# A New Genetic Algorithm Based Control Method Using State Space Reconstruction

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## ABSTRACT

A new control method using Genetic Algorithms (GAs) to reduce the structural response under seismic excitation is proposed. The proposed control method uses state space reconstruction technique to obtain the full state performance from the available reduced order feedback. The controller is optimized using GAs without making simplifying assumptions. The method has been used on a benchmark problem – an active mass driver system. The results and advantages of the proposed method are discussed in this paper. The robustness of the controller developed by this method has also been examined.

## 1. INTRODUCTION

Most optimization methods used in control design are traditional gradient based search methods. With this approach there are difficulties associated in selecting the suitable continuous differentiable cost function and in considering nonlinearities (Gray et al. 1995).

Compared with the traditional gradient based search methods, Genetic Algorithms (GAs) efficiently find an optimal value from the complex and possibly discontinuous solution space because the fitness function is the only information required of the problem. GAs do not require reformulating the problem into a suitable form unlike traditional gradient based search techniques. As a result, GAs provide a lot of flexibility in the controller design and optimization.

In the field of control design, GAs have been used successfully to obtain gains for optimal controller (Kundu and Kawata 1996), tune the weights of neuro–controllers (Lewis and Fagg 1992), and scale parameters of fuzzy controllers (Kim et al. 1995).

For the control of the civil structures, a new GA based control method has been proposed (Kim and Ghaboussi, 1997). The proposed control method estimates the system states from the observed time series data using the state space reconstruction technique which is based on the embedding theorem.

For the verification, the method has been examined on the benchmark problem – active mass driver system (Spencer 1997). The robustness of the proposed control method has also been examined.

## 2. GENETIC ALGORITHMS

GAs are computational methods which are inspired by natural Darwinian evolution. In GAs organisms or chromosomes evolving under a certain environment are represented by bit strings. A string is composed of genes which are an encoding of the parameters of the problems which are being optimized. Strings evolve over generations to adapt to a given environment by GA operators, which are designed to model the evolutionary forces such as competition, natural selection, reproduction, recombination and mutation.

In every generation, a set of strings is selected into the mating pool based on their relative fitness. The fitter strings are given more chance of passing their genes into the next generation. This process of natural selection, i.e. survival of the fittest is operated by selective reproduction. New strings are created by exchanging the genes between two old strings (cross–over). Mutation operator is applied at a specified low rate to change the randomly selected genes in the new generation. While these operations are randomized, GAs are no simple random walk. They are probabilistic search techniques which explore new search points as well as keep the historical information of the search space (Goldberg 1989).

GAs are very simple but powerful methods compared with the traditional gradient based search methods because GAs do not need the reformulation of the problem to search a nonlinear and non–differentiable space. The flexibility in the formulation of the fitness function is also one of the advantages of GAs. The fitness function can be formulated as a polynomial function of the output of the system to be optimized. Therefore, by using GAs, multiple optimal design criteria can be considered by simply including them in the fitness function.

## 3. STATE SPACE RECONSTRUCTION FOR SYSTEM ESTIMATION

A new method to estimate the states from the observed time series data suitable to GA based control has been developed. The method is based on Takens' embedding theorem (Takens 1981), which states that the observed time series data can be used to reconstruct the state space and dynamical information (attractor) of the underlying system. The $n$–dimensional reconstructed state space at time step t is defined as $\mathbf{W}^n(t)$ by the following equation in terms of the one–dimensional observed time series w(t) with time delay $\tau$.

$$\mathbf{W}^n(t) = \{w(t)\ w(t-\tau)\ ....\ w(t-[n-1]\tau)\}^{\mathbf{T}} \tag{1}$$

The reconstructed state space is not the same as the original state space. However, it can characterize the attractor of the original system for sufficiently large value of $n$. When the dimension of the original state space is $k$, Takens has suggested a value of $n > 2k$. However, the embedding often works well for smaller value of $n$ (Langi and Kinsner 1995). The optimum value of the time delay $\tau$ is still an open question. Generally, $\tau$ depends on the statistical correlation among samples. Larger value of $\tau$ can be used when the statistical correlation is high (Langi and Kinsner 1995).

## 4. THE PROPOSED GA BASED CONTROL METHOD

The control law used in the proposed GA based control method is given in the following equation.

$$\Delta\mathbf{u}(t) = f[\mathbf{u}(t-\tau), \mathbf{u}(t-2\tau), \ldots\ldots, \mathbf{u}(t-m\tau), \mathbf{y}(t), \mathbf{y}(t-\tau), \ldots\ldots, \mathbf{y}(t-[n-1]\tau)] \quad (2)$$

In this equation the control signal increment $\Delta\mathbf{u}$ is a function of the reconstructed states of the control input vector $\mathbf{u}$ and measured response vector $\mathbf{y}$. In the proposed GA based control method, the characterization of the attractor of the original system and the optimization of the controller's parameters are implemented simultaneously.

## 5. CONTROLLER DESIGN

### 5.1. Evaluation Model and Control Constraints

The proposed GA based control method has been evaluated on a benchmark problem. The structure considered in the benchmark problem is a scale model of a three story building using an active mass driver as a control device. The state space parameters of this structural system, including the actuator and sensor dynamics, have been obtained from the experiment. More details on the benchmark problem can be found in the reference (Spencer et al. 1997).

Control constraints are placed on the system for a realistic numerical simulation. The primary constraints (hard constraints) depend on the physical characteristics of the experimental setup and the capacity of the actuator. The RMS constraints and the peak response constraints are listed in Eqs. (3) and (4) respectively. In Eq. (3) $\sigma$ represent the RMS value of its subscript.

$$\sigma_u \le 1 \text{ volt}, \; \sigma_{\ddot{x}_{am}} \le 2 \text{ g and } \sigma_{x_m} \le 3 \text{ cm} \quad (3)$$

$$\max|u| \le 3 \text{ volt}, \; \max|\ddot{x}_{am}| \le 6 \text{ g and } \max|x_m| \le 9 \text{ cm} \quad (4)$$

The additional constraints (control implementation constraints), which depend on the sensors and the controller computer, are as follows.

**Table 1.** Control implementation constraints on the benchmark problem – AMD

| Sampling Time | Computational Time Delay | A/D & D/A Converter | Sensor Noise |
|---|---|---|---|
| 0.001 seconds | 200 μ seconds | 12 bit precision<br>±3 volts span | 0.01 RMS noise |

### 5.2. GA Based Controller

For control feedback, we have chosen to use four sensors which measure the absolute accelerations of the three floors, $\ddot{x}_{a1}$, $\ddot{x}_{a2}$, $\ddot{x}_{a3}$, and the absolute acceleration of the AMD mass, $\ddot{x}_{am}$. The feedback vector $\mathbf{y}(t)$ contains the following four sensor readings at time t.

$$\mathbf{y}(t) = \{\ddot{x}_{a1} \; \ddot{x}_{a2} \; \ddot{x}_{a3} \; \ddot{x}_{am}\} \quad (5)$$

By using the reconstructed state feedback, we are using the current vector of sensor reading, $\mathbf{y}(t)$, plus the previous samples of sensor readings. Therefore, the dimension of the reconstructed state space of the four sensor feedback will be equal to $4 \times n$ with $n$–1 previous time histories.

$$\mathbf{Y}^{4\times n}(t) = \{\mathbf{y}(t)\ \mathbf{y}(t-\tau)\ ....\ \mathbf{y}[t-(n-1)\tau]\}^{\mathbf{T}} \tag{6}$$

The proposed controller also uses previous values of control signals as a feedback. One role of this feedback is to make the control signal not deviate too much from the zero signal in the incremental form of the control law used in this study as in Eq. (2).

$$\mathbf{U}^{m}(t-\tau) = \{u(t-\tau)\ u(t-2\tau)\ ....\ u(t-m\tau)\}^{\mathbf{T}} \tag{7}$$

Currently, there is no rigorous method to determine the values of $m$ and $n$. For this study we have chosen to use the trial and error method. In the remainder of this study, we have used the 23–dimensional reconstructed state space (m=3, n=5) which consists of 20–dimensional reconstructed state space vector $\mathbf{Y}^{4\times 5}(t)$ and the 3–dimensional reconstructed state space vector $\mathbf{U}^{3}(t-\tau)$.

The control input is calculated from Eq. (8) with the additional constraint from the saturation of the actuator which requires that $|u| \leq +3$ volts as a limit.

$$u(t) = u(t-\tau) + \Delta u(t), \text{ where } \Delta u(t) = \mathbf{G}_R \begin{Bmatrix} \mathbf{Y}^{4\times 5}(t) \\ \mathbf{U}^{3}(t-\tau) \end{Bmatrix} \tag{8}$$

The controller gain matrix $\mathbf{G}_R$ has 23 elements as follows. The elements of the gain matrix $\mathbf{G}_R$ are optimized through evolution by using GAs.

$$\begin{aligned} &\mathbf{G}_R = [\ \mathbf{G}_1\ \mathbf{G}_2\ ] \\ &\mathbf{G}_1 = [g_1\ g_2\ ....\ g_{19}\ g_{20}] \qquad \mathbf{G}_2 = [g_{21}\ g_{22}\ g_{23}] \end{aligned} \tag{9}$$

## 5.3. GA Parameters

The simple GA (Goldberg 1989) is used to optimize the feedback gains. Ten bits are used to represent each gain as a real number by mapping, making the string length equal to 230 bits. The population size was 50 and the evolution was continued up to 1000 generations. Genetic operators used are: fitness proportional (roulette wheel type) random reproduction, two point cross–over at a rate of 0.8 and mutation at a rate of 0.003.

## 5.4. Fitness Function

The fitness function F is a nonlinear polynomial which consists of powered products of the normalized peak and RMS values of the responses of floors and the AMD. Each criterion in $C_1 - C_3$ has been designed to converge to 1.0 when the corresponding system response is reduced to zero.

$$C_1 = \prod_{i=1,3} \left(1 + \frac{|\ddot{x}_{ai}|_{max}}{\beta_i}\right)^{\alpha_i} \cdot \left(1 + \frac{|x_i|_{max}}{\delta_i}\right)^{\gamma_i} \cdot \left(1 + \frac{\sigma_{\ddot{x}_{ai}}}{\zeta_i}\right)^{\epsilon_i} \cdot \left(1 + \frac{\sigma_{x_i}}{\theta_i}\right)^{\eta_i} \tag{10}$$

$$C_2 = \left(1 + \frac{P_1(|\ddot{x}_{am}|_{max})}{\beta_m}\right)^{\alpha_m} \cdot \left(1 + \frac{P_2(|x_m|_{max})}{\delta_m}\right)^{\gamma_m} \cdot \left(1 + \frac{\sigma_{\ddot{x}_{am}}}{\zeta_m}\right)^{\epsilon_m} \cdot \left(1 + \frac{\sigma_{x_m}}{\theta_m}\right)^{\eta_m} \tag{11}$$

$$C_3 = \left(1 + \frac{\sigma_u}{\omega}\right)^{\psi} \tag{12}$$

For the evaluation of the fitness, peak accelerations, peak displacements, RMS accelerations and RMS displacements of the three floors and active mass driver and RMS value of control signal are used as the parameters of the cost function as in Eqs. (10) – (12). The denominators $\beta$, $\delta$, $\zeta$, $\theta$, and $\omega$ are the normalization factors, and powers $\alpha$, $\gamma$, $\varepsilon$, $\eta$, and $\psi$ are the exponential weight factors used to adjust the weight of responses which are to be reduced according to the control objective. In this study the factors are chosen by trial and error as follows: $\beta_i$=2.0, $\beta_m$=1.0, $\delta_i$=$\delta_m$=1.0, $\zeta_i$=2.0, $\zeta_m$=1.0, $\theta_i$=$\theta_m$=1.0, and $\omega$=1.0 for normalization, and $\alpha_i$=1.0, $\alpha_m$=3.0, $\gamma_i$=1.0, $\gamma_m$=2.0, $\varepsilon_i$=$\varepsilon_m$=1.5, $\eta_i$=$\eta_m$=1.0, and $\psi$=1.0 for the exponential weight factors.

$$F = \frac{C_{ref}}{C_T} \qquad C_T = \prod_{i=1,3} C_i \tag{13}$$

$C_T$ in Eq. (13) is the total cost, and the fitness F is the inverse of the total cost with a normalization factor $C_{ref}$ (=1.0 in this study).

## 5.5. Penalty Function

The penalty function has been successfully used for solving the constrained optimization problem by several researchers (Homaifar et al. 1994; Gray et al. 1995). In the early stages of evolution, the penalty function confines the search space by adding a large penalty value to the cost function. As a result, GAs search the fittest solution within the space that satisfies constraints after a few generations.

This penalty function is employed to impose the benchmark problem's hard constraints, i.e. maximum displacement and acceleration of AMD. Functions $P_1$ and $P_2$ in Eq. (11) are the penalty functions.

$$P_1(|\ddot{x}_{am}|_{max}) = \begin{cases} |\ddot{x}_{am}|_{max} & \text{for } |\ddot{x}_{am}|_{max} \le 6\text{ g}, \\ 50 & \text{for } |\ddot{x}_{am}|_{max} > 6\text{ g} \end{cases} \tag{14}$$

$$P_2(|x_m|_{max}) = \begin{cases} |x_m|_{max} & \text{for } |x_m|_{max} \le 9\text{ cm} \\ 50 & \text{for } |x_m|_{max} > 9\text{ cm} \end{cases} \tag{15}$$

## 6. EVALUATION CRITERIA

Root mean square and peak responses are used as the evaluation criteria of control efficiency. Ten criteria are defined in the benchmark problem (Spencer et al. 1997), and they are normalized by the corresponding worst–case responses of the third floor. These criteria are summarized in Table 2.

From $J_1$ to $J_5$ in Table 2, $\sigma_{x_{3o}}$ = 1.31 cm, $\sigma_{\dot{x}_{3o}}$ = 47.9 cm/sec and $\sigma_{\ddot{x}_{a3o}}$ = 1.79g are the worst case stationary RMS displacement, velocity and acceleration of the third floor of the uncontrolled building with parameters of $\omega_g$ = 37.3 rad/sec, and $\zeta_g$ = 0.3. RMS responses are computed using MATLAB/SIMULINK up to 300 seconds.

Evaluation criteria for the peak responses are non–dimensionalized with respect to the corresponding uncontrolled peak third floor responses. For the El Centro earthquake, $x_{3o}$ = 3.37 cm, $\dot{x}_{3o}$ =131 cm/sec, and $\ddot{x}_{a3o}$ = 5.05 g . For the Hachinohe Earthquake, $x_{3o}$ = 1.66 cm, $\dot{x}_{3o}$ = 58.3 cm/sec, and $\ddot{x}_{a3o}$ = 2.58 g are used.

**Table 2.** Evaluation criteria of benchmark problem

| $J_1$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ |
|---|---|---|---|---|
| $\max\limits_{\omega_g,\zeta_g,i=1,3}\left\{\dfrac{\sigma_{d_i}}{\sigma_{x_{3o}}}\right\}$ | $\max\limits_{\omega_g,\zeta_g,i=1,3}\left\{\dfrac{\sigma_{\ddot{x}_{ai}}}{\sigma_{\ddot{x}_{a3o}}}\right\}$ | $\max\limits_{\omega_g,\zeta_g}\left\{\dfrac{\sigma_{x_m}}{\sigma_{x_{3o}}}\right\}$ | $\max\limits_{\omega_g,\zeta_g}\left\{\dfrac{\sigma_{\dot{x}_m}}{\sigma_{\dot{x}_{3o}}}\right\}$ | $\max\limits_{\omega_g,\zeta_g}\left\{\dfrac{\sigma_{\ddot{x}_{am}}}{\sigma_{\ddot{x}_{a3o}}}\right\}$ |
| $J_6$ | $J_7$ | $J_8$ | $J_9$ | $J_{10}$ |
| $\max\limits_{\substack{t,\, i=1,3\\ \text{El Centro}\\ \text{Hachinohe}}}\left\{\dfrac{\lvert d_i(t)\rvert}{x_{3o}}\right\}$ | $\max\limits_{\substack{t,\, i=1,3\\ \text{El Centro}\\ \text{Hachinohe}}}\left\{\dfrac{\lvert \ddot{x}_{ai}(t)\rvert}{\ddot{x}_{a3o}}\right\}$ | $\max\limits_{\substack{t\\ \text{El Centro}\\ \text{Hachinohe}}}\left\{\dfrac{\lvert x_m(t)\rvert}{x_{3o}}\right\}$ | $\max\limits_{\substack{t\\ \text{El Centro}\\ \text{Hachinohe}}}\left\{\dfrac{\lvert \dot{x}_m(t)\rvert}{\dot{x}_{3o}}\right\}$ | $\max\limits_{\substack{t\\ \text{El Centro}\\ \text{Hachinohe}}}\left\{\dfrac{\lvert \ddot{x}_{am}(t)\rvert}{\ddot{x}_{a3o}}\right\}$ |

$\sigma$ – RMS value of its subscript $d_i$ – Inter–story drift at *i*th floor

## 7. NUMERICAL RESULTS

Numerical simulations of the proposed GA based controllers have been performed on the benchmark problem. Two controllers have been developed in this study. They have the same architecture, and have been developed with the same GA parameters, fitness and penalty function. However, one has been developed without sensor noise (Case A), and the other considers sensor noise (Case B) while optimizing control gains to improve the robustness of the controller. The measurement noise used in Case B is white noise ranging from –0.1 to 0.1 Volts. The RMS values of the white noise is 0.0577 Volts, which is approximately 1.9% of the full span of the A/D converters. It is about 5.8 times larger than the RMS noise used in the benchmark problem as the implementation constraint.

To develop each controller, only the El Centro earthquake ground motion data provided by the benchmark problem has been used. The time delay $\tau$=0.001 seconds (Eq. (2)) has been used for the state space reconstruction, which is the sampling period specified in the benchmark problem.

**Table 3.** Comparisons of results using evaluation criteria

| | $J_1$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ | $J_6$ | $J_7$ | $J_8$ | $J_9$ | $J_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Case A | 0.164 | 0.255 | 0.956 | 0.896 | 0.871 | 0.356 | 0.625 | 2.271 | 2.075 | 1.643 |
| Case B | 0.194 | 0.289 | 0.807 | 0.769 | 0.697 | 0.367 | 0.673 | 1.814 | 1.516 | 1.063 |
| Sample LQG | 0.283 | 0.440 | 0.510 | 0.513 | 0.628 | 0.456 | 0.711 | 0.670 | 0.775 | 1.340 |
| Uncontrolled* | 0.589 | 0.999 | 0.072 | 0.082 | 1.068 | 0.620 | 0.718 | 0.077 | 0.083 | 1.142 |

*Uncontrolled – zero control signal

The results of the GA based controllers (Cases A and B) have been compared with those of the sample LQG and uncontrolled case in Table 3. Cases A and B satisfy all the constraints in Eqs. (3) and (4). In the table $J_{10}$ is the criterion of the required peak control force which is related to the peak acceleration of the AMD. The constraint on the peak acceleration of the AMD has been known to be the most critical condition among other constraints in the benchmark problem in our experience. It is very interesting that $J_{10}$ in Case B is even lower than the uncontrolled result. The GA based controller in Case B reduces the response of the structure as well as the required peak control force very effectively. In terms of reducing the structural response ($J_1$, $J_2$, $J_6$ and $J_7$), the controllers in both cases perform much better than the sample LQG controller. The controller in Case A performs better than the controller in Case B because Case A is designed to minimize structural responses using the maximum available control effort while satisfying all of the constraints of the benchmark problem. The results seem to coincide with this design objective.

Figure 1 shows the transfer function from the ground acceleration $\ddot{x}_g$ to absolute acceleration at the first floor $\ddot{x}_{a1}$. In this figure, Case A and Case B are compared with the uncontrolled results. The GA based controllers in both cases reduce the response at the first three modes of the structure.

The loop gain transfer function is used to examine the closed loop stability of the system. The sample LQG controller was considered to be robust in the design if the magnitude of the loop gain was below –5 dB at all frequencies above 35 Hz (Spencer et al. 1997). The GA based controller satisfies the same stability and robustness criteria used in the sample LQG controller design in both cases (Figure 2). It can be seen that the robustness is much improved in Case B as expected.

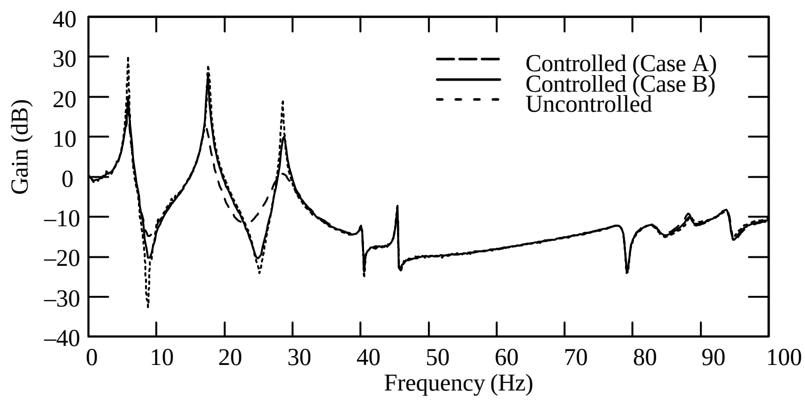


**Figure 1.** Transfer function from ground acceleration to 1st floor absolute acceleration

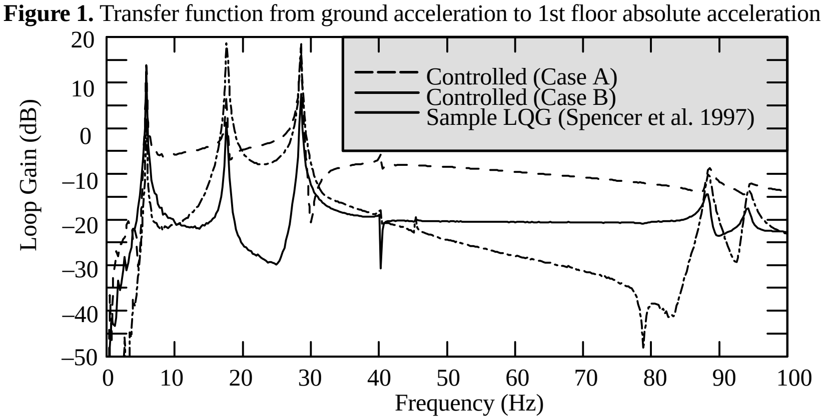


**Figure 2.** Loop gain transfer function

## 8. CONCLUSIONS

A new control method using GAs has been proposed. The proposed method uses the state space reconstruction technique to estimate the system states. Using this technique, the system states are estimated from the observed time series of the structural response. The advantages of the proposed method are primarily in its simplicity and flexibility. The fitness function in GAs doesn't require reformulating the problem into a suitable form unlike traditional gradient based optimization methods. There is considerable flexibility in the formulation of the fitness function. Different weights can be assigned to the multiple objectives and constraints in order to fine–tune the control as desired.

The proposed method has been applied to a benchmark problem. It has been shown that the new method's performance in the response reduction is far superior to that of the sample optimal control. The robustness of the GA based controller has also been examined by the loop gain transfer function. The GA based controller satisfies the same stability and robustness criteria used in the sample LQG controller design in both cases. In Case B the robustness has been much improved by adding measurement noise while optimizing the controller by GAs.

## ACKNOWLEDGEMENTS

The research reported in this paper was funded by National Science Foundation Grant CMS–95–003209. This support is gratefully acknowledged.

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