1984-2004 – 20 Years of Multiobjective Metaheuristics. But what about the Solution of Combinatorial Problems with Multiple Objectives?

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Abstract. After 20 years of development of multiobjective metaheuristics the procedures for solving multiple objective combinatorial optimization problems are generally the result of a blend of evolutionary, neighborhood search, and problem dependent components. Indeed, even though the first procedures were direct adaptations of single objective metaheuristics inspired by evolutionary algorithms or neighborhood search algorithms, hybrid procedures have been introduced very quickly. This paper discusses hybridations found in the literature and mentions recently introduced metaheuristic principles.

1 Multiobjective Optimization

A multiobjective optimization problem is defined as

$$\min_{x \in X} (z_1(x), \ldots, z_p(x)), \tag{MOP}$$

where $X \subset \mathbb{R}^n$ is a feasible set in the *decision space*, and $z : \mathbb{R}^n \to \mathbb{R}^p$ is a vector valued objective function. By $Z = z(X) \subset \mathbb{R}^p$ we denote the image of the feasible set in the *objective space*. We consider optimal solutions of (MOP) in the sense of *efficiency* (or Pareto optimality), that is, a feasible solution $x \in X$ is called efficient if there does not exist $x' \in X$ such that $z(x') \leq z(x)$, i.e., $z_k(x') \leq z_k(x)$ for all $k = 1, \ldots, p$ and $z_j(x') < z_j(x)$ for some j. In other words, no solution is at least as good as x for all objectives, and strictly better for at least one.

Efficiency refers to solutions x in decision space. In terms of the objective space, with objective vectors $z(x) \in \mathbb{R}^p$ we use the notion of *non-dominance*: If x is an efficient solution then $z(x) = (z_1(x), \ldots, z_p(x))$ is a non-dominated vector (or point). The set of efficient solutions is X_E, the set of non-dominated vectors is Z_N. We may also refer to Z_N as the *non-dominated frontier* or the trade-off surface or the Pareto front. For $x^1, x^2 \in X$ we shall use the notation $x^1 \succ x^2$ if x^1 dominates x^2, i.e., if $z(x^1) \leq z(x^2)$.

In case of multiple feasible solutions $x, x' \in X$ mapping to the same non-dominated point $z(x) = z(x')$, the solutions are said to be *equivalent* [24]. A *complete set* X_E [24] is a set of efficient solutions such that all $x \in X \setminus X_E$ are either dominated or equivalent to at least one $x \in X_E$. I.e., for each nondominated point $z \in Z_N$ there exists at least one $x \in X_E$ such that $z(x) = z$. To solve a multiobjective optimization problem often means to find a complete set of efficient solutions. The computation of a set of efficient solutions is a major challenge of multiobjective optimization. But to precisely characterize the ability of an algorithm to solve an MOP the definition of complete set is refined as follows:

- [24] A *minimal complete set* X_{E_m} is a complete set without equivalent solutions. Any complete set contains a minimal complete set.
- [36] The *maximal complete set* X_{E_M} is a complete set including all equivalent solutions, i.e., all $x \in X \setminus X_{E_M}$ are dominated.

Multiobjective combinatorial optimization problems form a particular class of MOPs, which can be formulated as follows:

$$\min \{Cx : Ax \geq b, x \in \mathbb{Z}^n\}. \tag{MOCO}$$

Here C is a $p \times n$ objective function matrix, where c_k denotes the k-th row of C. A is an $m \times n$ matrix of constraint coefficients and $b \in \mathbb{R}^m$. Usually the entries of C, A and b are integers. The feasible set $X = \{Ax \geq b, x \in \mathbb{Z}^n\}$ may describe a combinatorial structure such as, e.g., spanning trees of a graph, paths, matchings etc. We shall assume that X is a finite set. By $Z = CX$ we denote the image of X under C in $\mathbb{R}^p$.

2 Approximation Methods for MOCO

As in the single objective case, reasonable alternatives to exact methods for solving difficult MOCOs are approximation methods. An *approximation method* in a multiobjective optimization context is a method which finds either sets of locally potentially efficient solutions that are later merged to form a set of potentially efficient solutions – the approximation denoted by X_{PE} – or globally potentially efficient solutions according to the current approximation X_{PE}.

2.1 The Question of Quality of an Approximation

The quality of a solution of a combinatorial optimization problem can be estimated by comparing lower and upper bounds on the optimal objective function value. In multiobjective optimization the concept of bounds is not well developed. The best possible lower and upper bounds on values of all non-dominated points are given by the ideal and nadir point z^I and z^N defined by

$$z_k^I = \min_{x \in X} z_k(x), \quad k = 1, \ldots, p$$

and

$$z_k^N = \max_{x \in X_E} z_k(x), \quad k = 1, \ldots, p,$$

respectively. We sometimes refer to a utopian point $z^U = z^I - \varepsilon \mathbf{1}$, where $\mathbf{1}$ is a vector of all ones and ε is a small positive number. However, the ideal and nadir points are usually far away from non-dominated points and do not provide a good estimate of the non-dominated set. In addition, the nadir point is hard to compute for problems with more than two objectives, see [13].

To better capture the multiobjective nature of the problems and the fact that we are looking for a set of efficient solutions it is natural to generalize the notion of bounds to bound sets. Ehrgott and Gandibleux report first results on lower and upper bound sets in for the biobjective assignment, knapsack, traveling salesman, set covering, and set packing problems [11, 14]. Fernández and Puerto [15] use bound sets in their exact and heuristic methods to solve the multiobjective uncapacitated facility location problem.

There are a few other ideas in the literature. Kim et al. [30] propose a new measure, the integrated convex preference (ICP), to compare the quality of algorithms for MOCO problems with two objectives. Sayin [37] proposes the criteria of *coverage*, *uniformity*, and *cardinality* as quality measures. Although developed for continuous problems the ideas may be interesting for MOCO problems. However, the methods proposed in [37] can be efficiently implemented for linear problems only. Other authors propose *distance based measures* [44] and *visual comparisons* of the generated approximations. The latter are restricted to bi-objective problems. Jaszkiewicz [28] also distinguishes between *cardinal* and *geometric quality measures*. He gives further references and suggests preference-based evaluation of approximations of the non-dominated set using outperformance relations. Tenfelde-Podehl [42] proposes volume based measures.

None of these measures have been universally adopted in the multiobjective optimization literature, and further research is clearly needed.

2.2 Multiobjective Heuristics and Metaheuristics for MOP

Multiple objective heuristics (MOH) and *multiple objective metaheuristics (MOMH)* are methods that aim to provide a good tradeoff between an approximation of the set of efficient solutions and the time and memory requirements to obtain it. These methods may manipulate a complete or incomplete single solution or a collection of solutions at each iteration.

Heuristics are generally problem-specific, so that a method which works for one problem cannot be used to solve a different one. In contrast, metaheuristics are universal methods applicable to a large number of problems. A metaheuristic is a solution *concept*. The adaptation to a specific problem uses heuristics as solution *methods*. The family of metaheuristics includes, but is not limited to, constraint logic programming, genetic algorithms, evolutionary methods, neural networks, simulated annealing, tabu search, non-monotonic search strategies, greedy randomized adaptive search, ant colony systems, particle swarm optimization, noising methods, variable neighborhood search, scatter search, etc.

From a historical perspective, the pioneer approximation methods for multiobjective problems have appeared since 1984, in the following order: Genetic Algorithms (GA, Schaffer 1984 [38]), Artificial Neural Networks (ANN, Malakooti 1990 [32]), Simulated Annealing (SA, Serafini 1992 [39]), and Tabu Search (TS, Gandibleux 1996 [16]). The pioneer methods have three characteristics. First, they are inspired either by *Evolutionary Algorithms (EA)* or by *Neighborhood Search Algorithms (NSA)*. Second, the early methods are direct derivations of single objective optimization metaheuristics, incorporating small adaptations to integrate the concept of efficient solution for optimizing multiple objectives. Third, almost all methods were designed as a solution concept according to the principle of metaheuristics.

2.3 Evolutionary Algorithms versus Neighborhood Search Algorithms

Evolutionary Algorithms manage a solution population $\mathcal{P}$ rather than a single feasible solution. In general, they start with an initial population and combine principles of self adaptation, i.e., independent evolution (such as the mutation strategy in genetic algorithms), and cooperation, i.e., the exchange of information between individuals (such as the "pheromone" used in ant colony systems), to improve approximation quality. The usual components of an evolutionnary algorithm are:

- a population of solutions
- evolutionary operators (crossover, mutation)
- an archive of elite solutions
- a ranking method
- a guiding method
- a clustering method
- a fitness measure
- a penalty strategy for infeasible solutions, etc.

Because the whole population contributes to the evolutionary process, the generation mechanism is parallel along the frontier, and thus these methods are also called *global convergence-based methods*. This characteristic makes population-based methods very attractive for solving multiobjective problems.

In *Neighborhood Search Algorithms*, the generation of solutions relies upon one individual, a current solution x_n, and its neighbors $\{x\} \subseteq \mathcal{N}(x_n)$. Using a local aggregation mechanism for the objectives (often based on a weighted sum), a weight vector $\lambda \in \Lambda$, and an initial solution x_0, the procedure iteratively projects the neighbors into the objective space in a search direction λ by optimizing the corresponding parametric single objective problem. A local approximation of the non-dominated frontier is obtained using archives of the successive potentially efficient solutions detected. This generation mechanism is sequential along the frontier, producing a local convergence to the non-dominated frontier, and so such methods are called *local convergence-based methods*. The principle is repeated for diverse search directions to completely approximate the non-dominated frontier. The elementary components of a neighborhood search algorithm are:

- a neighborhood structure (moves)
- an exploration strategy (partial, exhaustive)
- an acceptation rule (SA principle, TS principle)
- a list of candidates
- a scalarizing function
- an oscillation strategy
- a greedy (randomized) strategy
- a path-relinking strategy, etc.

NSAs are well-known for their ability to locate the non-dominated frontier, but they require more effort in diversification than EA in order to cover the efficient frontier completely.

While the first adaptation of metaheuristic techniques for the solution of multiobjective optimization problems has been introduced 20 years ago, the MOMH field has clearly mushroomed over the last ten years. The first approximation methods proposed for MOCO problems were "pure" NSA strategies and were straightforward extensions of well-known metaheuristics for dealing with the notion of non-dominated points. Simulated annealing (the MOSA method [43]), tabu search (the MOTS method [16], the method of Sun [41]), or GRASP (the VO-GRASP method [18]) are examples.

2.4 Hybrid Algorithms and Problem-dependent Algorithms

The methods that followed the pioneer ones, designed to be more efficient algorithms in the MOCO context, have been influenced by two important observations.

The first observation is that one the one hand, NSAs focus on convergence to efficient solutions, but must be guided along the non-dominated frontier. On the other hand, EAs are very well able to maintain a population of solutions along the non-dominated frontier (in terms of diversity, coverage, etc.), but often converge too slowly to the non-dominated frontier. Naturally, methods have been proposed that try to take advantage of both EA and NSA features by combining components of both approaches, introducing *hybrid algorithms* for MOPs.

The second observation is that MOCO problems contain information deriving from their specific combinatorial structure, which can be advantageously exploited by the approximation process. Single objective combinatorial optimization is a very active field of research. Many combinatorial structures are very well understood. Thus combinatorial optimization represents a useful source of knowledge to be used in multiobjective optimization. This knowledge (e.g., cuts for reducing the search space) are more and more taken into account when designing a very efficient approximation method for a particular MOCO. It is not surprising to see an evolutionary algorithm – for global convergence – coupled with a tabu search algorithm – for the exploitation of the combinatorial structure – within one approximation method.

Problem dependent components which can be advantageously used are, for example:

- specific crossover operators (like in [31] for the bi-objective CARP)
- specific neighborhood structures
- bound sets on the non-dominated frontier
- handling constraints in a relaxation strategy
- numerical properties of the objective functions
- properties of subsets of exact solutions (easily computed), etc.

Modern approximation methods for MOCO problems appear more and more as a problem-oriented techniques, i.e., selections of components that are advantageously combined to create an algorithm which can tackle the problem in the most efficient way. By nature the algorithm is hybrid, including evolutionary components, neighborhood search components, and problem dependent components.

3 Steps in Hybridation Schemes

The hybridations introduced in multiobjective approximation methods are:

1. **EA components integrated in NSA**.
   The use of a population of individuals allows to have global information about the current approximation and to let that information *drive local search processes* in order to "guarantee" a good coverage of the non-dominated frontier. Using, for example, mechanisms based on notions of repulsion between non-dominated points, the search is guided toward subareas of the frontier containing (i) a high density of solutions or (ii) areas not yet explored. This is the principle of the PSA method [7] and the TAMOCO method [10] (see Figure 1).
2. **EA as master strategy, NSA as secondary strategy**.
   EA is the pilot of the search procedure, and activates an NSA. The main idea here is *to make the evolutionary algorithm very aggressive* in improving as far as possible good solutions resulting from the evolutionary operators. The NSA can be a depth first search method, a basic (or truncated) tabu search, etc. This is the principle behind the memetic version of MOGA [35], MOGLS [27], MGK [17], GTS^{MOKP} [1] (see Figure 2).

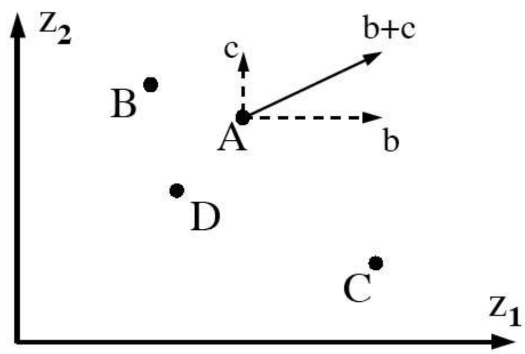

Fig. 1. The positions of four solutions A, B, C and D in objective space are shown. Solution A should be improved to move towards the nondominated frontier but at the same time it should move away from other current solutions, which are non-dominated with respect to A (solutions B and C). Solution B pushes solution A away and this is shown by an optimization influence in the direction of vector b. Likewise does solution C influence solution A to move away from it in direction c. The final optimization direction for solution A is found by adding these weighted influence vectors. (From [10])

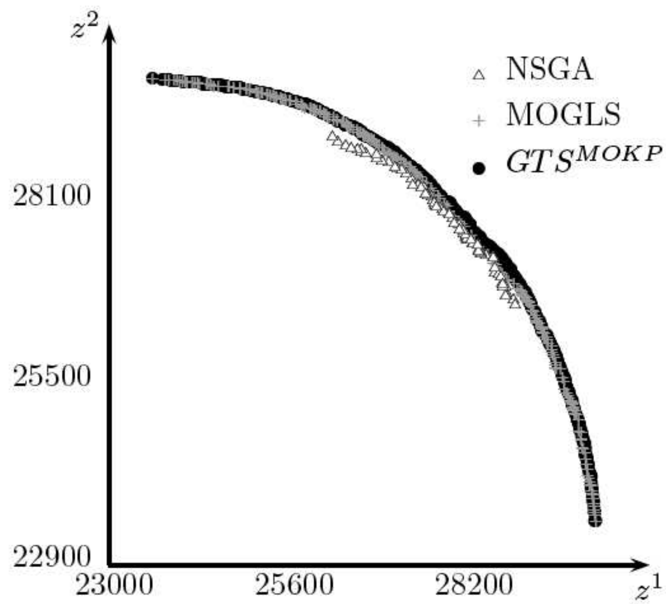

Fig. 2. The approximations obtained with NSGA, MOGLS and GTS^{MOKP} for a bi-objective binary knapsack problem with two constraints and 750 items. (From [3])

3. **Alternating schemes based on EA and NSA as a blackbox**.
   Ben Abdelaziz et al. [5] propose a hybrid algorithm using both EA and NSA independently. The goal of the EA (a genetic algorithm) is to produce a first diversified approximation, which is then improved by the NSA (a tabu search algorithm). Results have been reported on the multiobjective knapsack problem.
   Delorme et al. [8] design a scheme based on an NSA interfaced with an EA for solving the bi-objective set packing problem. The idea is to take advantage of an efficient heuristic known for the single objective problem in order to compute an initial set of very good solutions $\mathcal{P}_0$ in a first phase. The heuristic (a GRASP algorithm) is encapsulated in a basic generation procedure, for example using a convex combination of the objectives: λ-GRASP. The second phase works on a population of individuals $\mathcal{P} \supseteq \mathcal{P}_0$ derived from the initial set and performs an EA (a modified version of SPEA dealing with all potential efficient solutions and integrating a local search: A-SPEA) in order to consolidate the approximation of the non-dominated frontier (see Figure 3).
   In Target Aiming Pareto Search (TAPaS) [29], the search directions of the procedure are given by the current set X_{PE}, similar to the principle of almost all the tabu search adaptations for MOP [16, 25, 41]. A series of goals is deduced from X_{PE} and a scalarizing function is used for guiding an NSA, defining a two phase strategy. This scheme has been applied to two vehicle routing problems. For one problem (the covering tour problem), TAPas is coupled with a EA plus a branch and cut algorithm specifically designed for the single objective version of the problem.
4. **EA + NSA + problem dependent components**.
   The most recent hybrid procedures integrate EA and NSA components as well as problem-dependent components in order to design a powerful approximation method for a MOCO problem. Gandibleux et al. [20, 21] propose a population based method where a crossover uses a "genetic" map of the population, and that includes a path-relinking operator. Path-relinking generates new solutions by exploring the trajectories that connect elite solutions. Starting from one solution – the initiating solution – a path is generated through the neighborhood space that leads to the other solution – the guiding solution [22] (see Figure 4). Only potentially efficient solutions compose the population at any time and bound sets limit the triggering of a local search. This procedure has been applied for approximating the non-dominated frontier of assignment and knapsack problems with two objectives.
5. **Approximation method and exact procedure in a hybrid method**.
   The combination of EA and NSA can be more accurately integrated than by a "simple" switch between the two mechanisms. Gandibleux and Fréville [19] propose a procedure for the biobjective knapsack problem combining an exact procedure for reducing the search space with a tabu search process for identifying the potentially efficient solutions. The reduction principle is based on cuts which eliminate parts of the decision space where (provably) no

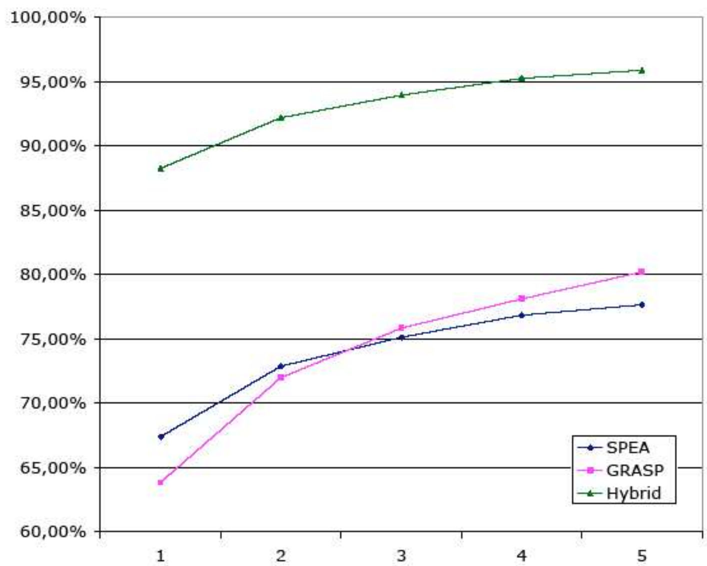

Fig. 3. The figure illustrates the average percentage of exact solutions found using λ-GRASP, A-SPEA, and the hybrid for the set packing problem with two objectives, when all three methods are allowed the same computational effort. (From Delorme et al. [8])

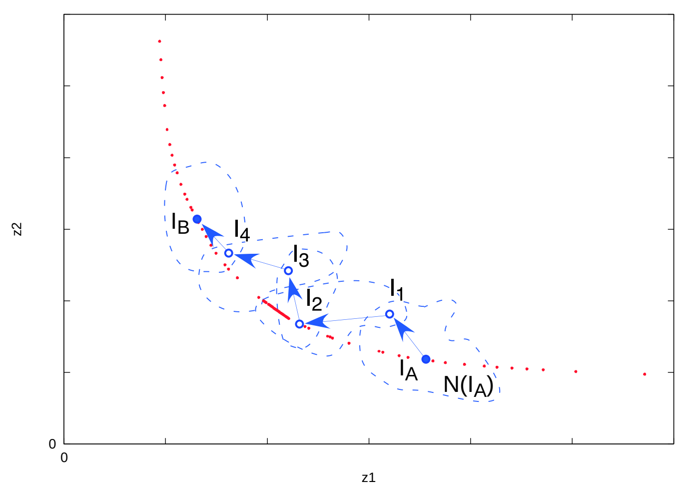

Fig. 4. Illustration of a possible path construction (see [21]). I_A and I_B are two individuals randomly selected from the current elite population (small bullets). I_A is the initiating solution, and I_B is the guiding solution. $\mathcal{N}(I_A)$ is the feasible neighborhood according to the move defined. $I_A - I_1 - I_2 - I_3 - I_4 - I_B$ is the path that is built

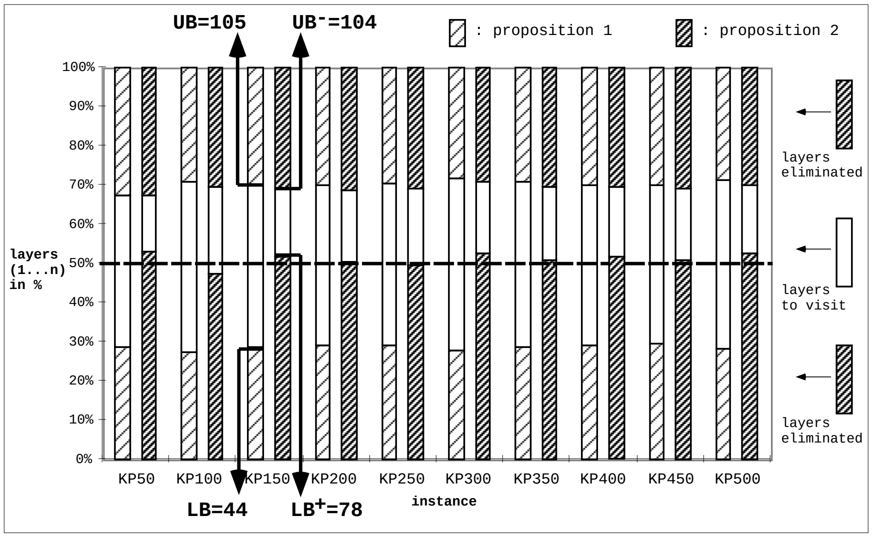

Fig. 5. A decision space reduction technique for the bi-objective knapsack problem uses an additional constraint on the cardinality of an optimal solution for computing a utopian reference point and an approximation set for verifying if the reference point is dominated. As output a *strategic map* is established eliminating all parts of the seach space where no efficient solution exists. A heuristic (a tabu search for example) can then be triggered inside the reduced decision space [19].

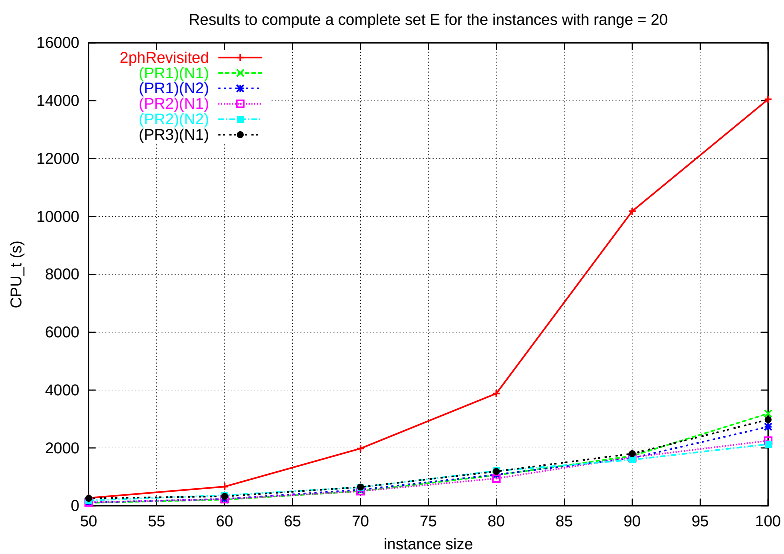

Fig. 6. CPU time used by an exact method for solving the assigment problem with two objectives without (the upper curve) and with (the lower curves) the use of approximate solutions for the pruning test inside the method [36].

exact efficient solution exists (see Figure 5). The tabu search is triggered on the reduced space and dynamically updates the bounds in order to guarantee the tightest value at any time.
This category also applies if in an exact method for generating the non-dominated points the exact method needs bounds of good quality. For example in the *seek and cut method* for solving the assignment problem [36] the "seek" computes a local approximation of the non-dominated frontier (i.e., bounds are computed by a population-based algorithm coupled with path-relinking) which is then used for "cutting" the search space of an implicit enumeration scheme (see Figure 6).

Other well-known principles of metaheuristics using have also been applied to MOP problems, but few concern MOCO problems:

1. The ant colony optimization principle, based on the behavior or real ants, is another EA principle (see [9, 23, 26, 40] for examples).
2. The scatter search principle is based on a linear combination of solutions selected from a candidate list, i.e., it uses a population of solutions [4, 33].
3. The particle swarm principle is based on the elementary moves of particles, i.e., a population of solutions [6, 34].
4. Constraint programming (CP) techniques have been introduced in the solution procedure of MOP problems with the PICPA method [2]. The main goal of CP in PICPA is to build a sharp bound in the the decision space around the efficient frontier using value propagation mechanisms over variables.

4 Conclusion

To be efficient, an approximation method for solving an MOP seems to be necessarily a hybrid algorithm, i.e., a combination of EA, NSA, and problem specific components. This is in particular true for MOCO, where the adapation of a universal method to a problem cannot compete with a method specifically designed for this problem. Because one main challenge is the scalability problem – the efficient solution of large scale problem instances – the future methods will be specifically designed methods, "recycling" the 50 years of knowledge of (single objective) optimization.

We are convinced that questions like how to reduce of the search space will become even more important than they are today. Constraint programming, cuts, and bounds appear to be possible answers as do the adaptation of intensification components identified as efficient for combinatorial problems, like path-relinking.

These challenges promise many future papers.

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