FUZZY MULTI-OBJECTIVE AND MULTI-STAGE OPTIMIZATION
- AN APPLICATION OF FUZZY THEORY TO ARTIFICIAL LIFE -

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ABSTRACT : This paper presents two new methods of multi-objective optimization. One is an application of simplified genetic algorithm in which membership functions are employed as usual objective functions, and maximizing decision is performed for optimization. The other is a method of membership control in growth processes in which selection is performed also on the way to the final growth step. In such a case of the optimization in regard to the growth of trees, the latter method is proved to be more effective than the former one.

Key Words : Fuzzy Theory, Artificial Life, L-Systems, Genetic Algorithm, Tree Formation, Multi-Objective Optimization, Multi-Stage Optimization

1. INTRODUCTION

Langton proposed a very wide and progressive concept of artificial life [9] in which local interaction in genotypes (ex. chromosome) produces phenotypes (ex. behavior, form) through emergence. This theory is very effective to simulate nonlinear and complicated phenotypes in the real world. On the other hand, Zadeh proposed fuzzy set theory [15] which is very suitable for modeling and simulation of complicated events in the real world. The first purpose of this paper is to apply fuzzy theory to artificial life.

Cellular automata by von Neumann [11], L-Systems by Lindenmayer [10], and genetic algorithms by Holland [5] and Goldberg [4] can be included in artificial life. Fuzzy theory has been already applied to automata theory [14], cellular automata [1], and genetic algorithms [13,3,12]. As for L-Systems, a stochastic approach has been already tried [7]. However, these researches can not be attributed to total systems in which fuzzy theory is applied to artificial life.

The author is now studying on an application system of cellular automata and L-Systems to structural design [6] in which structures are considered to be growing systems. Furthermore, when genetic algorithms are applied to it, we can get an ideal and typical artificial life system. Bellman and Zadeh proposed a very simple optimization method called maximizing decision [2] which is considered to be applicable to genetic algorithms [12].

In this paper, based on the above theories, a very simple case study on fuzzy multi-objective and multi-stage optimization for tree formation is shown, in which scale-, mechanics-, reproduction-, and energy-conditions can be taken into account.

In artificial life, it is very essential that complicated creatures have their own histories of growth and / or evolution from primitive states. So, optimization and selection can be performed also on the way of growth and / or evolution.

The second purpose of this paper is to compare the usual genetic algorithm method at the final growth step with a proposed multi-stage optimization method in which selection is performed also on the way to the final growth step.

2. TREE FORMATION

In this paper, a very simple growth model of trees is employed. Every cell is originated from a seed and only terminal cells produce newer terminal ones. In the growth of cells, two extreme types can be assumed, i.e. axial growth (A-Type) and branching growth (B-Type) such as shown in Fig.1. According to the serial combinations of A and B-Types, trees grow up with various figures.

When four steps of growth are supposed, we can get 16 ($=2^4$) kinds of genotypes such as shown in Fig.2. The phenotypes of trees corresponding to the above genotypes are illustrated in Fig.3.

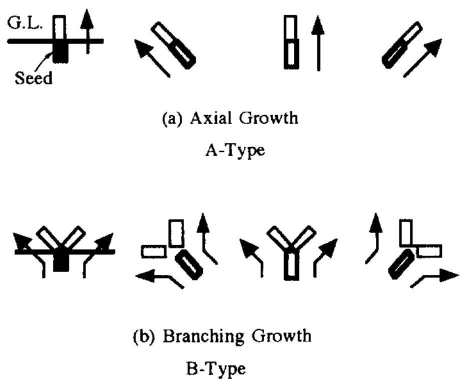


Fig. 1 Growth Types

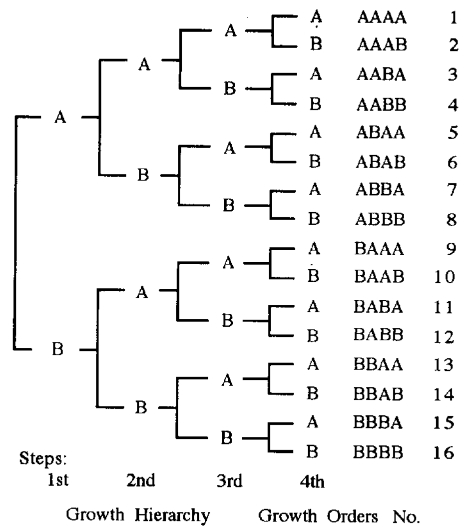


Fig. 2 Genotypes for Growth

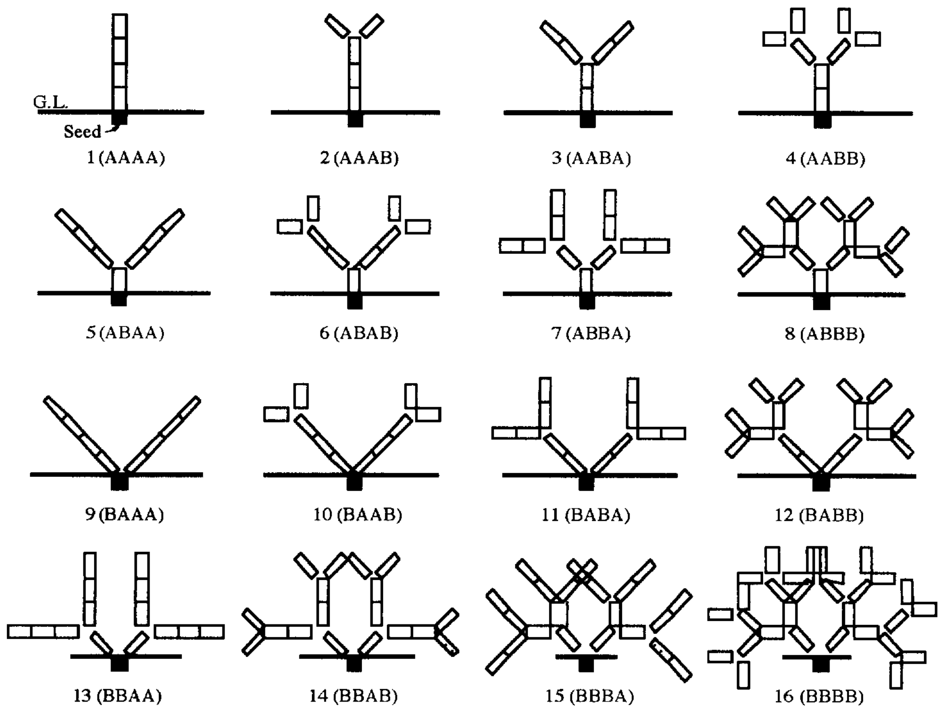


Fig. 3 Phenotypes of Trees

Two kinds of optimization processes, i.e. simplified genetic algorithm at the final growth step and preceding control by M.F. such as shown in Table1 are described in the next chapter.

3. FUZZY OPTIMAL FORMATION OF TREES

3.1. Simplified Genetic Algorithm

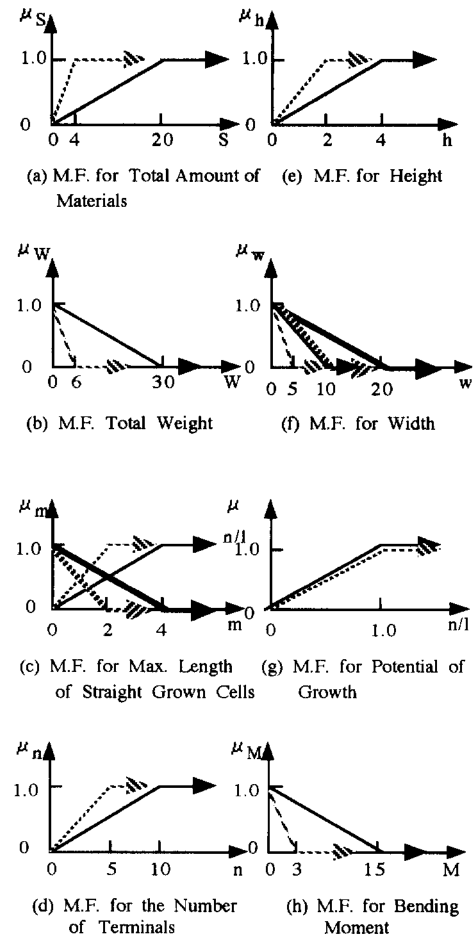
According to the systematized genetic algorithms [4], the following key words are needed: Objective Function, Coding, Reproduction, Crossover, and Mutation.

In this chapter, aiming at multi-objective optimization [12] objective functions are given by means of 8 kinds of membership functions employed in fuzzy theory [15] as shown in Fig.4, where thin and thick solid lines are used in this chapter (dotted lines will be used in the next chapter). Here, the following variables of membership functions are taken into account:

- S: total amount of materials
- W: total weight acting on seeds
- m: maximum length of straight grown cells
- n: number of terminals
- h: height
- w: width
- n/l: potential of growth (l is total number of cells)
- M: bending moment acting on initial branching point symmetrically

These values are measured by the fundamental size and weight of a cell unit. For simplicity, the length and weight of a cell unit are assumed to be the unity and its width zero. Total amount of material S is proportional to l, so S is assumed to be equal to l. Of course, W is also equal to l. The potential of growth n/l is defined as the possibility of getting energy through leaves supposed to be located on terminals.

The membership functions of S, m, n, h, and n/l (solid thin lines in Fig.4) are assumed according to the thought that the more the better. The membership



(a) M.F. for Total Amount of Materials (e) M.F. for Height

(b) M.F. Total Weight (f) M.F. for Width

(c) M.F. for Max. Length of Straight Grown Cells (g) M.F. for Potential of Growth

(d) M.F. for the Number of Terminals (h) M.F. for Bending Moment

Fig. 4 Membership Functions for Multi-Objectives

- —— for Final Optimization ; Practical Use
- ----- for Pre. and Final Optimizations ; Practical Use
- ▬▬ for Final Optimization ; Appreciation
- \\\\\\\\ for Pre. and Final Optimizations, for Appreciation

Table 1 Two Optimization Methods of Simplified Genetic Algorithms and Preceding Control by M.F.

Step	1 →	2 →	3 →	4
Genotype	□	□□	□□□	□□□□
Simplified Genetic Algorithm (Number of Different Phenotypes)	— (2)	— (4)	— (8)	Optimization (16)
Preceding Control by M.F. (Number of Different Phenotypes)	— (2)	Optimization (4)	— (2)	Optimization (4)

functions of W, w, and M are determined according to the thought that the less the better.

The membership functions of m and w intend to make trees slender and straight for practical use, e.g. availability as structural elements. On the other hand, when tree are evaluated for appreciation, people prefer branching growth to straight growth and the width of trees may be allowed to be more wider than for practical use. So, for reference, the membership functions of m and w are modified as shown by the solid thick lines, and optimal formation of trees can be performed in the two cases, i.e. for practical use and for appreciation.

As for codings, in this paper, binary genetic codings are employed corresponding to genotype numbers 1~16 as shown in Table 2.

Reproduction, crossover, and mutation are performed as shown in Fig.5, in which population size is four and □ shows 1 or 0. Optimization processes are shown as follows:

1. 1 or 0 is selected 16 times randomly and four binary genetic codings are determined.
2. Referring to Table 2, genotype numbers are given and their phenotypes' characteristic values S, W, m, n, h, w, n/l, and M can be calculated.
3. By using the membership functions in Fig.4, membership values, $\mu_S \sim \mu_M$, can be obtained, and let the minimum of these membership values in regard to each genotype be μ^1_j (j=1,2,3,4).
4. The genotype with the minimum $\mu^1_4(\mu^1_4 \leqq \mu^1_1, \mu^1_2, \mu^1_3)$ is submitted to mutation and the others to crossover as shown in Fig.5.
5. In regard to seven genotypes reproduced,

Table 2 Binary Genetic Coding for Genotype Nos.

Genotype Nos.	Binary Genetic Codings	Genotype Nos.	Binary Genotype Codings
1	0000	9	1000
2	0001	10	1001
3	0010	11	1010
4	0011	12	1011
5	0100	13	1100
6	0101	14	1101
7	0110	15	1110
8	0111	16	1111

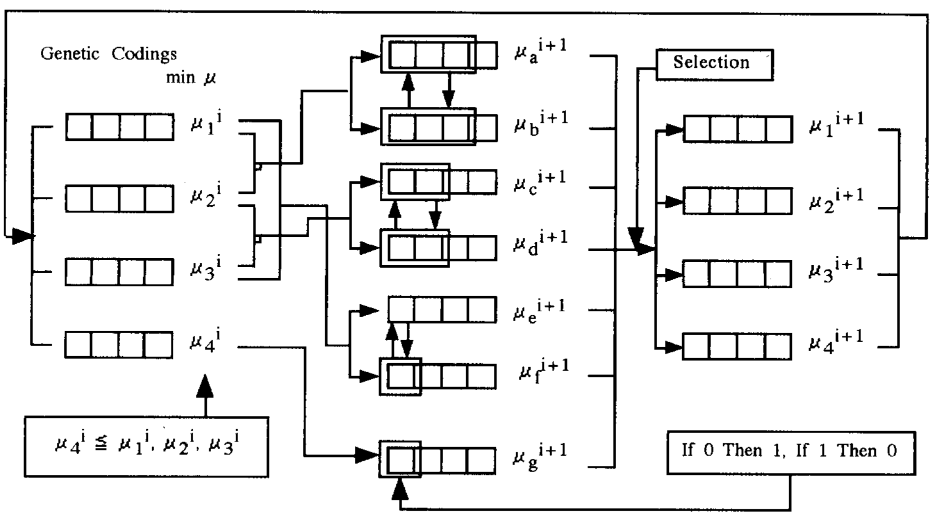


Fig. 5 Reproduction, Crossover, and Mutation
(in Case of Population Size 4)

minimum membership values, $\mu^2{}_a \sim \mu^2{}_g$, can be calculated by using the membership functions in Fig.4.

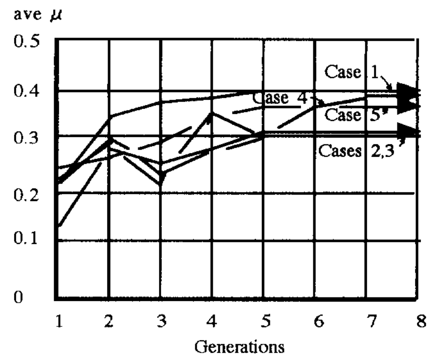
6. Finally, we can select four genotypes with the first to fourth largest membership values, $\mu^2{}_1 \sim \mu^2{}_4$, among $\mu^2{}_a \sim \mu^2{}_g$, and this 6th step can replace the first step.
7. The superscript i in Fig.5 means the ordinal number of generations, and the alternation of generations has to be continued until selected genotypes remain unchanged.

Fig.6 shows optimization processes expressed by generation membership values, aveμ(=($\mu^i{}_1+\mu^i{}_2+\mu^i{}_3+\mu^i{}_4$)/4). Fig.6 (a) shows 5 cases of the processes for practical use, and Fig.6 (b) for appreciation. The finally selected genotypes in each case are shown in Table 3. For reference, the ideal optimal genotypes are also shown in Table 3, and they are derived from the whole lists of characteristic values for genotypes at the fourth step (Table 4) and of maximizing decision of optimal genotypes (Table 5).

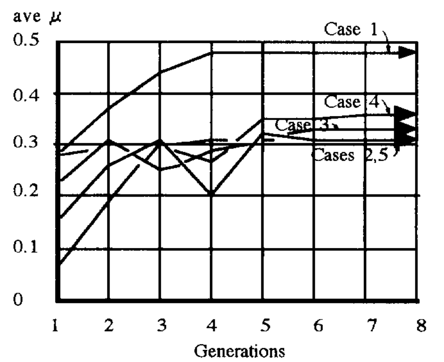
These results show that the employed simplified genetic algorithm gives true or near optimal genotypes which are caused by accident.

3.2. Preceding Control by Membership Functions

In the previous section, a simplified genetic algorithm is applied to the optimization of geno- and phenotypes at the final step. However, in reality, selection should be carried out on the way to the final growth step, too. Furthermore, in this case study, the number of different figures of trees increases with the increase of the number of steps and it is reasonably suggested that the selection on the way may be easy



(a) for Practical Use



(b) for Appreciation

Fig. 6 Optimization Processes by Generation Average Membership Values

Table 3 Selected Optimal Genotypes
(by Fuzzy Multi-Objective Genetic Algorithm)

	Selected Genotypes(No.) for Practical Use	Number	Selected Genotypes(No.) for Appreciation	Number
Case1	ABAB (6)	4	ABBB (8)	4
Case 2	BABA (11)	4	ABAB (6) ABAA (5) BBAA (13)	2 1 1
Case 3	BABA (11)	4	BABA (11)	4
Case 4	ABAB (6)	4	ABBA (7)	4
Case 5	BAAB (10)	4	ABBB (7) BAAB (10)	1 3
Ideal Optimal Genotypes	AABB (4) ABAB (6)	2 2	ABBB (8)	4

Table 4 Characteristic Values for Genotypes (at the Fourth Step)

Genotype Numbers	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Total Number of Cells l	4	5	6	8	7	9	11	15	8	10	12	16	14	18	22	30
Total Amount of Materials S(=l)	4	5	6	8	7	9	11	15	8	10	12	16	14	18	22	30
Total Weight W(=l)	4	5	6	8	7	9	11	15	8	10	12	16	14	18	22	30
Max. Length of Straight Lines m	4	3	2	2	3	2	2	1	4	3	2	2	3	2	2	1
Number of Terminals n	1	2	2	4	2	4	4	8	2	4	4	8	4	8	8	16
Height h	4	3.71	3.42	3.71	3.13	3.42	3.71	3.42	2.84	3.13	3.42	3.13	3.71	3.42	3.13	3.42
Width w	0	1.42	2.84	3.42	4.26	4.84	5.42	4.84	5.68	6.26	6.84	6.26	7.42	6.84	6.26	6.84
Potential of Growth n/l	0.25	0.40	0.33	0.50	0.29	0.44	0.36	0.53	0.25	0.40	0.33	0.50	0.29	0.44	0.36	0.53
Bending Moment M	0	0.35	1.42	2.27	3.18	4.76	5.19	7.81	5.68	7.94	9.10	13.14	9.11	12.73	14.79	21.33

Table 5 Maximizing Decision of Optimal Genotypes (at the Fourth Step)

Genotype Nos.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
μ_S	0.2	0.25	0.3	0.4	0.35	0.45	0.55	0.75	0.40	0.50	0.60	0.80	0.70	0.90	1.0	1.0
μ_W	0.87	0.83	0.80	0.73	0.77	0.70	0.63	0.50	0.73	0.67	0.60	0.47	0.53	0.40	0.27	0
μ_m	1.0 (0)	0.75 (0.25)	0.5 (0.5)	0.5 (0.5)	0.75 (0.25)	0.5 (0.5)	0.5 (0.5)	0.25 (0.75)	1.0 (0)	0.75 (0.25)	0.5 (0.5)	0.5 (0.5)	0.75 (0.25)	0.5 (0.5)	0.5 (0.5)	0.25 (0.75)
μ_n	0.1	0.2	0.2	0.4	0.2	0.4	0.4	0.8	0.2	0.4	0.4	0.8	0.4	0.8	0.8	1.0
μ_h	1.0	0.93	0.86	0.93	0.78	0.86	0.93	0.86	0.71	0.78	0.86	0.78	0.93	0.86	0.78	0.86
μ_w	1.0 (1.0)	0.86 (0.93)	0.72 (0.86)	0.66 (0.83)	0.57 (0.79)	0.52 (0.76)	0.46 (0.73)	0.52 (0.76)	0.43 (0.72)	0.37 (0.69)	0.32 (0.66)	0.37 (0.69)	0.26 (0.63)	0.32 (0.66)	0.37 (0.69)	0.32 (0.66)
$\mu_{n/l}$	0.25	0.40	0.33	0.50	0.29	0.44	0.36	0.53	0.25	0.40	0.33	0.50	0.29	0.44	0.36	0.53
μ_M	1.0	0.98	0.91	0.85	0.79	0.68	0.65	0.48	0.62	0.47	0.39	0.12	0.39	0.15	0.01	0
minμ	0.1 (0)	0.2 (0.2)	0.2 (0.2)	0.4 (0.4)	0.2 (0.2)	0.4 (0.4)	0.36 (0.36)	0.25 (0.48)	0.2 (0)	0.37 (0.25)	0.32 (0.33)	0.12 (0.12)	0.26 (0.25)	0.15 (0.15)	0.01 (0.01)	0 (0)

[( ) in case of optimization for appreciation]

and effective because of the small total population size (4 in this case).

In this paper, multi-stage selections are performed twice, i.e. at the second and fourth steps.

Before such selections, we have to adjust the membership functions in Fig.4 for the fourth step to the ones for the second step. Generally speaking, the characteristic values of phenotypes of trees, S, W, and M, increase exponentially with the increase of steps, and m, n, h, and w increase proportionally to the number of steps. So, n/l decreases with the increase of steps. Here, for simplicity, reduction factors are assumed to be 1/5 for S, W, and M, 1/2 for m, n, h, and w, and 1 for n/l. The membership functions assumed for the second step are shown in Fig.4 (dotted lines).

The characteristic values for genotypes at the second step is shown in Table 6 and the corresponding membership values are shown in Table 7, which, after maximizing decision, implies that genotype Nos. 5~8 are suitable for survival in both the cases for practical use and for appreciation.

The characteristic values and membership values of genotype Nos.5~8 at the fourth step are shown in Tables 3 and 4. After maximizing decision, genotype Nos.5 and 8 are suitable for survival in the case for practical use and for appreciation, respectively. These selected results are shown and compared with the ideal optimal genotypes in Table 8.

These results show that the proposed method of preceding control by membership functions is very effective for optimization. In this case study, each population size at each optimization step is so small (four) that it is not necessary to use genetic algorithms.

Table 6 Characteristic Values for Genotypes (at the Second Step)

Genotype Nos.	1~4	5~8	9~12	13~16
l	2	3	4	6
S(=l)	2	3	4	6
W(=l)	2	3	4	6
m	2	1	2	1
n	1	2	2	4
h	2	1.71	1.42	1.71
w	0	1.42	2.84	3.42
n/l	0.5	0.67	0.5	0.67
M	0	0.35	1.42	2.27

Table 7 Maximizing Decision of Optimal Genotype (at the Second Step)

Genotype Nos.	1~4	5~8	9~12	13~16
$\mu_{S(=l)}$	0.5	0.75	1.0	1.0
$\mu_{W(=l)}$	0.67	0.5	0.33	0.0
μ_m	1.0 (0)	0.5 (0.5)	1.0 (0)	0.5 (0.5)
μ_n	0.2	0.4	0.4	0.8
μ_h	1.0	0.86	0.71	0.86
μ_w	1.0 (1.0)	0.72 (0.86)	0.43 (0.72)	0.32 (0.66)
$\mu_{n/l}$	0.5	0.67	0.5	0.67
μ_M	1.0	0.88	0.53	0.24
minμ	0.2 (0)	0.4 (0.4)	0.33 (0)	0 (0)

[( ) in case of optimization for appreciation]

Table 8 Selected Optimal Genotypes (by Membership Control in Growth Processes)

	Selected Genotypes(No.) for Practical Use	Selected Genotypes(No.) for Appreciation
2nd Step	AB (5~8)	AB(5~8)
4th Step	ABAB (6)	ABBB (8)
Ideal Optimal Genotypes	AABB (4) ABAB (6)	ABBB (8)

4. DISCUSSIONS AND CONCLUSIONS

The proposed method to optimize the figures of trees has the following features.

(1) Multi-objective optimization is performed.
(2) Membership functions are employed as objective functions for genetic algorithms. Membership values mean the possibility for survival and not the probability for survival.
(3) A new optimization method of preceding control by membership functions on the way of growth processes is proposed and it is proved that this method is more effective than genetic algorithms performed only at the final step.
(4) Generally speaking, it is very effective to employ the preceding control method and genetic algorithms at the same time. Although the case study carried out in this paper is very primitive, the fundamental ideas mentioned above are applicable to the optimization of bigger and more complicated systems.

In this paper, a very effective application method of fuzzy theory to artificial life could be shown, and it may be useful for the intel-life co-generating system for the design of architectural structures proposed by the author [8].

ACKNOWLEDGMENTS

The author is very grateful to Dr. Akinori Tani and Miss Kazuko Kamada for their help in preparing this paper. This study was supported by the National Science Foundation of the Ministry of Education of Japan.

REFERENCES

[1] Adamatzky, A. I. : Hierarchy of Fuzzy Cellular Automata, Fuzzy Sets and Systems, 62, 1994, pp.167-174.
[2] Bellman, R. E., and Zadeh, L.A. : Decision-Making in a Fuzzy Environment, Management Science, 17, 4, 1970, pp.141-164.
[3] Buckley, J. J. : Fuzzy Genetic Algorithm and Applications, Fuzzy Sets and Systems 61, 1994, pp.129-136.
[4] Goldberg, D. E. : Genetic Algorithms, in Search, Optimization & Machine Learning, Addison-

Wesley Pub. Comp., 1989.
[5] Holland, J. H. : Adaptation in Natural and Artificial Systems, University of Michigan Press 1975, MIT Press, 1992.
[6] Ichikawa, S., Kawamura, H. and Tani, A. : Automatic Formation of Rigid Frames by Genetic Rules (An Application of Artificial Life), Extended Abstracts, International KATACHI U SYMMETRY, 1994. (to be published)
[7] Jürgensen, H., and Matthews, D. E. : Stochastic OL Systems and Formal Power Series, The Book of L (ed. by Rozenberg, G., and Salomaa, A.), Springer-Verlag, 1986, pp.167-177.
[8] Kawamura, H. : An Intel-life Co-Generating System for the Design of Architectural Structures and Forms, Proc. A, Annual Meeting of Architectural Institute of Japan, Sept. 1994, pp.1679-1680. (in Japanese)
[9] Langton, C. G. : Artificial Life, in Artificial Life (ed. by Langton, C. G.), Addison-Wesley Pub. Comp., 1988, pp.1-47.
[10] Lindenmayer, A. : Mathematical Models for Cellular Interactions in Development, I. and II., J. Theoret. Biol., 18, 1968, pp.280-315.
[11] von Neumann, J. : Theory of Self-Reproducing Automata, University of Illinois Press, 1966.
[12] Sakawa, M., Inuiguchi, M., Sunada, H., and Sawada, K. : Multiobjective Combinatorial Optimization through Revised Genetic Algorithms, J. of Japan Society for Fuzzy Theory and Systems, Vol.6, No.1, Oct. 1993, pp.177-186. (in Japanese)
[13] Sanchez, E. : Fuzzy Genetic Algorithms in Soft Computing Environment, Fifth International Fuzzy Systems Association World Congress'93, Vol.I, pp.XLIV-XLX.
[14] Wee, W. G. and Fu, K. S. : A Formulation of Fuzzy Automata and its application as a Model of Learning Systems, IEEE Trans. on Systems Science and Cybernetics, Vol.ssc-5, No.3, July 1969.
[15] Zadeh, L. A. : Fuzzy Sets, Information and Control, Vol.8, 1965, pp.338-353.