EVOLUTIONARY METHODS FOR DESIGN, OPTIMISATION AND CONTROL
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COMBINING GLOBAL AND LOCAL SEARCH OF NON-DOMINATED SOLUTIONS IN INVERSE ELECTROMAGNETISM.

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Abstract. A hybrid local-global and deterministic-evolutionary strategy is proposed for the reduction of objective function calls when Pareto Optimal front approximation is considered in multiobjective optimization problems arising from electromagnetic shape design. Both analytical and real-life test cases are discussed stressing the key-point of switching criteria.

Key words: Electromagnetics, Shape design, Multiobjective optimization, Hybrid global-local search.

1 Introduction

Several evolutionary methodologies[7,8] are successfully used, nowadays, for Pareto-Optimization problems, in a wide variety of fields.[1] Yet, the problem of computational costs stands still, as one of the main concerns for evaluating the effectiveness of a method. Amongst the possible definitions of cost, one will be referred to throughout this paper, in terms of the number of evaluations of the objective functions. This proves reasonable, for design optimization of electromagnetic devices, under fairly general conditions; in particular, it is assumed that any evaluation requires a time-consuming field computation, e.g. by non-linear F.E.M.
The other mandatory point is the accuracy achieved in sampling the Pareto Optimal Front (POF). In particular, the POF should be reached by as many individuals as possible, within a conveniently slight error distance, and with the most uniform spread of individuals along its profile, i.e. avoiding crowding along some spots and poor sampling of others. This, in turns, generally conflicts with the task of reducing the cost.

2 Hybrid strategy

When multiobjective optimization is concerned, in analogy to single-objective optimization, stochastic search is indeed not supposed to give a precise solution but to avoid local fronts and to identify individuals close to the POF region in a few

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BEGIN
◇ Build a random starting population (npop indiv.)
◇ Run NSESA up to partial conv.  (ε_STOP = K_2 ≪ K_1)
◇ Build npop scalar preference functions
◇ Run npop CGA or NMA up to full conv.(ε_STOP = K_1)
END
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```
BEGIN
◇ Build a random starting population (npop indiv.)
◇ Run NSESA up to full conv.  (ε_STOP = K_1)
END
```

Figure 1: Proposed (Combined global-local) and conventional strategy (Fully global).

iterations; the enhancement of solution precision is left to the deterministic search. The idea is analogous to that of hybrid single-objective optimization but switching criteria from global to local search have to be developed especially suited for Pareto front approximation . The strategy we propose is based on a Nondominated Sorting Evolutionary algorithm NSESA (see[5] and[6] for a description of the method) for the stochastic search and on a Pareto Gradient Based Algorithm (PGBA) for the deterministic search.

Two are the key-points of such a strategy; the first one is the switching criterion to be used for stopping the evolutionary search and moving to local search, i.e. set up the value of K_2, ε_{STOP} being the maximum normalised distance between individuals for successive iterations. The second one is the metrics to be used in order to assign *npop* search directions to individuals when moving to local search. When convergence towards POF has to be represented, the following two convergence indexes can be evaluated for each iteration of the optimization process

$$C_x(iter) = \frac{1}{npop} \sum_{i=1}^{npop} \sqrt{\sum_{j=1}^{ndof} (x_{ij}^{iter} - (x_{ij}^{iter-1})^2} \tag{1}$$

$$C_f(iter) = \frac{1}{npop} \sum_{i=1}^{npop} \sqrt{\sum_{l=1}^{nobjf} (f_{il}^{iter} - (f_{il}^{iter-1})^2} \tag{2}$$

where x_{ij}^{iter} is the $j-th$ component of the $i-th$ individual of population at iteration iter while f_{il}^{iter} is the $l-th$ objective value for the $i-th$ individual at iteration $iter$. The two indexes monitor the convergence toward POS (in the design variable space) and toward POF (in the objective function space) respectively. Due to the generally highly complex, and problem dependent relationship, between the two spaces, both are to be evaluated separately. The global search is based on Pareto-ranking and thus does not require preference functions.

3 Switching criteria

The criterion based upon indexes 1 is met when the population of solutions has not been undergoing remarkable improvements for a certain number of iterations. This, however, could also mean that some individuals are trapped about points on some local front. If so, the subsequent local search will likely be unable to draw

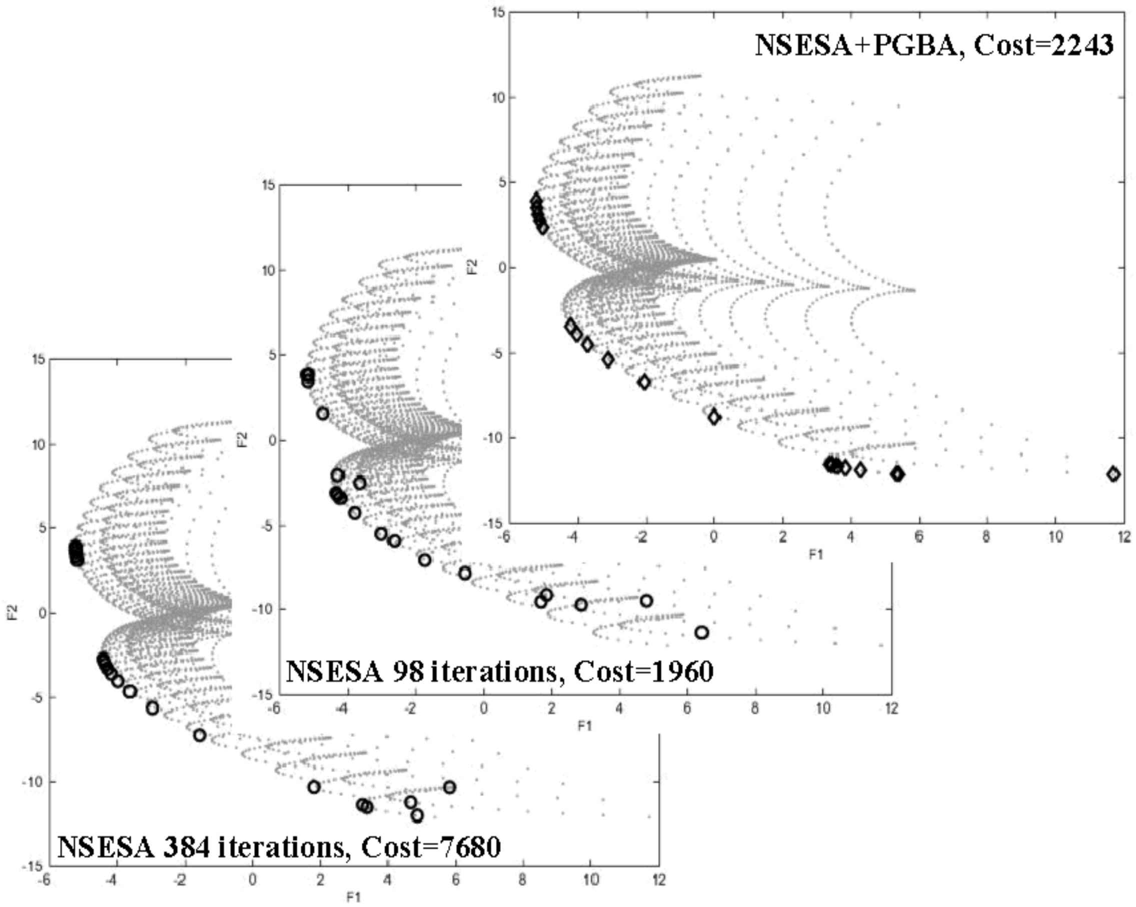

Figure 2: 20 individual solutions: Fully global, Stopped global , Combined global-local

them out; rather, it will probably retrieve the local extremes precisely, preventing the individuals involved from escape, what translates to time waste and poorer sampling of the final front. Another criterion is proposed here, trying to identify the most likely trapped individuals. The idea is to approximate the POF by a polynomial function of convenient degree. In a 2-dimensional case, for instance, the values of the objective functions (say f_1 and f_2) should be regarded as the values of an independent and a dependent function, respectively. The polynomial, then, should interpolate just those points which are non-dominated at switching-test time which are assumed to yield good indication of the front's profile. Afterward, an esteem of the interpolation error is to be taken; possibly by taking the mean square error:

$$s_p = \sqrt{\frac{\sum_{i=1}^{n}(f_2(i) - p(f_1(i)))^2}{n}} \tag{3}$$

where $p(x)$ is the interpolating polynomial, and n is the number of individuals belonging to the first front. Now, the displacements between single individuals and the curve can be evaluated and compared to the mean square error: when the ratio exceeds some relative threshold, the point is to be considered 'far' from the asymptotic zone, i.e. probably trapped. A possible definition of distance is simply the difference between the individual's 'dependent' coordinate (f_2 in this case) and the value of the polynomial at the same value of the 'independent' variable (f_1); the restart condition so becomes:

$$\frac{(f_2(j) - p(f_1(j)))}{s_p} > \sigma \tag{4}$$

where σ is the relative threshold. The subsequent run lasts a fixed, reasonably low number of iterations; the switching criterion is then re-tested, and other supplementary runs can be performed until either setting all the individuals free or reaching a maximum of iterations.

4 Test cases

As a preliminary result the proposed strategy has been tested on the following 2D analitycal test problem

$$\begin{cases} \min\limits_{-6<x_1,x_2<6} (f_1, f_2) \\ f_1(x_1,x_2) = \frac{1}{100} \sum\limits_{i=1}^{2} (x_i + 0.5)^4 - 30x_i^2 - 20x_i \quad f_2(x_1,x_2) = (10 - x_1^3 - x_2^2) \end{cases} \tag{5}$$

Fig 2 shows 20 individual solutions. The first picture shows a fully stochastic search while the last picture represent results of an hybrid approach where the local search is started from the population shown in the second picture which is the result of 98 NSESA iteration; stopping criterion tolerances have been set to $K_1 = 10^{-6}$ and $K_2 = 10^{-2}$. As evident the hybrid strategy solution is much better in terms of both precision, and diversity of individuals. The cost in terms of objective function evaluations is smaller for the hybrid search (2243 against 7680 for the fully stochastic one).

A typical inductor for TFH (Transverse Flux induction Heating) system, is composed of two parts, each facing one side of the workpiece (metal strip), and is characterised by any number of poles, with different dimensions and supply.[9] The design variables (see cross section of coil in figure 3) are the half-internal width of the coil in the longitudinal direction **a**, the half-internal height in the transversal direction **b**, the width of the coil conductor **d**, and the working frequency **f** . The number of coils has been chosen equal to 4 i.e. two butterflies (a butterfly being a couple of inductor sections). The inductor is supplied by a current equal to 700 A, the velocity of the strip is v = 0.4 cm s-1. The material of the strip is silver; its width of the strip to be heated is fixed and equal to 100 mm. The following two objective functions are defined:

- the **electrical efficiency** F_1 (to be maximised) of the inductor defined as the ratio between power transferred to the workpiece and power supplied to the inductor.

- the **maximum temperature gap** F_2 (to be minimised) in the y direction in the same instant. Congruency bounds have been imposed to design variables.

The analytical solution of the Helmholtz's equation in three dimensions gives the expressions of the electric field in the strip and in the air; the power density distribution and all integral parameters of the system can be thus calculated. The analysis of the thermal transient is based on finite difference method starting from the solution of the electromagnetic problem described above.

5 CONCLUSION

The combination of global-evolutionary and local-deterministic search seems to be a promising strategy for the approximation of Pareto Optimal Front in real-life

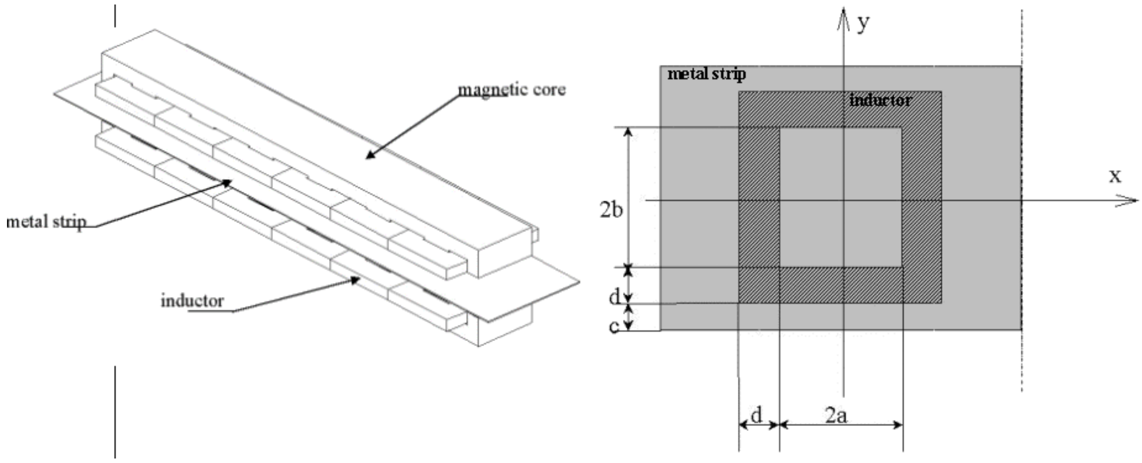

Figure 3: TFH inductor

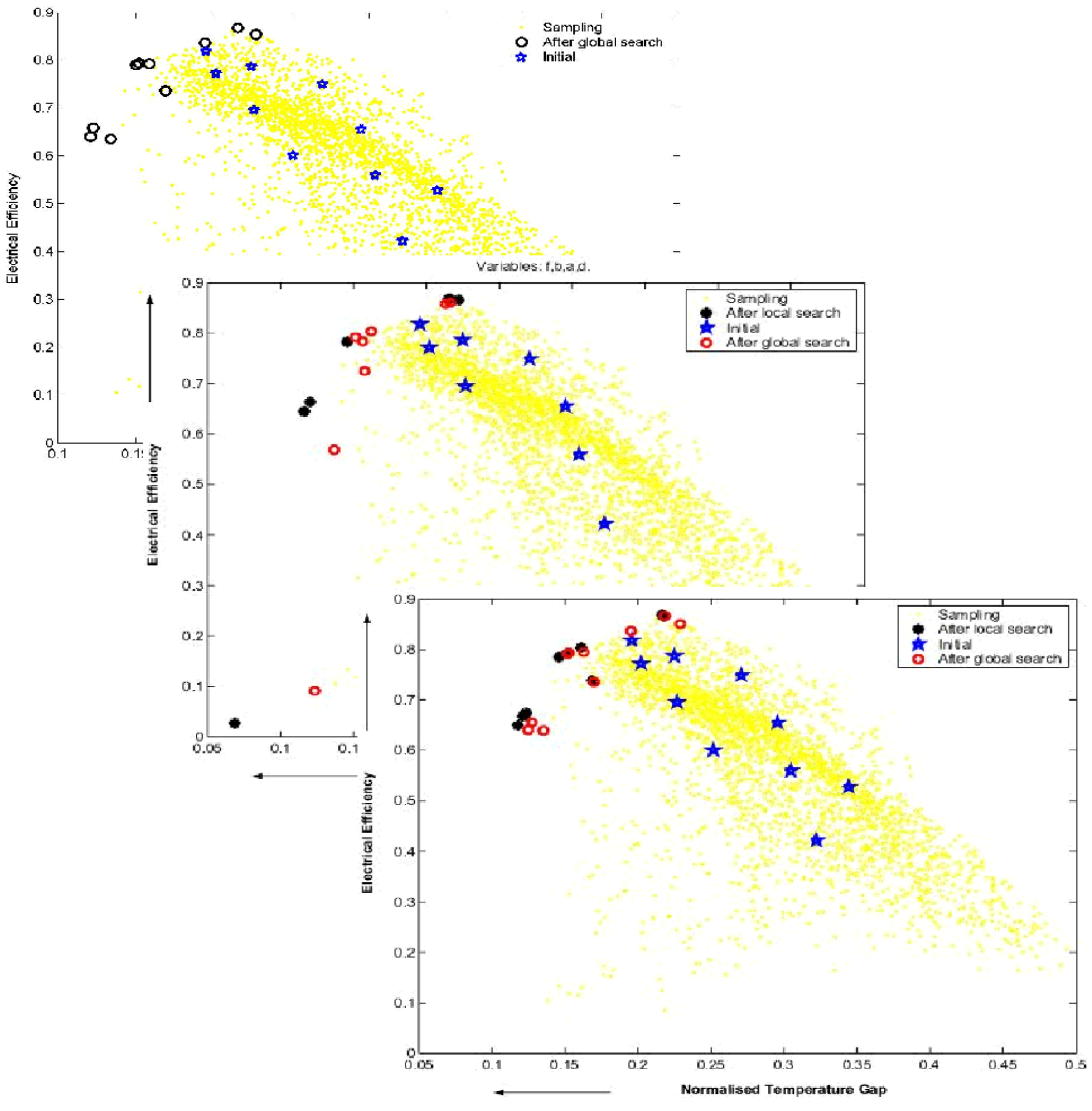

Figure 4: Optimization results for the TFH device (C,A,B cases described in table 1)

Figure	A	B	C
Strategy	NSESA+PSA	NSESA+PCGA	NSESA
global s.	128	150	206
local s.	422	56	-
Total cost	550	206	206
cpu-time [h]	17	7	7

Table 1: Number of objective function calls and cpu-time for TFH inductor optimization.

multiobjective Optimization arising in Electromagnetic industrial design. A significant reduction of computational cost is achieved with a satisfactorily approximation quality in terms of both precision and diversity of solutions. Analytical and real-life test cases shows that that the switching strategy is crucial for the approximation quality on one hand and for the reduction in computational cost on the other.

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