

DESIGNING CONTROL SYSTEMS WITH MULTIPLE OBJECTIVES

Peter Fleming

University of Sheffield

Introduction

Many problems arising in control and systems engineering require the simultaneous optimisation of multiple, often conflicting, design criteria, such as performance, reliability, and cost (Fig. 1). Unlike in single-objective optimisation, the global solution to such problems is seldom a single point, but a family of compromise solutions known as the Pareto-optimal set, such as illustrated by the trade-off surface in Fig. 1. These solutions are optimal in the sense that improvement in any objective can only be achieved at the expense of degradation in at least one of the remaining objectives.

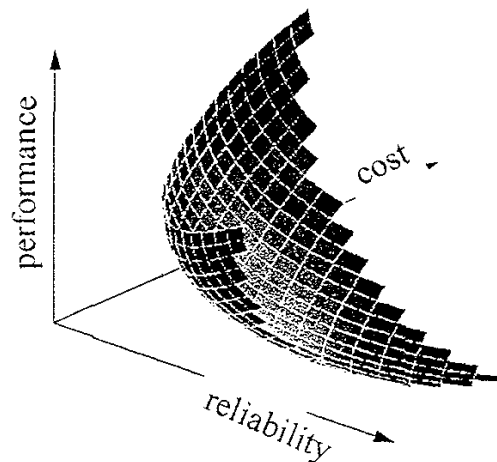


Fig. 1 Trade-off surface depicting competing system performance objectives

Multiobjective Optimisation

Consider the following multiobjective optimisation (MO) problem:

$$\begin{aligned} \min F(\mathbf{p}) \quad & \dots (1) \\ \mathbf{p} \in \Omega \end{aligned}$$

where $\mathbf{p}=[p_1, p_2, \dots, p_q]$, Ω defines the set of free variables, \mathbf{p} , subject to any constraints and $\mathbf{F}(\mathbf{p})=[f_1(\mathbf{p}), f_2(\mathbf{p}), \dots, f_n(\mathbf{p})]$ are the design objectives to be minimised.

Clearly, for this set of functions, $\mathbf{F}(\mathbf{p})$, it can be seen that there is no one ideal 'optimal' solution, rather a set of Pareto-optimal solutions for which an improvement in one of the

design objectives will lead to a degradation in one or more of the remaining objectives. In Fig.2 there are two objectives, f_1 and f_2 , to be simultaneously minimised. These objectives are competing with one another such that there is no single solution. Candidate solution point A has a lower value of f_2 , but a higher value of f_1 , than candidate solution point B. Thus, it is not possible to state that one point on the trade-off curve shown in Fig. 2 is better or worse than another. Such solutions are known as Pareto-optimal solutions (alternatively as *non-inferior* or *non-dominated* solutions) to the multiobjective optimisation problem.

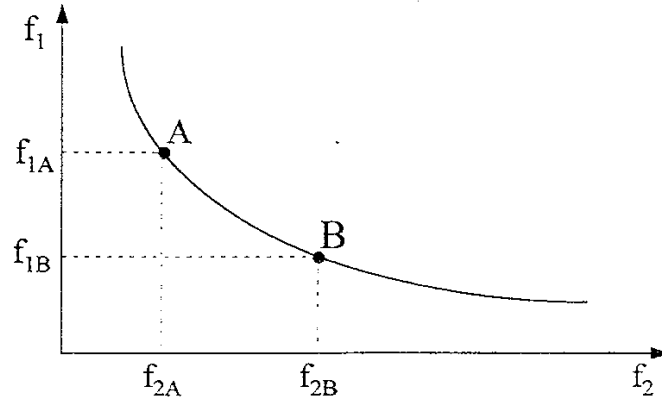


Fig. 2 Pareto-optimal set of solutions for 2-objective problem

Generally, members of the Pareto-optimal solution set are sought through solution of an appropriately formulated non-linear programming (NP) problem. A number of approaches are currently employed including the ϵ -constraint, weighted-sum and goal attainment methods (Hwang and Masud, 1979). However, such approaches require precise expression of a, usually not well understood, set of weights and goals.

If the trade-off surface between the design objectives is to be better understood, repeated application of such methods will be necessary. In addition, NP methods cannot handle multimodality and discontinuities in function space well and can thus only be expected to produce local solutions.

Genetic algorithms (GAs) are population-based methods, unlike NP schemes which seek to improve single-point estimates of a solution. This enables the evolution of a Pareto-optimal set of solutions. Also, because of the stochastic nature of the search mechanism, GAs are capable of searching the entire solution space with more likelihood of finding the global optimum than conventional optimisation methods. Indeed, conventional methods usually require the objective function to be well behaved, whereas the generational nature of GAs can tolerate noisy, discontinuous and time-varying function evaluations. Moreover, GAs allow the use of mixed decision variables (binary, n -ary and real-values) permitting a parameterisation that matches the nature of the design problem more closely.

Multiobjective Genetic Algorithms (MOGAs)

The multiobjective genetic algorithm approach proposed by Fonseca and Fleming (1993) uses a rank-based fitness assignment, where the rank of a certain individual x_i at generation t is

related to the number of individuals $p_i(t)$ in the current population by which it is dominated. This is given by

$$rank(x_i, t) = p_i(t). \quad \dots(2)$$

All non-dominated individuals are assigned rank 0 and remaining individuals are penalised according to Eqn. (2).

Fitness is assigned by interpolating from the best individual (rank=0) to the worst, and then the fitness assigned to individuals with the same rank is averaged where the global population fitness is kept constant. However, such fitness assignment tends to produce premature convergence due to the fact that all non-dominated (best rank) points are considered equally fit (Fig. 3). In order to overcome this deficiency, Fonseca and Fleming have used a niche induction method to promote the distribution of the population over the Pareto-optimal front in order to maintain diversity. This is achieved by a method of fitness sharing which encourages the reproduction of isolated individuals and favours diversification.

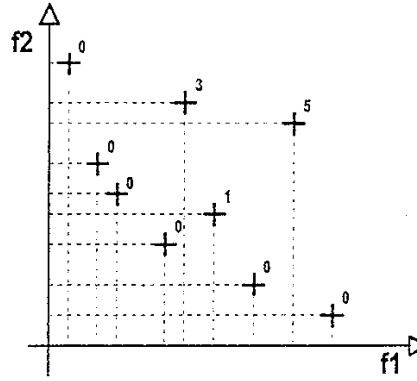


Figure 3. Pareto-Ranking without preference information.

Preference Information

Preference information is also introduced in the form of a goal vector, which provides a means of evolving only a specific region of the search space. This allows the decision maker to focus on a region of the Pareto front by providing external information to the selection algorithm. A typical set of design trade-offs resulting from a MOGA design exercise is shown in Fig. 4. In this "parallel co-ordinates representation" of more than two objectives (eight objectives, in fact, for this flight control example) each line in the graph represents a potential solution to the design problem, indicating the achieved objective values for that solution. All solutions are both non-dominant **and** satisfy the prescribed *goals* as represented by the "x" marks. The decision-maker (DM) must select a suitable compromise from this set of solutions. DM ~~may~~ interact with the MOGA as it runs to "tighten" or "slacken" the *goals*, in order to ~~target~~ a specific compromise solution.

Through such a representation, the DM is informed of conflicts, or otherwise, between objectives. For example, in Fig. 4, solution lines between Objectives 2 and 3 clearly cross one another, indicating that improvement in one objective can only be achieved at the expense of the other objective. Other refinements at the disposal of DM include the ability to specify "hard" constraints for objectives.

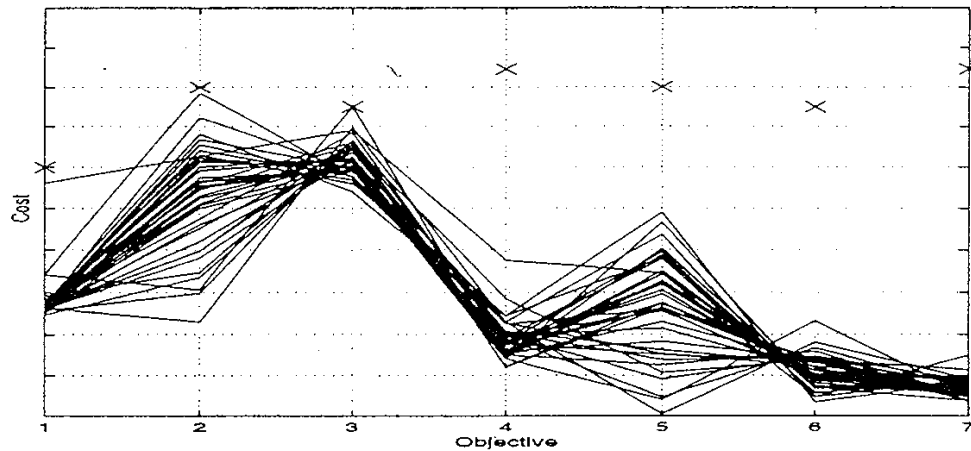


Fig. 6 Parallel Co-ordinates Representation: Design Objective Trade-Offs

Design Examples

MOGAs are, therefore, a powerful decision-making aid for the control system designer. It is possible to search for many Pareto-optimal solutions concurrently, while concentrating on relevant regions of the Pareto set. Also, a human decision maker may interactively supply preference information to the algorithm as it runs. Applications to be described will include the design of controllers for flight dynamics, gas turbine engines and active magnetic bearings. Design problem characteristics will include non-linear system descriptions, incorporation of H-infinity approaches and on-line use of the MOGA tool. Examples of the use of the method may be found in Fonseca & Fleming (1998a; 1998b), Dakev *et al.* (1997), Chipperfield & Fleming (1996) and Schroder *et al.* (1998).

References

- Chipperfield AJ and Fleming PJ (1996): Multiobjective gas turbine engine controller design using genetic algorithms, *IEEE Transactions on Industrial Electronics*, vol 43, no 5, pp 583-589.
- Dakev, NV, Whidborne JF, Chipperfield AJ and Fleming PJ (1997): Evolutionary H_∞ design of an EMS control system for a Maglev vehicle, *Proc Inst Mech Engrs, Part I: J of Sys and Contr. Eng*, vol 211, pp345-355.
- Fonseca CM and Fleming PJ (1993): Genetic algorithms for multiobjective optimization: formulation, discussion and generalization, *Proc Int Conf Genetic Algorithms*, Illinois, pp 416-423.
- Fonseca CM and Fleming PJ (1998a): Multiobjective optimization and multiple constraint handling with evolutionary algorithms - Part I: A unified formulation, *IEEE Trans Systems, Man and Cybernetics*, vol 28, no 1, pp26-37.
- Fonseca CM and Fleming PJ (1998b): Multiobjective optimization and multiple constraint handling with evolutionary algorithms - Part II: Application example, *IEEE Trans Systems, Man and Cybernetics*, vol 28, no 1, pp38-47.
- Hwang C-L and Masud ASM (1979): Multiple Objective Decision Making - Methods and Applications, Vol. 164 of *Lecture Notes in Economics and Mathematical Systems*, Springer-Verlag, Berlin.
- Schroder P, Green B, Grum N and Fleming PJ (1998): On-line auto-tuning of mixed H_2/H_∞ optimal magnetic bearing controllers, *Proc UKACC International Conference on CONTROL '98*, Swansea, UK, pp 1123-1128.