Design of Multiobjective Robust Controllers Using Genetic Algorithms*

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Abstract

In this paper we present an approach to design robust controllers. The control design problem is formulated as a multiobjective optimization problem and solved by GAs.

1 Introduction

The design of a control system is a multiobjective problem. In a control application, a plant and a set of specifications to be satisfied are given. The designer has to determine a controller such that the feedback interconnection of the plant and the controller satisfies the specifications. The specifications are usually competitive, and the controller synthesis method requires a trade-off between conflictive objectives such as fast time response, disturbance attenuation, command following, stability, robustness, etc.

In addition, not every specification can be represented by the same performance measure. Different performance measures or norms are appropriate for different objectives. For example, $\mathcal{H}_2$ norms can be used for requiring attenuation of stochastic white noise disturbances, while $\mathcal{H}_\infty$ norms could represent robustness against unstructured dynamic uncertainties [6].

There are many useful control system design techniques, but more of them deal with a single kind of performance measure. A most powerful design technique should permit the designer to deal with different objectives and performance indexes. Here, we present an approach for multiobjective control design by using the GA paradigm.

*This research was supported in part by Junta de Castilla y León under grant Programa de Apoyo a la Investigación

2 Problem Formulation

We shall use the concept of Linear Fractional Transformation (LFT) [6] to represent input/output relationships of the closed-loop interconnection of the plant and the controller to be designed. A control system design problem can be formulated in the LFT framework as a multiobjective problem.

Let the lower LFT $\mathcal{F}_l(G,K)$ represent a relevant input/output relationship of the closed-loop system, where G is the generalized plant and K is the controller to be designed. The closed-loop specifications are defined in terms of merit functions of the LFT $J_i(\mathcal{F}_l(G,K))$. Therefore, the synthesis problem can be posed as follows:

Find a stabilizing controller K such that

$$J_i(\mathcal{F}_l(G,K)) < \gamma_i, \quad i \in \{1,\ldots,n\}$$

In modern control design methods, the objective functions J_i are norms such as the $\mathcal{H}_\infty$, $\mathcal{H}_2$ or $\mathcal{L}_1$ norm, but any other (positive) closed-loop system property can be accounted for.

The synthesis problem can be alternatively formulated as the multiobjective optimization problem:

$$\min_K \begin{bmatrix} J_1(\mathcal{F}_l(G,K)) \\ \vdots \\ J_n(\mathcal{F}_l(G,K)) \end{bmatrix} \tag{1}$$

A second important problem is considered in this work. In a real-world problem, perfect plant modeling is not available. Physical modeling and system identification provide linear models that represent the plant in a given limited range of operation. These models are only approximations to the true plant. Therefore, one should account for the uncertainty in the plant model in order to design controllers that are valid and useful not only for the plant model (or nominal plant) but for the true plant.

Let us assume that the plant model has parametric uncertainty. In this case, the generalized nominal plant of the lower LFT that represents the feedback system interconnection can be expressed as an upper LFT $G = \mathcal{F}_u(G_n, \Delta_G)$ where G_n is the nominal generalized plant and Δ_G is a diagonal uncertainty matrix. Now, the robust parametric synthesis problem can be formulated as the following min-max multiobjective problem:

$$\min_K \max_{\Delta_G} \begin{bmatrix} J_1(\mathcal{F}_l(\mathcal{F}_u(G_n, \Delta_G), K)) \\ \vdots \\ J_n(\mathcal{F}_l(\mathcal{F}_u(G_n, \Delta_G), K)) \end{bmatrix} \qquad (2)$$

3 Description of the GA.

Two GAs have been developed, the MRCD algorithm for solving the Pareto minimization problem (1) and the min-max MRCD algorithm for solving the min-max problem (2). Both algorithms exploit different techniques to get diversity and both use the ranking function to search individuals along the whole Pareto front, see [5].

The MRCD has an adaptive search space. This is motivated by two reasons: the selection of the initial population and the delimitation of the best search space. Every individual of the initial population should be stabilizing. In fact, this is a very difficult problem, because the set of stabilizing fixed structure controllers may be disconnected. Moreover, the determination of this set is an open problem in control theory.

The selection method used in the MRCD algorithm is the Horn and Nafpliotis method [3, 4], with ranking function and fitness shaping. This algorithm incorporates a coarse-grained parallel strategy [1] to exploit the adaptive search space bounds at each subpopulation. The external migration operation introduces new information about the search space. The MRCD has crossover operator with mate restriction to get diversity, and optimization problem constraints are all considered as new objective functions to optimize. This algorithm uses elite, but it is filtered with a niche count criterion.

The min-max MRCD, as its name indicates, is a modified version of the MRCD algorithm to solve the min-max problem 2. The phenotype of an individual is composed of two parts: the minimizing parameters m, and the maximizing parameters, M. The adaptive search space, the initial population, the selection method, and the coarse-grained parallel strategy are essentially the same as for the MRCD algorithm. The elite filter operator is the central difference between the MRCD algorithm and the min-max algorithm. At each generation and during a number of the cycles, the elite of the population is manipulated by changing their M parts. As a consequence, if an individual remains in the elite after several generations then it minimizes the vector objectives for the worst case of the M part.

4 Conclusion.

We have applied both the MRCD and mix-max MRCD algorithm to several control design problems and the results are very promising. The MRCD algorithm has been compared with linear matrix inequality (LMI) approaches [2] for multiobjective $\mathcal{H}_2/\mathcal{H}_\infty$ control problems and the results are clearly better.

The min-max MRCD algorithm addresses a design problem where no efficient approaches exist so far. As a real-world application, we have applied this algorithm to the design of decentralized multi-loop PID controllers, that are frequently used in the process industry. Our results are also very satisfactory.

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