A new evolutionary technique for detecting Pareto continuous regions

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Abstract

In this paper, a new evolutionary technique for detecting continuous Pareto optimal sets is proposed. The technique is designed for functions of one real variable but it can be extended for several variables functions. In this approach an individual (a solution) is either a closed interval or a point. Each solution in the final population corresponds to a decision region of Pareto optimal set. Proposed technique is called Pareto Evolutionary Continuous Regions Algorithm (PECRA). In the considered experiment the picture of Pareto set supplied by PECRA has a better accuracy than that given by other techniques.

1 INTRODUCTION

Usually, Pareto evolutionary algorithms supply a discrete picture of the Pareto optimal set (and of the corresponding Pareto frontier). But, there are some cases when the Pareto optimal set is a continuous region in the search space. In those cases the Pareto continuous regions are represented by a discrete set. When continuous decision regions are represented by discrete solutions there is a loss of information. Moreover, reconstructing continuous Pareto set from a discrete picture is not an easy computational task (Veldhuizen, 1999).

In this paper a new evolutionary multiobjective optimization approach is proposed. This technique is called *Pareto Evolutionary Continuous Regions Algorithm* (PECRA) and it is able to handle Pareto continuous regions.

2 PECRA DESCRIPTION

PECRA technique is designed for coping with objective functions of one real variable.

Let us note that when the search space is a subset of $\Re$, the Pareto optimal set may be represented as a set of points, a set of disjoint intervals or a set of disjoint intervals and a set of points. Within the proposed approach continuous decision regions may be detected. A solution (individual) is either a closed interval (a nondegenerated interval, i.e. an interval for which the extremities are not equal) or a point (considered as a degenerated interval, i.e. an interval for which the left extremity value is equal to the right extremity value).

The algorithm starts with a random population of degenerated intervals (i.e. a population of points). The variation operators are mutation and crossover. Mutation consists of normal perturbation of interval extremities. Mutation is also applied to point-solutions (considered as degenerated intervals). By crossover new solutions are added to the current population.

The solutions are detected in two stages. In the first stage all (local and global) Pareto solutions are detected. In the second stage detected solutions are refined by a fine-tuning process when the sub-optimal regions are removed from population.

2.1 REPRESENTATION AND DOMINATION

We consider solutions are represented as nondegenerated or degenerated intervals in the search space Ω (where Ω is a nonempty subset of $\Re$). Each interval-solution k is encoded by an interval $[x_k, y_k] \subset \Re$. Degenerated intervals ($y_k = x_k$) are allowed. In this case the solution is a point. In order to deal with interval representation of solutions a new domination concept is needed. This domination concept is given by the next definition.

Definition 1. An interval-solution $[x, y]$ is said to be *interval-nondominated* if and only if all points of that interval $[x, y]$ are nondominated (in Pareto sense) point-wise solutions. An interval-nondominated solution will be called a *Pareto-interval*.

If $x = y$ (degenerated intervals) this dominance concept reduces to the ordinary non-domination notion. If no ambiguity arises in what follows we will use the term *nondominated* instead of *interval-nondominated*.

Definition 2. (*sub-optimal Pareto regions*). A search space region S, S $\subset \Omega$ is said to be *sub-optimal* iff all the points from the set S are local Pareto optima.

Definition 3. An interval that contains optimal and possibly sub-optimal sub-intervals is called an *unhomogeneous interval*.

True Pareto set comprises distinct points which do not dominate each other. But, due to the representation capabilities these points could be represented by rounded versions that are equal or are dominated. For instance the number $x = 0.17$ may be represented either as $x' = 0.1$ or as $x'' = 0.2$, according to a given convention representation. Let us consider that x represent a nondominated solution. However it is possible that either x' or x'' or both of them are dominated solutions.

Therefore representation limitations may induce a falsification of domination relationship. As a consequence some points belonging to the true Pareto set could be lost during the search process based on domination.

A procedure for avoiding the loose of some dominated solutions is highly needed. In this way a new concept of dominance that takes into account the representation precision and called ε *-dominance* is defined in what follows:

Definition 4. (ε*-dominance*)

Consider a maximization problem. Let x, y be two decision vectors (solutions) from the search space Ω.

Solution x (ε*-dominates)* solution y if $f_i(x) \geq f_i(y)$, $\forall\ i = 1, 2, \ldots, n$, and $\exists\ j \in \{1, 2, \ldots, n\}$: $f_j(x) > f_j(y) + \varepsilon$.

A similar ε*-dominance* definition is given in (Laumans, 2002).

2.2 GENETIC OPERATORS

The genetic operators used by PECRA are mutation and crossover. Both of them are described in what follows.

2.2.1 Mutation Operator

For coping with the proposed solution encoding a new mutation operator is designed. Each interval extremity is mutated. The left extremity of an interval is always mutated towards left and the right extremity is mutated only towards right. Points are considered as representing degenerated intervals and they are mutated in a similar way.

Therefore, two cases are considered.

a) Degenerated interval.

An offspring is obtained by mutation towards left. The obtained point represents the offspring. Parent and offspring compete for survival.

If the offspring dominates the parent then the offspring is added to the new population. If the parent dominates the offspring then the parent is mutated again toward right. The best, in the sense of domination, enter the new population.

If parent and offspring are not comparable with respect to domination relation then the two points define an interval solution which is included in the new generation. The point solution representing the parent is discarded.

b) Nondegenerated interval.

Left extremity of the interval $[u, v]$ is mutated towards left. A point u' is obtained. Consider the case when the offspring-point u' and the parent-point u do not dominate each other. In this situation a new interval solution $[u', v]$, having the point u' as its left extremity and v as its right extremity is generated. If the offspring-point u' dominates the parent-point u, then the interval solution $[u, v]$ enters the new population. A similar mutation procedure is applied to the right interval extremity of the solution ($[u, v]$, or $[u', v]$) obtained above.

2.2.2 Crossover Operator

Mutation operator performs a local search of solutions. By contrast, the crossover operator is used for generating new solutions within the unexplored regions of the search space. Only two points in the search space are needed as parents for crossover. Therefore, if one (or both) parents are nondegenerated intervals then only one of its (their) extremities is considered as the true parent for crossover. An offspring is obtained by crossover of two parents. In our implementation convex crossover (Goldberg, 1999) has been used. Other crossover operators may also be used.

2.3 POPULATION MODEL

Problem solutions are detected in two stages. In the first stage (evolution stage) all (global and local) solutions are detected. In the second stage (*fine tuning* or *refinement stage*) the sub-optimal Pareto regions are removed from the final population.

Most of the multiobjective optimization techniques based on Pareto ranking use a secondary population (an archive) for storing nondominated individuals. Archive members may be used to guide the search process. As dimension of secondary population may dramatically increase several mechanisms for reducing archive size have been proposed. In (Zitzler, 1999) a population decreasing technique based on a clustering procedure is considered. But preserving only one individual from each cluster implies a loss of information.

The proposed approach uses a unique population.

A dynamically size population model is considered. In this approach the population size may increase or decrease depending on the number of Pareto optimal points found during the search process. The algorithm starts with a random population of degenerated intervals (i.e. a population of points). The variation operators are mutation and crossover.

Several pairs of individuals are randomly selected for crossover. The offspring obtained by crossover are added to the current population only if the population size does not exceed a given threshold. Note that by crossover the population size could increase.

Then, each individual in the current population is mutated. Parents and offspring directly compete for survival in a binary tournament. The tournament winner enters the new population.

For detecting the correct number of Pareto optimal regions it is necessary to have, in the final population, only one solution per Pareto optimal region. If two interval solutions partially overlap the shortest interval solution is discarded. Degenerated solutions included into non-degenerated interval-solutions are removed too. If two degenerated solutions are closer than a fixed threshold *r* then the worst solution is discarded.

The algorithm stops after a specified number of generations.

2.4 FINE TUNNING

Final population may comprise *unhomogeneous* interval solutions (solutions representing global as well as local optima). The aim is to detect optimal sub-solutions and discarding sub-optimal ones. Idea of fine-tuning is to isolate and discard from each final interval-solution those sub-intervals of S representing local optima. Each continuous Pareto region represented by a final solution is mapped into a discrete set.

Consider a final solution $[x,y]$. Discretized version of $[x,y]$ is obtained considering points with a fixed step size. Let D be the set of points obtained by discretizing the solution interval $[x,y]$. Discretized solutions are compared by using *ε-dominance* concept. Let us denote by ss the step size. From solution $[x,y]$ consider the points x_j fulfilling the conditions: $x_j = x + j \cdot ss$, $j = 0, 1, \ldots$ and $x_j \le y$.

These points represent the discretized version D of the interval solution $[x, y]$. Each point x_j within the discretized set is checked. If a certain point from the set D dominates the point x_j then x_j is removed from the Pareto interval $[x, y]$ together with a small neighboring region R. The size of the removed region is equal with ss.

The following mechanism is used to decrease population size:

(i) The shortest interval solution is discarded if two interval solutions partially overlap. Degenerated solutions included into non-degenerated interval-solutions are removed too.

(ii) The worst solution is discarded if two degenerated solutions are closer than a fixed threshold *r* then.

The intervals obtained after the fine tuning stage are considered as the true Pareto sets.

3 A NUMERICAL EXPERIMENT

In this experiment we compare PECRA with NSGA II (Deb, 2000), PAES (Knowles, 1999) and SPEA (Zitzler, 1999). The biobjective optimization (minimization) problem used for comparison is given by the formulas (1) and (2).

These two functions have been chosen in order to obtain a discontinuous Pareto set (made of a sequence of points and intervals. The corresponding Pareto front is also discontinuous.

$$f_1(x) = sin(x), \tag{1}$$

$$f_2(x) = \begin{cases} \dfrac{-4 \cdot x}{\pi} + 8 \cdot k, \\ \text{if } \; 2 \cdot k \cdot \pi \le x < 2 \cdot k \cdot \pi + \dfrac{\pi}{2}, \\ \dfrac{4 \cdot x}{\pi} - 4 \cdot (2 \cdot k + 1), \\ \text{if } \; 2 \cdot k \cdot \pi + \dfrac{\pi}{2} \le x < (2 \cdot k + 1) \cdot \pi, \\ \dfrac{-2 \cdot x}{\pi} + 2 \cdot (2 \cdot k + 1), \\ \text{if } \; 2 \cdot k + 1) \cdot \pi \le x < (2 \cdot k + 1) \cdot \pi + \dfrac{3 \cdot \pi}{2}, \\ \dfrac{2 \cdot x}{\pi} - 4 \cdot (k + 1), \\ \text{if } \; 2 \cdot k \cdot \pi + \dfrac{3 \cdot \pi}{2} \le x < 2 \cdot (k + 1) \cdot \pi \end{cases} \tag{2}$$

$$k \in Z^{+}.$$

The correct Pareto set (denoted P_c) is:

$$P_c = [\frac{\pi}{4}, \frac{3\pi}{4}] \cup \{\frac{3\pi}{2}\} \cup [\frac{9\pi}{4}, \frac{11\pi}{4}] \cup \{\frac{7\pi}{2}\} \cup$$

$$[\frac{17\pi}{4}, \frac{19\pi}{4}] \cup \{\frac{11\pi}{2}\} \cup [\frac{25\pi}{4}, \frac{27\pi}{4}] \cup \{\frac{15\pi}{2}\}.$$

An approximation P_{ca} of the correct Pareto P_c set may be written as:

$P_{ca} = [0.78, 2.35] \cup \{4.71\} \cup [7.06, 8.63] \cup \{10.99\}$

$\cup [13.35, 14.92] \cup \{17.27\} \cup [19.63, 21.20] \cup \{23.56\}$.

The number of generations used by NSGA II, SPEA and PECRA is 250. PAES used 25000 functions evaluations. ε-dominance value for PECRA is 0.003. PECRA, SPEA and NSGA II use a population of 100 individuals. PAES archive size is 100.

The result obtained by the compared algorithms is depicted in Figure 1.

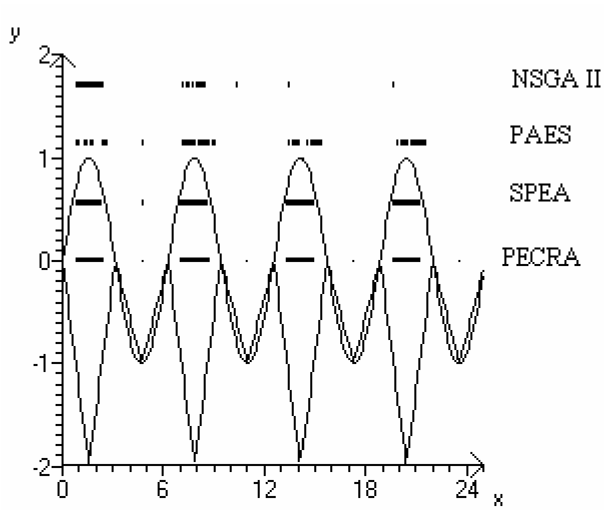

Figure 1. PECRA detects a very good approximation of the Pareto set. The other considered algorithms detect a subset of the correct Pareto set.

The solution obtained by PECRA is:

$P_{PECRA} = [0.81, 2.34] \cup \{4.71\} \cup [7.06, 8.63] \cup \{10.99\} \cup [13.37, 14.9] \cup \{17.28\} \cup [19.65, 21.18] \cup \{23.56\}$.

From Figure 1 we can see that the others three algorithms (NSGA II, PAES and SPEA) detect only a discrete set of Pareto set. By contrast, PECRA is able to detect the continuous regions and the singular solutions in a single run.

4 CONCLUSION

A new evolutionary technique for solving multiobjective optimization problems involving one variable functions has been proposed in this paper. A new encoding type and specific genetic operators have been used. Solutions in the final population represent the Pareto optimal region. The proposed evolutionary multiobjective optimization technique uses only one population. This is a dynamic size population consisting of local nondominated solutions already found.

Evolutionary technique proposed in this paper supplies directly a continuous picture of Pareto optimal set and of Pareto frontier. This makes our approach very appealing for solving problems where very accurate solutions detection is needed.

5 FUTURE WORK

Further research will focus on the possibilities to extend the proposed technique to deal with multidimensional domains.

Another research direction is to exploit the solution representation as intervals for solving inequality systems and other problems for which this representation seems to be natural.

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