A New Genetic Algorithm using Pareto Partitioning Method for Robust Partial Model Matching PID Design with Two Degrees of Freedom

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Key Words: Genetic Algorithm, Pareto patitioning method, Minimax optimization, Partial model matching, PID controller with two degrees of freedom

Abstract: In this paper we present a new genetic algorithm (GA) using pareto partitioning method and apply this GA to solving the design problem of the robust PID controller with two degrees of freedom based on the partial model matching approach. This design problem is formulated as two objective minimax optimization problem. Therefore we need to generate a pareto optimal set that is properly distributed in the neighborhood of the trade-off surface. The proposed GA is able to uniformely control the convergence of this pareto optimal solutions. Some numerical examples show the effectiveness of the present method.

1. Introduction

Most industrial processes are controlled by PID (that has *P*ropotional, *I*ntegral and *D*erivative actions) or I-PD controllers, because these controllers have simple construction and easy to tune. Many auto-tuning method of PID controller based on Ziegler-Nichols and so on have been developed consequently[1, 2]. These methods of tuning parameters of PID controllers, however, are experimental and heuristic. In addition to, most of them do not explicity examin robust property. One of the reason of these facts is that industrial processes are too complex to obtain precise dynamics of a plant.

About this point, recently we have developed a minimax design method based on a generalized ISE, which includes a penalty on the derivative of the control variable[3]. This method, but, is unable to apply the multiple specification design problem, such as the case required the optimization of both reference response and disturbance response at the same time[2]. And if we tried to apply the method proposed in [3] to the PID controller with two degrees of freedom, that will be failure. The reason why is that the generalized ISE diverge in this case. Hence we have also developed a new design method for robust PID controller with two degrees of freedom, which optimize bothe reference and disturbance responses of the closed-loop system at the same time, based on the PMM (partial model matching) approach[4, 5].

In this method, the design problem for robust PID controller with two degrees of freedom is formulated as a two objective minimax optimization problem, which minimizing the disturbance and reference performance measure maximized by plant parameters with perturbations at the same time. Namely, this design problem is one of the multiobjective optimization problem. Multiobjective optimization seeks to optimize the components of a vector-valued cost function. Unlike single objective

optimization, the solution to this problem is not a single point, but a family of points known as the Pareto-optimal set, also called non-dominated set. Each point in the trade-off surface is optimal in the sense that no improvement can be achieved in one cost vector component that does not lead to degradation in at least one of the remaining components.

A Pareto-based approach was first proposed by Goldberg[6], as a means of assigning equal probability of reproduction to all non-dominated individuals in the population. Fonseca and Fleming [7, 8] proposed a multiobjective ranking based on the Pareto-based fitness assignment. In this method, an individual's rank corresponds to the number of individuals in the current population by which it is dominated. Multiobjective ranking correctly assigns all non-dominated individuals the same fitness, but that, on its own, does not guarantee that the Pareto-optimal set be uniformly sampled. When presented with multiple equivalent solutions, finite population member tend to converge to only one of them, due to stochastic errors in the selection process. This phenomenon, known as genetic drift, has been observed in natural as well as in artificial evolution, and can also occur in the Pareto-based evolutionary optimization.

Therefore, in this paper, we propose a new GA using the pareto partitioning method for the design problem of robust partial model matching PID controller with two degrees of freedom. The purpose of this new GA is to generate a Pareto-optimal set that is properly distributed in the neighborhood of the trade-off surface. Some numerical results indicate that proposed GA is effective for solving this design problem and that proposed robust partial model matching PID controller with two degrees of freedom is effective for the process control.

2. Problem Formulation

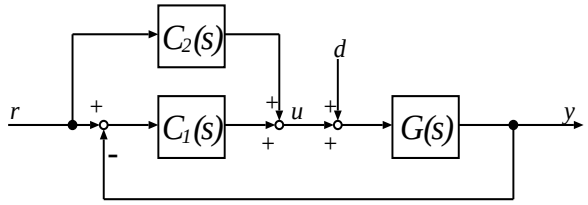

Figure 1 Closed-loop system with two degrees of freedom

Consider the control system shown in Figure 1, where $G(s) = P(s)e^{-Ls}$ is the plant and L is a delay.

We use the first or second order Padé approximation to the delay, so that the plant is approximated as

$$G(s) \cong P(s)\frac{1-\frac{Ls}{2}}{1+\frac{Ls}{2}} ,$$

$$G(s) \cong P(s)\frac{1-\frac{Ls}{2}+\frac{L^2s^2}{12}}{1+\frac{Ls}{2}+\frac{L^2s^2}{12}}$$

where $P(s)$ is a rational transfer function. Let θ be the parameters in $P(s)$. Taking account of the plant parameter uncertainty, we assume that θ belongs to a bounded set Θ, which is a priori given to the designer.

Next, C_1 and C_2 are controllers given by

$$C_1(s) = K_c(1+\frac{1}{T_is}+T_ds) ,$$
$$C_2(s) = -K_c(\alpha+\beta T_ds).$$

From Figure 1, the transfer function of step disturbance response is given by

$$H_{yd}(s) = \frac{P(s)}{1+C_1(s)P(s)}$$

$$= \frac{T_is}{K_c+\{(a_0+K_c)T_i\}s+\{(a_1+K_cT_d)T_i\}s^2+A(s)} \quad (1)$$

where

$$A(s) = \sum_{k=3}^{\infty}(a_{k-2}T_i)s^k$$

and where

$$a_k = \frac{1}{k}\left[\frac{d^k}{ds^k}\frac{1}{P(s)}\right]_{s=0} \quad (2)$$

Let the standard transfer function of step disturbance response for PMM (partial model matching) be

$$M_d(s) = \frac{T_is}{K_c}\frac{1}{1+\gamma_1\tau s+\gamma_2\tau^2s^2+\cdots+\gamma_n(\tau s)^n} \quad (3)$$

On the other hand, the transfer function of step reference response is

$$H_{yr}(s) = \frac{\{C_1(s)+C_2(s)\}P(s)}{1+C_1(s)P(s)} \quad (4)$$

and the standard transfer function of step reference response is

$$M_r(s) = \frac{1}{1+\gamma_1\tau s+\gamma_2(\tau s)^2+\cdots+\gamma_n(\tau s)^n} \quad (5)$$

Hence the objective of PMM[4, 5] is

$$H_{yd} \simeq M_d(s) \quad \text{and} \quad H_{yr} \simeq M_r(s)$$

We assume, therefore, that the performance measures for this objective of PMM are given by

$$J_d = e_1^2+e_2^2+e_3^2+e_4^2 \ , \quad (6)$$
$$J_r = e_5^2+e_6^2 \quad (7)$$

where J_d is the measure for the disturbance response and J_r is the measure for the reference response. And where

$$e_1 = \gamma_1\tau - \frac{(a_0+K_c)T_i}{K_c}$$
$$e_2 = \gamma_2\tau^2 - \frac{(a_1+K_cT_d)T_i}{K_c}$$
$$e_3 = \gamma_3\tau^3 - \frac{a_2T_i}{K_c}$$
$$e_4 = \gamma_4 - \frac{a_2+K_c\tau_2\{(1-\beta)\gamma_1T_d-(1-\alpha)\gamma_2\tau_2\}T_i}{K_c{\tau_2}^3}$$
$$e_5 = \gamma_1 - \frac{(a_0+\alpha K_c)T_i}{K_c\tau_2}$$
$$e_6 = \gamma_2 - \frac{(a_1+\beta K_cT_d-(1-\alpha)\gamma_1K_c\tau_2)T_i}{K_c{\tau_2}^2}$$
$$\tau = \frac{\gamma_3a_3}{\gamma_4a_2}$$

As[4], the recommended values are

$$\gamma_1 = 1.00 \ , \ \gamma_2 = 0.50 \ , \ \gamma_3 = 0.15 \ , \ \gamma_4 = 0.03.$$

So we use these values.

Now the design problem of robust PID controller with two degrees of freedom is conveniently formulated as a two objective minimax optimization problem

Design problem :

$$\min_{K_c,T_i,T_d} \max_{\theta} J_d \quad (8)$$
$$\min_{\alpha,\beta} \max_{\theta} J_r \quad (9)$$

s.t. closed-loop system is stable for all $\theta \in \Theta$

Solving this two objective optimization problem, we propose the new GA using pareto partitioning method in next section.

We note that this optimization problem has a significant constraint, that is the closed-loop system should be robustly stabel for all plant parameters in the set Θ. We can check the robust stability by Kharitonov's Theorem[9, 10].

3. New Genetic Algorithm

3..1 Pareto partitioning method

To prevent a partial convergence of non-dominated solutions in the trade-off surface, we propose the Pareto partitioning method which uniformly controls a distribution of non-dominated solutions. The proposed method assigns all non-dominated individuals the differ fitness on the basis of the distribution in pre-specified regions.

The proposed method consists of following procedure. First, the objective space is divided prespecified regions. The edge points of the whole region correspond the best solutions for each objective function. Then, the fitness f_i of the individual p_i is defined as $f_i = 1/n_i$. The value of n_i denotes the number of non-dominated solutions in the identical region with the individual p_i.

The continuous generation model with a pareto-optimal preservation strategy is employed, and is illustrated in Figure 2.

In this model, a pareto-optimal set is always forced to appear in the following generation.

The proposed procedure consists of the following steps:

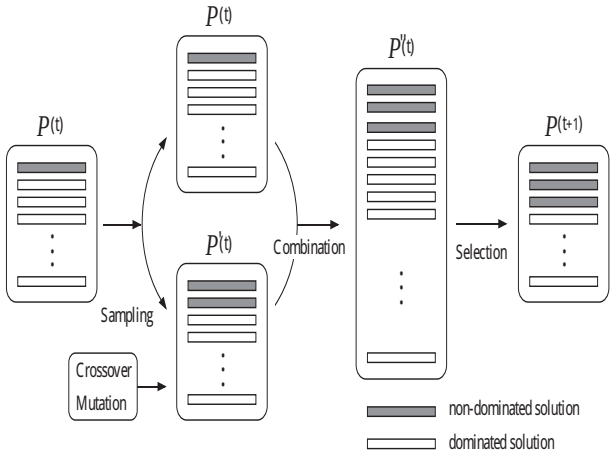

Figure 2 Generation model with a pareto -optimal preservation strategy

Step 1 Set a generation number $t = 0$. Randomly generate an initial population $P(t)$ of M individuals.

Step 2 Calculate the fitness of each solution in the current population according to the multiobjective ranking. If the non-dominated solutions occupy the current population, apply the pareto partitioning method for the fitness assignment of all non-dominated individuals.

Step 3 Generate a new population $P'(t)$ from $P(t)$ by using a crossover operator.

Step 4 Apply a mutation operator to the newly generated population $P'(t)$ a ccording to mutation rate.

Step 5 Calculate the fitness both of $P(t)$ and $P'(t)$.

Step 6 Select M individuals from all population member on the basis of the fitness. If the non-dominated solutions is over M, select M individuals from whole population according to the proposed fitness assignment.

Step 7 If a terminal condition is satisfied, stop and return the best individuals. Otherwise set $t = t + 1$ and go to *Step 2*.

In this procedure, update of the current population size is always constant M. There is no question that selection of individuals for the next generation should be proportional to their fitness values. However, when a pure proportional approach is adopted, in practice the population is rapidly dominated by a few super-individuals, resulting in a dramatic decrease in genetic diversity. Here, to avoid the rapid loss of genetic diversity, multiple equivalent individuals are eliminated from the current population.

3..2 GA formulation

To design a GA for the robust partial model matching PID design problem, certain problem-dependent algorithm elements need to be defined. They influence both the efficiency of the algorithm and the quality of its results.

3..2.1 Representation of individuals

Gray-code string representation is employed for candidate solutions. Each of the parameters x_1 and x_2 is subject to interval constraints. Two such strings are concatenated into a binary string, representing a point in the parameter space to be searched by the algorithm.

3..2.2 Fitness function

The linear scaling method based on the multiobjective ranking is employed as a fitness function. The fitness f_i of the individual p_i is computed through two steps. First, the rank R_i $(i = 1, \ldots, M)$ of each individual in the current population $P(t)$ is calculated by the multiobjective ranking. Then, the fitness f_i of the individual p_i is defined as follows:

$$\begin{aligned} f_i &= (F_{\max} - F_{\mathrm{s}} \times (R_i - 1)) \\ F_{\mathrm{s}} &= \frac{1}{M-1}(F_{\max} - F_{\min}) \end{aligned} \tag{10}$$

where M denotes the population size, $F_{\max}$ and $F_{\min}$ denote the maximum and the minimum fitness values. In computer simulations, the values of $(F_{\max}, F_{\min})$ are held constants (10.0, 1.0), namely, the fitness of best individuals is 10.0, on the contrary, the fitness of worst one is 1.0.

3..2.3 Genetic operators

The genetic operators applied in the algorithm are uniform crossover, bit mutation and roulette wheel

selection. Uniform crossover generates two offspring by exchanging a predefined number of alternate subsections between two parent strings. The recombination operator, mutation, is implemented as altering bit values at randomly selected string positions.

3..2.4 Control parameters

In this simulation, each parameter x_1, x_2 has 20 bits precision, namely, each population member consists of a 40-bit string. During all experiments, the population size is 50, the mutation rate is 0.1 and the stopping condition is 100 generations. In the Pareto partitioning method, the number of partitioning region is $50^2 = 2500$.

4. Design Examples

We consider these plants with transfer finctions

$$G_1(s) = \frac{K_p}{1+T_p s} e^{-Ls} \quad (11)$$

$$G_2(s) = \frac{K_p}{(1+T_{p1}s)(1+T_{p2}s)} e^{-Ls} \quad (12)$$

and assume that the set Q for controller is given by

$$Q = \{q \mid \underbrace{\begin{bmatrix} 0.10 \\ 1.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{bmatrix}}_{q_l} \le \underbrace{\begin{bmatrix} K_c \\ T_i \\ T_d \\ \alpha \\ \beta \end{bmatrix}}_{q} \le \underbrace{\begin{bmatrix} 15.90 \\ 100.00 \\ 100.00 \\ 1.00 \\ 1.00 \end{bmatrix}}_{q_u} \} \quad (13)$$

4..1 4.1 Numerical Result of $G_1(s)$

Let the uncertainty set of the plant parameters be given by

$$\Theta = \{\theta \mid \underbrace{\begin{bmatrix} 0.8 \\ 5.6 \end{bmatrix}}_{\theta_l} \le \underbrace{\begin{bmatrix} K_p \\ T_p \end{bmatrix}}_{\theta} \le \underbrace{\begin{bmatrix} 1.2 \\ 8.4 \end{bmatrix}}_{\theta_u} \} \quad (14)$$

We use the second order Padé approximation to the delay e^{-Ls} with $L = 1.0$.

The design result is given in Table 1.

Table 1 Design result ($G_1(s)$)

K_c	T_i	T_d	α	β
6.4277	3.0535	0.2638	0.8257	0.6958

Figures 3 and 4 show the step responses of closed-loop system controlled by the controller given in Table 1. Solid line shows the nominal case response. And the other dotted or dashed lines show the response of using the vertex values of Θ. We observe the good robust performance.

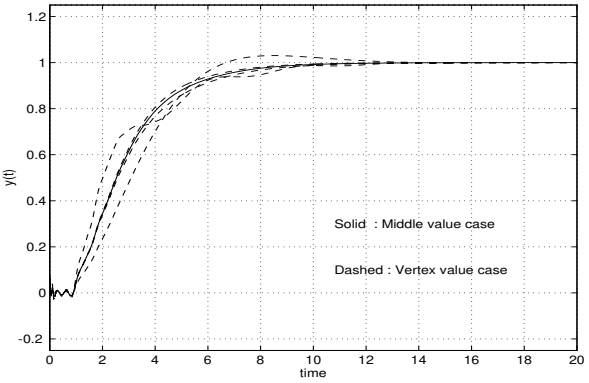

Figure 3 Step reference responses of closed-loop system ($G_1(s)$)

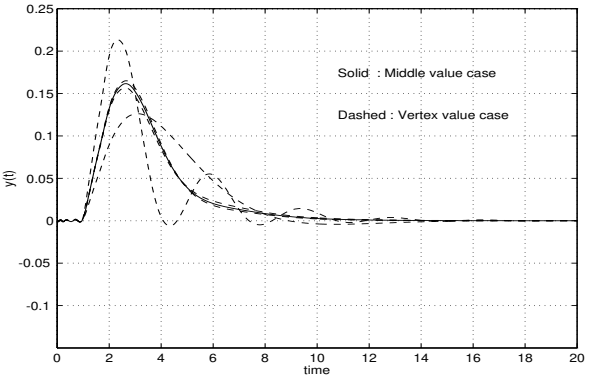

Figure 4 Step disturbance responses of closed -loop system ($G_1(s)$)

4..2 4.2 Numerical Result of $G_2(s)$

Let

$$\Theta = \{\theta \mid \underbrace{\begin{bmatrix} 0.8 \\ 4.57 \\ 1.16 \end{bmatrix}}_{\theta_l} \le \underbrace{\begin{bmatrix} K_p \\ T_{p1} \\ T_{p2} \end{bmatrix}}_{\theta} \le \underbrace{\begin{bmatrix} 1.2 \\ 6.85 \\ 1.74 \end{bmatrix}}_{\theta_u} \} \quad (15)$$

We use the first order Padé approximation to the delay e^{-Ls} with $L = 0.5$. The numerical result is shown in Table 2 and closed-loop responses are depicited in figures. 5 and 6.

Table 2 Design result ($G_2(s)$)

K_c	T_i	T_d	α	β
9.2382	4.7466	1.0857	0.4384	0.7957

Figures 5 and 6 show the step responses of closed-loop system controlled by the controller given in Table 2. We also observe the good robust preformance.

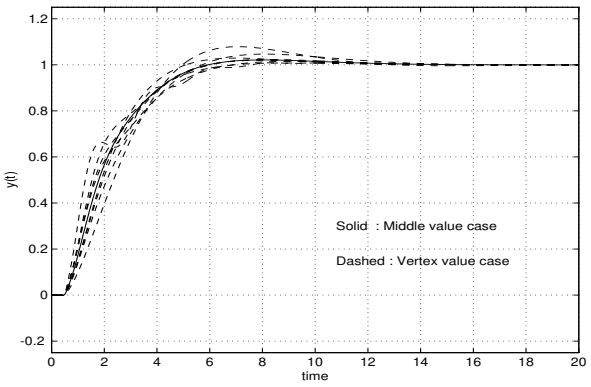

Figure 5 Step reference responses of closed-loop system ($G_2(s)$)

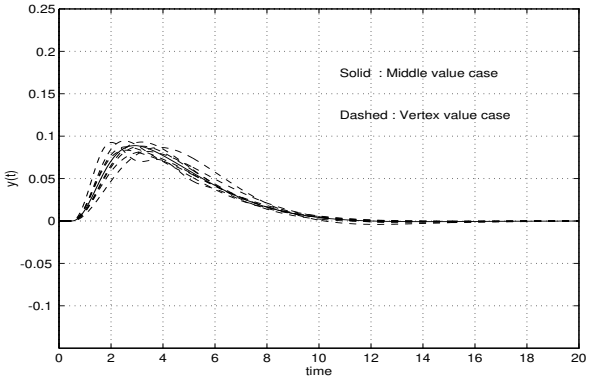

Figure 6 Step disturbance responses of closed -loop system ($G_2(s)$)

5. Conclusion

In this paper, we have proposed a new GA using pareto partitioning method in order to avoid the partial convergence of the solutions. The proposed Pareto partitioning method uniformly controls a distribution of non-dominated solutions in the objective space. And we also developed, using this new GA, the design method of robust partial model matching PID controller with two degrees of freedom. And we can recognize from several numerical simulations that the proposed method is effective for the process control.

Acknowledgement : Author would like to thank Prof. Tohru Katayama in Kyoto University for his steadfast encouragement and valuable advice.

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