$$

$$Y = k \int_{\lambda} P_{\lambda} \bar{y}(\lambda) d\lambda \tag{2}$$

$$Z = k \int_{\lambda} P_{\lambda} \bar{z}(\lambda) d\lambda \tag{3}$$

where $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ are color matching functions (weighting factors, Fig. 2), $P(\lambda)$ is the power at wavelength λ, and k is an application specific constant (set to 1 for the work presented here). The Y tristimulus value is proportionate to the brightness of the light source. Efficiency is defined as the ratio of the Y tristimulus value of a filtered light source to the Y tristimulus value of the same unfiltered light source (i.e., "how bright is the filtered light source compared to the original light source"). Note that because this is a measure of relative efficiency (rather than absolute efficiency), efficiency as defined here should not be directly compared between lamps.

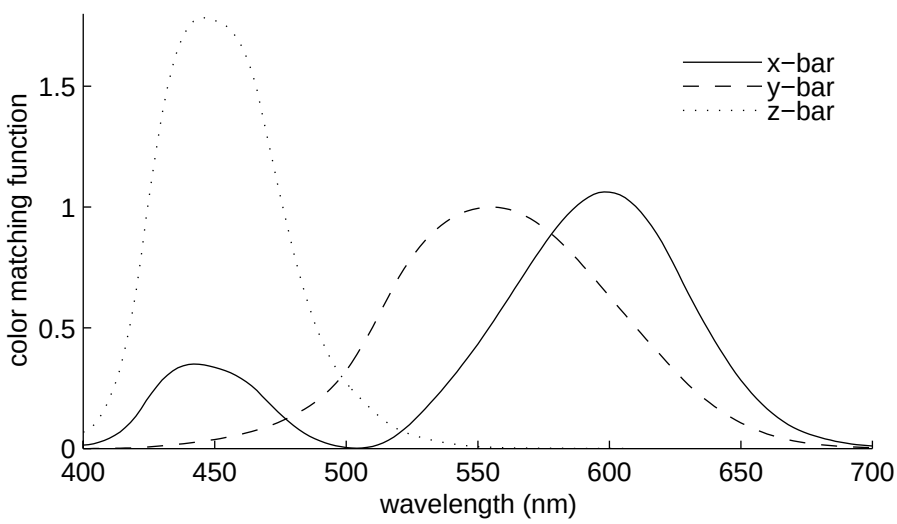

Fig. 2. CIE color matching functions.

The tristimulus values can be normalized:

$$x = \frac{X}{X + Y + Z} \tag{4}$$

$$y = \frac{Y}{X + Y + Z} \tag{5}$$

to yield the chromaticity coordinates, x and y, which describe the color of the light source. The universe of possible chromaticity coordinates is referred to here as 'color space'.

While a SPD has a unique set of chromaticity coordinates, a certain set of chromaticity coordinates can be produced by an infinite number of SPDs. Moreover, a light source might be filtered any number of ways to reach a point in color space - what will vary (among other properties, such as color rendering) is the brightness parameter of the light source, Y, and, therefore, the relative efficiency of the filtered light source. For any point in color space achievable by a light source, there is a way to filter the light source such that the efficiency at that color is maximized.

While only two factors, color and efficiency, are considered in this paper, because color is inherently two dimensional, the universe of non-dominated solutions results in a surface rather than a line. The surface is defined by the chromaticity coordinates and the maximum achievable efficiency at that point.

3 Optimization Approach

Genetic algorithms (GAs) were used to solve this multiobjective optimization problem. A sampling technique was used to fully explore color space, which ensures that areas near the spectrum locus are properly evaluated. However, this does induce a cost in computation time over other methods, which can be partially mitigated by sampling more frequently in areas of rapid change in efficiency and less frequently in areas of slower change. A grid of points in color space was cycled through, with each point acting as the target color of for a full run of a GA. The GA Optimization Toolbox [3] was used to implement the GAs for this research.

3.1 GA Implementation

The function:

$$\text{fitness} = \text{efficiency} - 2.5*(\text{distance}/0.08)^2 \tag{6}$$

was used to evaluate the fitness of chromosomes, where 'distance' is Euclidean distance in color space from the target color. This fitness metric incorporates both objectives (efficiently and color); the color portion of the fitness function is low in a small region near the objective (to give the GA room to search the efficiency

dimension) and quite high away from the objective (encouraging the GA to stay away from unfruitful areas in the color space). This has been shown [4] to be a good fitness function for finding an efficient filter near a given color.

A floating-point representation was used for this problem. The visible spectrum (400 to 700 nm) was partitioned in 151 bins, each 2 nm wide. Each chromosome consisted of 151 genes, where each gene represents the transmittance of the filter over a 2 nm band of the visible spectrum. The 2 nm bin width was chosen as a compromise between smoothness and computational tractability. Valid allele values for each gene could range from zero to one (i.e., 0% to 100% transmission at that wavelength). A population size of 50 chromosomes was used, which was initialized with uniform random gene values. A run was terminated after 500 generations.

Three mechanisms of crossover were applied simultaneously: (i) single point crossover [3]; (ii) arithmetic cross-over (produces two complimentary linear combinations of the parents) [3]; and (iii) heuristic cross-over (cross-over based on interpolation, moving in the direction of the fitter chromosome) [3]. Three problem specific mutation methods were applied: (i) boundary chunk mutation (BCM); (ii) push mutation (PM); and (iii) smooth mutation (SM).

BCM, PM, and SM are mutation methods that were developed specifically for the spectrum optimization problem, based on two properties observed in optimal solutions. First, many gene values are exactly at the limits of the allele (i.e., 100% transmission or 0% transmission) in relatively fit chromosomes. Second, adjacent wavelength-bins have nearly the same value (smoothness). These mutation methods were bases on the "chromosome smoothing" technique [4], which has been shown to increase the convergence rate dramatically for this type of problem. However, a limitation of this method is that it tends to drive the solutions toward a particular result - more or less "single smooth notch" solutions. While this is suitable for many applications, it is not an ideal result. Because the problem-specific mutations are implemented probabilistically (rather than to every chromosome), this unwanted side effect is mitigated, while still producing physically realizable results.

The boundary chunk mutation selects a random contiguous portion of the chromosome (up to 10% of the total length) and set it to the one of the limits (either 1 or 0). This mutation was expected to be effective because many of the genes of fit solutions were at the maximum or minimum value (i.e., 100% transmission, or 0%). Moreover, because most of the genes for any color could be expected to be at 100% transmission, the selection of which boundary to mutate to was biased slightly: 65% of the time it went to 1, 35% of the time it went to 0.

The push mutation selects a random contiguous portion of the chromosome (up to 20% of the total length) and scales the genes from their current value towards either 1 or 0 by a randomly chosen amount (up to 0.20).

The smooth mutation selects a random contiguous portion of the chromosome (up to 20% of the total length) and smoothes it - the value of each gene in the mutated portion is weighted by the value of its neighboring genes:

$$gene[i]=0.2*gene[i-1]+0.6*gene[i]+0.2*gene[i+1] \quad (7)$$

4 Results

Figure 3 is a plot of the transmittance of a typical filter developed using this technique, for a target point in color space of x=.4800, y=.3200. The color of filtered HPS spectrum is x=.4804, y=.3206 (which would be visually indistinguishable from the target color, [1]); the efficiency of this filter is 0.23.

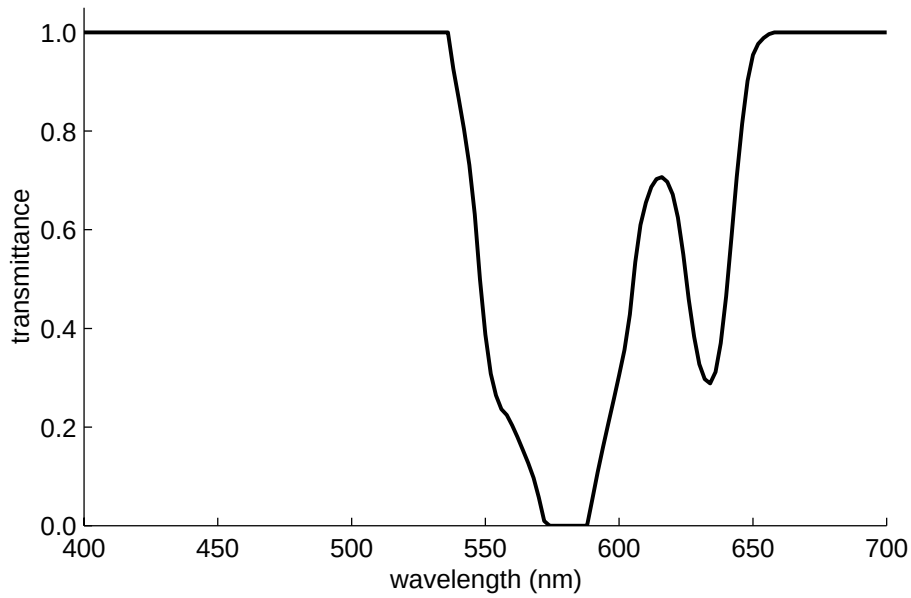

Fig. 3. The filter developed to move the HPS lamp to the point in color space, x = 0.48, y = 0.32 at high efficiency.

Figures 4-6 are contour plots of the color-efficiency Pareto-optimal surface for the three different sources. The heavy, sail shaped line is the spectrum locus - the set of chromaticity coordinates for monochromatic lights (and the straight line at the bottom of the "sail", connecting two end points of the spectrum locus, known as the purple line [1]). The spectrum locus defines color space - no light source can ever plot outside the spectrum locus. The heavy dashed line is the blackbody locus - the set of chromaticity coordinates for a blackbody radiator at various temperatures. Light sources near the blackbody locus in color space are frequently considered more desirable in the lighting industry, as they tend to appear more "natural". The heavy dot denotes the chromaticity coordinates of the unfiltered light source (and the only point where efficiency == 1.0).

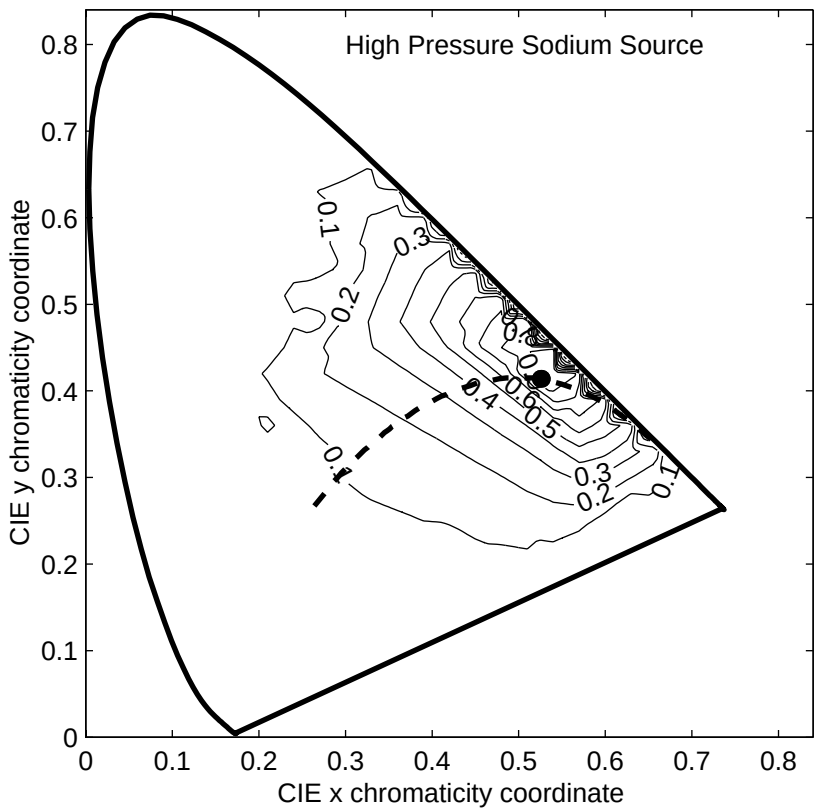

Fig. 4. Efficiency contours for the high pressure sodium source.

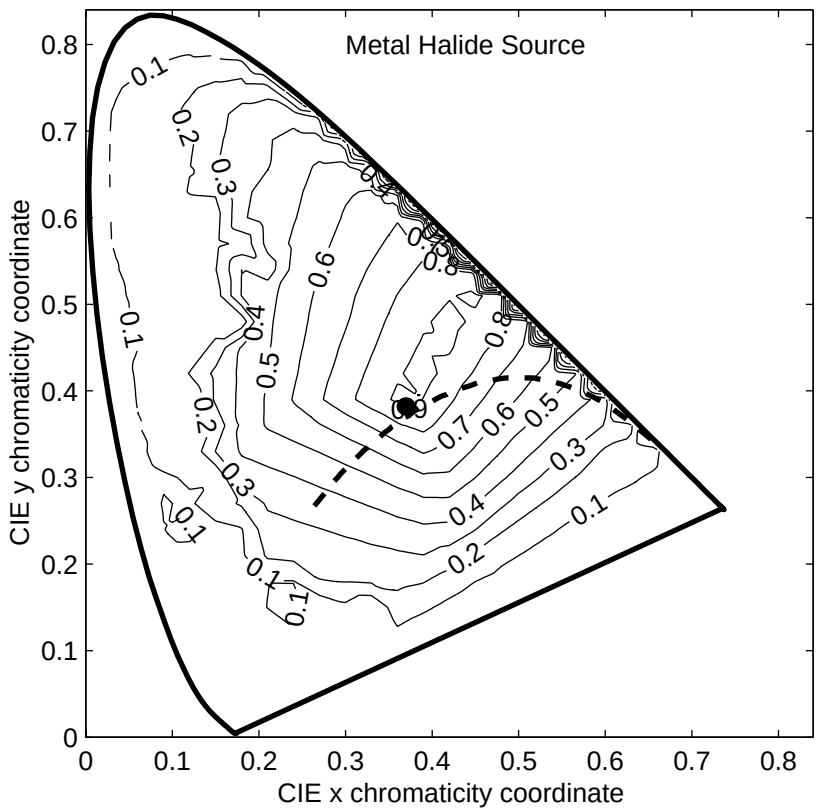

Fig. 5. Efficiency contours for the metal halide source.

The shape of the color-efficiency surface is consistent with what one might expect from the SPDs of the three sources. The amount of color space achievable at any level of efficiency by the HPS source is much smaller than the area achievable by the other two sources, due to the HPS lamp having most of its power in a relatively small region of the visible spectrum. Although both the HPS and incandescent sources have energy spread throughout the spectrum, the HPS source has a relatively narrow (compared to the incandescent) band of high efficiency, related to the relative spiky spectrum of the HPS lamp.

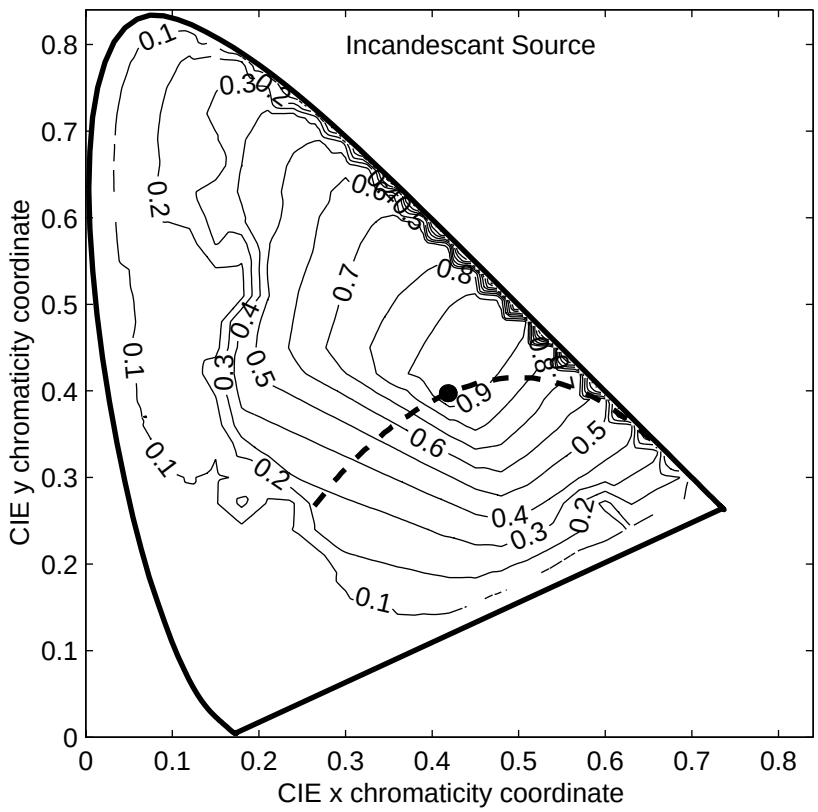

Fig. 6. Efficiency contours for the incandescent source.

5 Discussion

This paper presents a method for determining the color-efficiency Pareto optimal surface for filtered light sources, which has two major applications. First, as novel light sources are developed which are extremely efficient, but have a color that is unacceptable for some applications, filtering may adjust the color to an acceptable region while maintaining a relatively high level of efficiency, permitting a wider range of application for the lamp technology. For example the sulfur lamp [5] has a very high luminous efficacy, but its chromaticity coordinates (x=0.33, y=0.41) are relatively distant from the blackbody locus, which makes it unsuitable in appearance for many indoor applications. This lamp could be filtered to bring it much closer to the blackbody locus while still maintaining high luminous efficacy, which would make it much more desirable for indoor applications (and, consequently, more marketable).

A second major application is filtering existing light source technologies for use in new applications. For example, metal halide lamps are used extensively in theatre lighting where colored lighting is desired. Currently, the shift in color is achieved by modifying the composition of metal salts in the lamp, which tends to reduce the lamp life. A filtered standard (i.e., longer life) metal halide lamp might be an attractive alternative. The technique described here gives manufactures the ability to determine where in color-efficiency space filtered lamps might be competitive.

References

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