

Genetic optimization of two-material composite laminates

Laurent Grosset^{*}, Satchi Venkataraman[†] and Raphael T. Haftka[‡]
Department of Aerospace Engineering, Mechanics & Engineering Science
University of Florida, P.O.Box 116250, Gainesville, FL 32611, USA

Keywords: Composite laminates, Hybrid laminate, Minimum weight and cost optimization, Multi-criteria optimization, Genetic algorithms.

ABSTRACT

This paper describes the optimization of a composite laminate made from two materials. The use of two materials in the laminates design can offer improved designs, as it is possible to combine the desirable properties of the two materials. In this paper, we consider the use of graphite-epoxy, which has high stiffness properties, but is expensive, and glass-epoxy, which is not as stiff, but has a much lower cost. The multi-material composite optimization gives rise to a multi-objective optimization problem because weight and cost have to be minimized simultaneously. This requires the construction of a Pareto trade-off curve, which is done here by solving a series of optimization problems with different convex combinations of the two objectives. For each problem, the selection of material and ply orientations is performed using a genetic algorithm. An example of the design of a plate subject to a constraint on its first natural frequency is used to demonstrate the approach.

INTRODUCTION

Over the last few decades, laminated composite have found usage in aerospace, automotive, marine, civil, and sport equipment applications. This popularity is due to excellent mechanical properties of composites as well as their amenability to tailoring of those properties.

^{*} (corresponding author) Ph. D. Student (laurent@aero.ufl.edu)

[†] Post-Doctoral Associate (satchi@aero.ufl.edu)

[‡] Distinguished Professor, Fellow AIAA (haftka@ufl.edu)

^{*} Laurent Grosset, Ph. D. Student, laurent@aero.ufl.edu, Department of Aerospace Engineering, Mechanics & Engineering Science, University of Florida, P.O.Box 116250, Gainesville, FL 32611, USA

[†] Satchi Venkataraman, Visiting Assistant Professor, satchi@aero.ufl.edu, Department of Aerospace Engineering, Mechanics & Engineering Science, University of Florida

[‡] Raphael T. Haftka, Distinguished Professor, haftka@ufl.edu, Department of Aerospace Engineering, Mechanics & Engineering Science, University of Florida

Most applications involve a single material system, with ply orientations being the design variables used to tailor properties (e.g., Miki, [1]). However, in some applications it makes sense to combine more than one material system in order to combine desirable properties. The present paper investigates the combination of two materials for balancing cost and weight considerations.

A composite laminate is designed from several materials, and the optimization problem requires choice of material for each ply. This leads to a combinatorial optimization problem that is handled well by genetic algorithms (GA). Genetic algorithms have been used extensively (e.g., Callahan and Weeks, [2], Le Riche and Haftka, [3]) for laminate stacking sequence optimization when ply angles are limited to a discrete set of angles (e.g., 0° , $\pm 45^\circ$ and 90°). This paper employs an existing GA developed by McMahan et al. [4] and adapts it to improve its efficiency for the problems considered. The design of a composite plate with a frequency constraint is used to demonstrate the power of GAs for this application.

COMPOSITE LAMINATE DESIGN

Composite laminate optimization typically consists in determining the stacking sequence, that is the orientation and material properties of each layer, that gives the desired properties: stiffness, strength, frequency, buckling load, etc. The strength of composite materials is their very high stiffness-to-weight or strength-to-weight ratios, but the cost of those materials increases very fast with performance. Therefore it can be advantageous to use a combination of efficient but expensive, and less expensive but less stiff materials in order to reduce cost while ensuring a high level of performance. Combining the two material allows us to trade weight and cost. This can be done in the framework of bi-objective optimization. In this paper, a rectangular laminated plate optimized for minimum weight and minimum cost. A single constraint is specified, imposing a lower bound f_{\min} on the first natural frequency f of the plate. The laminate is symmetric and balanced (for every angle \mathbf{q} different from 0° and 90° , the angle $-\mathbf{q}$ is also present in the laminate). The multi-objective optimization problem can be formulated as follows:

Minimize: weight and cost

By changing: the orientation \mathbf{q}_i and the material m_i of the plies

Such that: $f > f_{\min}$

BASIC CONCEPTS OF MULTI-OBJECTIVE OPTIMIZATION

The purpose of multi-objective optimization is different from that of single-objective optimization: in the latter, the goal is to find *the best solution*, which is the design that minimizes (or maximizes) the objective function. In contrast, in multi-objective optimization there is no single solution that minimizes (maximizes) all the objective functions. Indeed, the objective functions often conflict, as a design that decreases one objective will increase another. The interaction between the objective functions gives rise to a set of compromise solutions called *Pareto set*. A solution

belongs to the Pareto set if there is no other design such that all the objective functions are lower at the same time. The designer will then need to use additional information to prioritize the objective functions in order to choose between the elements of the Pareto set. . There are genetic algorithms that construct the Pareto set in a single optimization (see Deb, [5]). In this paper the Pareto set is generated by optimizing a convex combination of the two objectives, weight W and cost C

$$F = \mathbf{a} W + (1 - \mathbf{a}) C \quad (1)$$

for a series of values of the multiplier α . Several values of \mathbf{a} were chosen successively and the combined objective function was minimized using a single-objective optimizer based on a genetic algorithm. If the Pareto set is convex, this procedure yields points that belong to the Pareto set.

OPTIMIZATION OF A RECTANGULAR LAMINATED PLATE

Problem description

Our design problem was the minimization of the weight and the cost of a rectangular laminated plate of length $a = 36$ in and width $b = 30$ in, subject to a constraint on the first natural frequency. The first natural frequency f of a rectangular plate is given by the following expression [6]:

$$f = \frac{\mathbf{P}}{2\sqrt{\mathbf{r}h}} \sqrt{\frac{D_{11}}{a^4} + 2\frac{D_{12} + 2D_{66}}{a^2 b^2} + \frac{D_{22}}{b^4}} \quad (2)$$

where D is the flexural stiffness matrix calculated using the Classical Lamination Theory, \mathbf{r} is the average density and h the total thickness of the plate.

A two-material composite made of graphite-epoxy and glass-epoxy was considered. The material properties are given in Table 1 (source <http://composite.about.com>). The stiffness-to-weight ratio of graphite-epoxy is about four times higher than that of glass-epoxy, with $E_1/\mathbf{r} = 345$ against $E_1/\mathbf{r} = 87.5$. However it is also more expensive, with a cost per pound that is 8 times higher than that of glass-epoxy. If the first priority is weight, then graphite-epoxy will be preferred; while if cost is paramount the optimum laminate will obviously contain glass-epoxy plies. The design of this simple rectangular plate leads us to study the trade-off between the two objective functions weight and cost. The ply orientation can take a set of 19 values ranging from 0° to 90° in steps of 5° .

TABLE 1: MATERIAL PROPERTIES OF GRAPHITE-EPOXY AND GLASS-EPOXY

	Graphite-epoxy	Glass-epoxy
Longitudinal modulus (Msi), E_1	20.01	6.3
Transverse modulus (Msi), E_2	1.30	1.29
In-plane shear modulus (Msi), G_{12}	1.03	0.66
Poisson modulus, ν_{12}	0.3	0.27
Density (lb/in ³), ρ	0.058	0.072
Thickness (in), t	0.005	0.005
Cost factor (lb ⁻¹), C	8.0	1.0

Several methods for constructing the Pareto-optimal front exist. The approach chosen here is the weighted sum method (cf. Deb, 2001) where the two objective functions are combined into one overall objective function as shown in Eq. (1).

The trade-off curve between weight and cost is constructed by running a succession of the following single-objective optimizations:

Minimize: F

By changing: the orientation \mathbf{q} and the material m_i of the plies

Such that: $g = \frac{f}{f_{\min}} - 1 \geq 0$

The single-objective optimizations resulting from the cost/weight minimization were performed using a Genetic Algorithm (GA) developed by McMahon et al. [4]. This algorithm works with a population of designs (individuals) represented by two strings (chromosomes): respectively describing the orientation and the material of each ply. One character of the string corresponds to a given ply in the laminate. Since symmetric laminates are considered, only one half of the stacking sequence needs to be represented. In addition, the requirement that the laminate be balanced can be easily enforced by using pairs of $\pm\mathbf{q}$ plies. Although 0° plies and 90° plies do not need to come in pairs, they are treated like other angles, but they are assigned half the normal thickness to simulate a single ply. For instance, the chromosomes:

Orientation: [1 5 8 2 19]

Material: [1 2 2 1 1]

correspond to the stacking sequence: $[0/\pm 20/\pm 35/\pm 5/90]_s$. Where plain numbers designate graphite-epoxy and underlined numbers designate glass-epoxy.

A fitness function is constructed using a penalty approach:

$$fitness = \begin{cases} -F(1 - 0.01g) & \text{if } g \geq 0 \\ -1.1F + pg & \text{otherwise} \end{cases} \quad (3)$$

where p denotes the penalty parameter, which is selected high enough to ensure that the design with the lowest value of F does not violate the constraint. The term 0.01 in the expression for feasible designs is used to reward designs that satisfy the constraints with larger margins.

Designs are selected based on a roulette wheel approach: the higher the rank in the population, the higher the probability of being selected to generate new designs. Then a population of new designs is created by applying a series of genetic operators that were devised specifically for composite optimization: two-point crossover, which swaps part of the stacking sequence between two designs, mutation, which changes the value of a random gene, and permutation, which inverts the order of a portion of a chromosome. These operators act separately on each of the two chromosomes. Two additional operators allow the number of plies (thickness) to vary during the optimization. Those are a deletion operator, which removes one ply selected at random and a counterpart addition operator, which adds a ply at a random location in the stacking sequence. The new population is analyzed, and the fitness of each individual is evaluated. Selection and recombination operators are then applied on the newly created population and a new population is generated. The process is repeated until convergence is reached. The algorithm used in the present work is an elitist algorithm, where the best individual is always passed on to the next generation.

Results

First the minimum of the objective functions weight and cost were found using the Genetic Algorithm. The optimization started with a population of 10 designs made of 44 plies whose orientation and material were set randomly. The GA was applied with the parameters shown in Table 2. These values were determined by trial and error in order to maximize reliability, which is the probability of reaching the optimum for a given number of function evaluations. They are used in all cases in this paper.

TABLE 2: PARAMETERS ASSOCIATED WITH THE GENETIC OPERATORS

Crossover probability	Mutation probability, orientation	Mutation probability, orientation	Permutation probability	Addition probability	Deletion probability
1.0	0.3	0.2	0.2	0.05	0.1

The minimum weight was found to be 6.89 lb. It is reached when all plies are made of graphite-epoxy. The optimum design is then $[\pm 50_5/0]_s$ and has a relative cost of 55.12. The minimum cost was 16.33, 70% less expensive than the optimum of the weight objective function. The optimum design is $[\pm 50_{10}/0]_s$ comprised

exclusively of glass-epoxy. Its weight is 16.33 lb, which represents a rise of 137% over the optimum of the weight objective function.

In order to construct the Pareto front, the weighting factor \mathbf{a} was varied from 0.0 to 1.0 and the composite objective function F was minimized using the GA. The optimum designs obtained as well as their weight and cost are summarized in Table 3. The optimum designs are mainly made of $\pm 50^\circ$ plies in order to maximize the first vibration frequency of the plate. In some cases a 0° ply is present in the core layers of the laminate. . Although 0° plies do not contribute much to the frequency, it is advantageous to use them because unlike other angles (with the exception of 90°) they do not have to come in pairs (to satisfy balance), thereby saving unnecessary additional weight and cost. The 0° plies always appear in the inner layers, where they are the least damaging for the performance of the plate. For the same reason, the less stiff glass-epoxy layers, when they have to be used, always appear in the inner layers. This creates a sandwich type composite where the structural function is assured by the stiff graphite layers, placed on the outside, where their contribution to the flexural properties of the laminate is maximal, while inner layers are merely used to increase the increase the distance of the outer plies from the neutral plane.

TABLE 3: OPTIMUM DESIGN OF MINIMIZATION OF THE COMPOSITE OBJECTIVE FUNCTION

Weighting factor \mathbf{a}	Cost	Weight	First natural frequency	Stacking sequence (plain numbers: graphite, underlined numbers: glass)
0.00	16.33	16.33	25.82	$[\pm 50_{10}/0]_s$
0.70	20.90	12.14	25.10	$[\pm 50/\pm 50_7]_s$
0.80	27.82	10.28	25.88	$[\pm 50_2/\pm 50_5]_s$
0.87	31.28	9.35	25.08	$[\pm 45_2/90/\pm 50_3/\pm 80]_s$
0.93	38.96	8.27	25.38	$[\pm 50_3/90/\pm 50_2/0]_s$
0.96	43.20	8.12	26.07	$[\pm 50_4/\pm 50_2]_s$
1.00	55.12	6.89	25.14	$[\pm 50_5/0]_s$

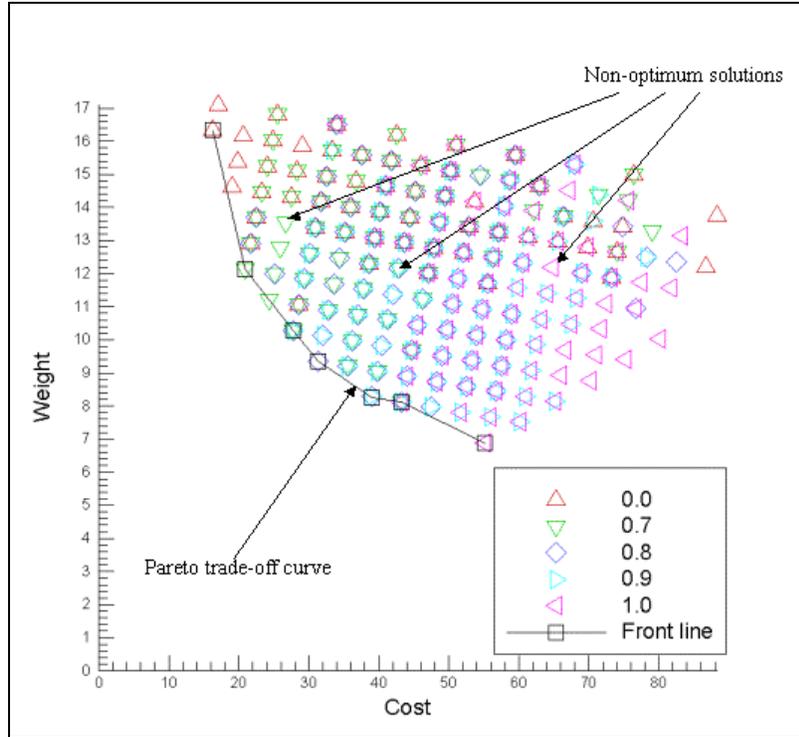


Figure 1: Pareto set of non-dominated solution obtained by the weighted sum method. The solid line shows the Pareto front, the symbols represent all the feasible solutions calculated by the algorithm during the optimization for the different values of the weighting factor α .

Figure 1 shows the Pareto front obtained using the weighed sum method. The solid black line represents the set of solutions of the composite objective function for the different values of α . The various symbols show all the feasible designs that were generated during the search. The Pareto front is the set of all the non-dominated solutions, which corresponds to the lower envelope of all the design points in the weight/cost plane. This confirms the validity of our method for constructing the Pareto front.

The Pareto trade-off curve can be used to help the designer determine the optimal configuration for his problem. The final choice of the best design will depend on additional information that will enable him to assign priorities to the two objectives. There is no single best design: depending on the application that is considered, the choice will be different. . For example, if weight and cost are of similar importance, the design obtained with $\alpha=0.93$ in Table 3 may be attractive. The weight is 20% above the minimum weight, but the cost is reduced by 30%.

CONCLUDING REMARKS

The method used in this paper for solving the multi-objective optimization problem that derives from multi-material composite design is a very simple method. It is based on a standard genetic algorithm that has been adapted to composite

optimization. The Pareto-optimal front was constructed by optimizing a series of composite objective function combining weight and cost. The method was shown to be able to capture the Pareto trade-off curve for that two-objective problem, providing the designer with a very helpful tool for decision-making.

An example of a plate designed to minimize weight and cost subject to a constraint on the first natural frequency illustrated the approach. When cost was a primary consideration the plate was made from glass-epoxy, and when weight was a primary consideration, it was made of graphite-epoxy. Compromise designs are easily selected from the Pareto trade-off curve.

However additional work remains to be done in order to improve the efficiency of the optimization. Due to their inherent parallelism, genetic algorithms are able to find multiple solutions in one single run thus saving computational time and simplifying the optimization procedure. Genetic algorithms that generate the entire Pareto curve in a single run may be attractive.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial support of Visteon Corporation and the technical advice of Dr. Naveen Rastogi, our project sponsor at Visteon.

REFERENCES

1. M. Miki (1982) "Material Design of Composite Laminates with Required In-Plane Elastic Properties," *Progress in Science and Engineering of Composites*, Vol. 2, 1725-1731. Hayashi, T. Kawata, K. and Umekawa, S. (eds.) ICCM-IV, Tokyo.
2. K.J. Callahan and G.E. Weeks, "Optimum Design of Composite Laminates Using Genetic Algorithms," *Composite Engineering*, **2(3)**, 149-160.
3. R. Le Riche and R.T. Haftka (1993) , "Optimization of Stacking Sequence Design for Buckling Load Maximization by Genetic Algorithms," *AIAA Journal*, **31(5)**, 951-956.
4. M.T. McMahon, L.T. Watson, G.A. Soremekun, Z. Gürdal, and R.T. Haftka (1998), "A Fortran 90 Genetic Algorithm Module for Composite Laminate Structure Design" *Engineering with Computers*, 14, pp. 260-273.
5. K. Deb (2001), *Multi-Objective Optimization Using Evolutionary Algorithms*, UK Wiley, Chichester.
6. Z. Gürdal, R. T. Haftka and P. Hajela (1998), "Design and Optimization of Laminated Composite Materials", Wiley Interscience, p. 300.
7. M. F. Ashby (2000), "Multi-objective optimization in material design and selection", *Acta Materialia* 48, pp. 359-369.
8. C. A. Coello (2000), A. D. Christiansen, "Multi-objective optimization of trusses using genetic algorithms", *Computers and Structures* 75, pp. 647-660.

APPENDIX: ENFORCEMENT OF THE BALANCE CONSTRAINT

Very often it is desirable to have balanced laminates, for which every occurrence of orientation θ is balanced by an occurrence of $-\theta$ in order to minimize shear-extension and bending-twisting effects. In this work two options for enforcing this requirement were explored. The first approach is a soft implementation that uses a penalty function to make unbalanced laminates less attractive to the genetic algorithm. During the analysis of laminates, the number of plies that are not balanced is counted and a penalty proportional to the number of unbalanced plies is applied. Since a ply of a given material can only be balanced by another ply of the same material, unmatched angles have to be counted separately for each material. The fitness function is therefore defined as follows:

$$fitness = \begin{cases} -F(1-0.01g) - (p_{gr} + p_{gl})b & \text{if } g \geq 0 \\ -1.1F + pg - (p_{gr} + p_{gl})b & \text{otherwise} \end{cases} \quad (4)$$

where p_{gr} and p_{gl} represent the penalty terms associated with one unbalanced ply for graphite-epoxy and glass-epoxy respectively, and b is a penalty multiplier. The advantage of this soft implementation of the balance constraint is that it allows laminates where conjugate plies are not necessary adjacent and makes the design more flexible.

The second approach only enables balanced laminates by using pairs of plies of opposite orientation. This implies that the coding can be further condensed: besides using symmetry to code only half of the laminate, a single gene will be decoded as a pair of $\pm\theta$ plies. For instance in the case of 19 angles ranging from 0° to 90° by steps of 5° , the allele 4 will be decoded as $\pm 15^\circ$ (instead of 15° in the previous approach). Since 0° -plies and 90° -plies do not cause coupling between shear and extension or bending and twisting, they do not need to be balanced by a conjugate ply. However, owing to the decoding method, they will always appear in pairs. This will have the disadvantage of possibly increasing the thickness of the laminate. To overcome this problem we facilitate the use of a material with half the nominal thickness for 0° -plies and 90° -plies; the pair of two half-thickness plies will behave identically to a single regular ply.

Figure 2 shows the evolution of the reliability versus the number of generations for the two approaches. In each case, the code was run 150 times with $\alpha = 1$ (cost minimization) and near optimum penalty parameters obtained through trial and error. The reliability, which is the probability of reaching the optimum after a given number of generations, was calculated. The solid line represents the evolution of reliability when the first approach (penalty) is used. The dashed line shows the same quantity for the case where balance is hard-coded in the decoding procedure. Obviously, the second approach is considerably more effective than the penalty method. After only approximately 75 generations the ply pairs approach yields the optimum design with a reliability of 100%. For the same number of analyses, the penalty approach finds the optimum in only about 40% of the runs. Those results demonstrate that the penalty approach is not efficient for enforcing the balance constraint. Indeed, genetic algorithms work well when two parents with a high

fitness function are likely to produce a children at least as good as them. This is not the case when unbalanced laminates are penalized, as the balance characteristic cannot be transmitted to the children with a large probability. Not only is the approach obviously ineffective but is also hampers the good progress of the algorithm and makes enforcement of the frequency constraint more difficult by adding noise to the fitness function. Finally in the penalty approach the optimizer has to deal with twice as many variables, which adds to the complexity of the problem. Consequently, the approach that was retained for the multi-objective optimization is the ply-pair method.

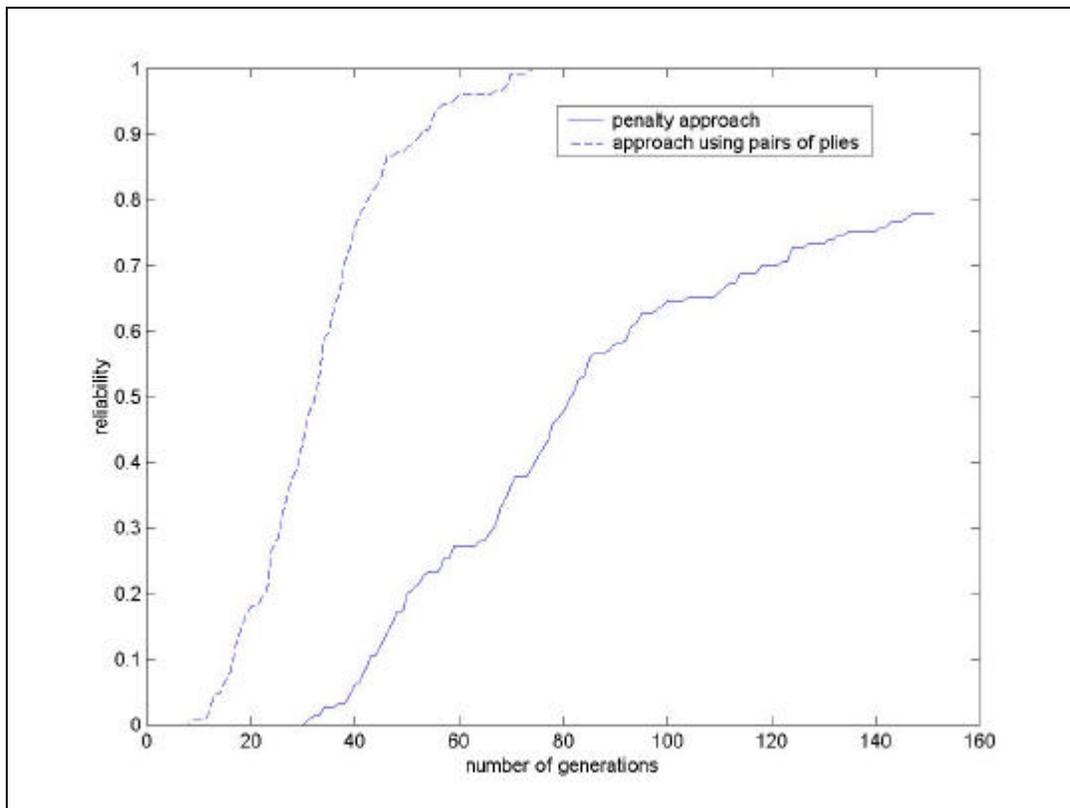


Figure 2 Evolution of the reliability for the two alternatives for enforcing the balance constraint. The approach that uses pairs of plies of opposite angles is more effective for finding the optimum design than the penalty approach.