Evolutionary Program for Multicriteria Solid Transportation Problem with Fuzzy Numbers

Yinzhen Li, Kenichi Ida, Mitsuo Gen
Department of Industrial and Systems Engg.
Ashikaga Institute of Technology
Ashikaga, 326, Japan
e-mail:{yzli ida gen}@genlab.ashitech.ac.jp

ABSTRACT

In this paper, we present an evolutionary program for solving the fuzzy mSTP(multicriteria solid transportation problem) in which the coefficients of objective function are represented as fuzzy numbers. The ranking fuzzy numbers with integral value are used in the evaluation and selection. The proposed algorithm is incorporated with problem-specific knowledge and conductive to find out the set of nondominated points in the criteria space based on decision maker degree of optimism.

Keywords: evolutionary program, fuzzy numbers, multicriteria optimization technique, solid transportation problem, ranking fuzzy numbers.

1. INTRODUCTION

The STP is a generalization of traditional transportation problem. The necessity of considering this special type of transportation problem arises when heterogeneous conveyances are available for shipment of products in public distribution systems. Furthermore, in more real-world cases transportation problems can be formulated as a multiobjective STP(mSTP). In a multiple objective context, the concept of optimal solution gives place to one of nondominated solutions. This decision problems entail analyzing tradeoffs among the objectives in order to get a satisfactory compromise solution from the set of nondominated solutions.

Vignaux and Michalewicz firstly discussed the use of genetic algorithm(GA) for solving linear transportation problem [8]. They used it as an example of constrained optimization problem, investigated how to handle such constraints with GA, and demonstrated the power of GAs that allow to use any data structure suitable for a problem together with any set of meaningful genetic operators. Gen *et al.* further extended Michalewicz's work to bicriteria linear transportation problem and multicriteria solid transportation problem. They have embedded the basic idea of criteria space approach in evaluation phase so as to force genetic search towards to exploiting the nondominated points in the criteria space.

In real-world situation, due to the complexity of the social and economic environment as well as some unpredictable factors such as weather, a common problem is the difficulty for determining the proper values of model parameters. One way of handling such uncertainty in decision making is fuzzy programming. Kaufmann and Gupta firstly examined fuzzy transportation problem [1].

In this paper, we present an evolutionary implementation of GA to solve the fuzzy mSTP(f-mSTP) in which the fuzzy numbers are used in the objective function. We use ranked fuzzy numbers with integral values to calculate evaluation and selection. The proposed algorithm is incorporated with problem-specific knowledge and conductive to find out the set of nondominated points in the criteria space based on decision maker degree of optimism. Finally, the computer simulation results show that the proposed algorithm is efficient for solving the mSTP with fuzzy numbers.

2. FORMULATION OF MSTP WITH FUZZY NUMBERS

Assume that there are m origins (or sources), n destinations and K conveyances. At each origin, let a_i be the amount of homogeneous products which are transported to n destinations to satisfy the demand for b_j units of the product there. Let e_k be the units of this product which can be carried by K different modes of transportation called conveyances. We consider that the coefficients of the objective function as fuzzy numbers are presented as $\tilde{c}_{ijk}^{q}(q = 1, 2, \cdots, Q)$. They are associated with transportation of a unit of the product from source i to destination j by means of the k-th conveyance for the q-th objective function. The fuzzy coefficient could represent transportation cost, delivery time, quantity of goods delivered, under-used capacity, and so on. They may be uncertain due to some environmental impacts. Therefore, it is necessary to treat these coefficients.

A variable x_{ijk} represents the unknown quantity to be transported from origin i to destination j by means of the k-th conveyance. The mSTP with fuzzy numbers can be formulated as follows:

$$\min \quad \tilde{z}_q(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} \tilde{c}_{ijk}^{q} x_{ijk}, \quad q = 1, 2, \cdots, Q \quad (1)$$

$$\text{s. t.} \quad \sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} = a_i, \quad i = 1, 2, \cdots, m \tag{2}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} = b_j, \quad j = 1, 2, \cdots, n \tag{3}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = e_k, \quad k = 1, 2, \cdots, K \tag{4}$$

$$x_{ijk} \geq 0, \ \forall i, j, k.$$

where $a_i \geq 0, \forall i$; $b_j \geq 0$, $\forall j$; $e_k \geq 0$, $\forall k$; $\tilde{c}_{ijk}^{q} \geq 0$, $\forall i, j, k, q$ and the balanced condition

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{K} e_k$$

is treated as a necessary and sufficient conditions for the existence of a feasible solution to the problem.

In a multiobjective context, criteria are usually conflicting each other in nature and the concept of optimal solution gives place to the concept of Pareto optimal solutions (efficient solutions, nondominated solutions, or noninferior solutions), for which nonimprovement in any objective function is possible without sacrificing on at least one of other objective function. Denoting the set of feasible solutions in decision space by F, for the mSTP we can get the following definition of nondominated solution:

Pareto optimal solution:

A solution $\bar{x} = (\bar{x}_{ijk})$ is said to be a Pareto optimal solution if and only if there does not exist another $x \in F$ such that

$$z_q(x) \leq z_q(\bar{x}) \ \forall q \quad \text{and} \quad z_p(x) \neq z_p(\bar{x}) \ \exists p.$$

When the coefficients of objectives are represented with fuzzy numbers, the objective function values become fuzzy numbers. Since a fuzzy number represents many possible real numbers, it is not easy to compare among solutions to determine the Pareto optimal solution. Fuzzy ranking techniques can help us to compare fuzzy numbers. Pareto optimal solutions are determined based on the ranked values of fuzzy objectives and genetic algorithms are used to search Pareto optimal solutions.

3. Ranking fuzzy numbers

Several methods of ranking fuzzy subsets have been proposed. Here, we introduce a simple and flexible method of ranking fuzzy numbers with integral value, which is developed by Liou and Wang [2]. This method ranks fuzzy numbers which can be triangular, trapezoidal, or their other forms with integral value instead of a relative value. The left integral value is used to reflect the pessimistic viewpoint and the right integral value is used to reflect the optimistic viewpoint of the decision maker. A convex combination of right and left integral values through an index of optimism is called the total integral value. It is used to rank fuzzy numbers.

Definition: A fuzzy number $\tilde{A}$ is a triangular fuzzy number(TFN) denoted by (a_1, a_2, a_3) if its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a_1)/(a_2-a_1), & a_1 \leq x \leq a_2, \\ (x-a_3)/(a_2-a_3), & a_2 \leq x \leq a_3, \\ 0, & \text{otherwise,} \end{cases} \tag{5}$$

where a_1, a_2, and a_3 are real numbers. Denote the left and right membership functions as $\mu_{\tilde{A}}(x)^L = (x - a_1)/(a_2 - a_1)$, $\mu_{\tilde{A}}(x)^R = (x - a_3)/(a_2 - a_3)$, respectively. Based on mathematical theorems, both inverse function of $\mu_{\tilde{A}}(x)^L$ and $\mu_{\tilde{A}}(x)^R$ exist (see Figure 1).

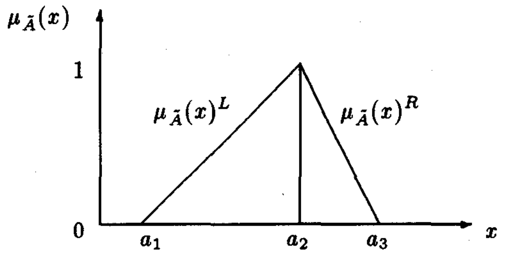

Fig. 1. The membership function $\mu_{\tilde{A}}$

The corresponding inverse functions of $\mu_{\tilde{A}}(x)^L$ and $\mu_{\tilde{A}}(x)^R$ can be expressed by

$$g_{\tilde{A}}(y)^L = a_1 + (a_2 - a_1)y, \tag{6}$$

$$g_{\tilde{A}}(y)^R = a_3 + (a_2 - a_3)y. \tag{7}$$

$y \in [0, 1]$, respectively. Thus the left and right integral value are formulated as follows:

$$I(\tilde{A})^L = \int_0^1 g_{\tilde{A}}(y)^L \mathrm{d}y = \frac{1}{2}(a_1 + a_2), \tag{8}$$

$$I(\tilde{A})^R = \int_0^1 g_{\tilde{A}}(y)^R \mathrm{d}y = \frac{1}{2}(a_2 + a_3). \tag{9}$$

The total integral value of the TFN $\tilde{A} = (a_1, a_2, a_3)$ is

$$\begin{aligned} I_T^{\alpha}(\tilde{A}) &= \alpha I(\tilde{A})^R + (1-\alpha) I(\tilde{A})^L \\ &= \frac{1}{2}[\alpha a_3 + a_2 + (1-\alpha)a_1] \end{aligned} \tag{10}$$

when a degree of optimism $\alpha \in [0, 1]$ is given. And when the decision degree of optimism α is 0.5, above integral value is same as ordinary representives [1].

For any distinct $\tilde{A}_i$, $\tilde{A}_j \in S$ (S is the set of convex fuzzy numbers), we use the following criteria for ranking fuzzy numbers.

(1)if $I_T^{\alpha}(\tilde{A}_i) < I_T^{\alpha}(\tilde{A}_j)$, then $\tilde{A}_i < \tilde{A}_j$.
(2)if $I_T^{\alpha}(\tilde{A}_i) = I_T^{\alpha}(\tilde{A}_j)$, then $\tilde{A}_i = \tilde{A}_j$.
(3)if $I_T^{\alpha}(\tilde{A}_i) > I_T^{\alpha}(\tilde{A}_j)$, then $\tilde{A}_i > \tilde{A}_j$.

4. Evolutionary program for fuzzy mSTP

4.1 Representation and initialization

Perhaps the natural representation of a solution for the solid transportation problem is a three-dimensional array and It creates a matrix of at most $m+n+K-2$ nonzero elements such that all constraints are satisfied. The initialization procedure is used to generated the *pop_size* initial population which satisfies all constraints in STP.

procedure: initialization
begin
  $\pi \Leftarrow \{1, 2, \cdots, m \times n \times K\}$
  repeat
    select a random number l from set π;
    calculate corresponding subscript indices;
      $i \Leftarrow \lfloor (l-1) \bmod (m \cdot n)/n + 1 \rfloor$;
      $j \Leftarrow (l-1)/m \bmod n + 1$;
      $k \Leftarrow \lfloor (l-1) \bmod (m \cdot n) \rfloor + 1$;
    assign available amount of units to x_{ijk};
      $x_{ijk} \Leftarrow \min\{a_i, b_j, e_k\}$;
    update data;
      $a_i \Leftarrow a_i - x_{ijk}$;
      $b_j \Leftarrow b_j - x_{ijk}$;
      $e_k \Leftarrow e_k - x_{ijk}$;
      $\pi \Leftarrow \pi \setminus \{l\}$;
  until(π becomes empty)
end

So, the produced initial solution-matrix has integer numbers in each element.

4.2 Genetic operators

We define two genetic operators, *mutation* and *crossover*.

mutation: The mutation is performed in the following three steps and so *m_size* offspring are generated.
step 1: Make a submatrix from parent solution-matrix. Randomly select $\{i_1, \cdots, i_x\}$, $\{j_1, \cdots, j_y\}$, and $\{k_1, \cdots, k_z\}$ to create a $(x \times y \times z)$ submatrix $W = (w_{ijk})$, where $\{i_1, \cdots, i_x\}$ is a proper subset of $\{1, 2, \cdots, m\}$ and $2 \leq x \leq m$, $\{j_1, \cdots, j_y\}$ is a proper subset of $\{1, 2, \cdots, n\}$ and $2 \leq y \leq n$, $\{k_1, \cdots, k_z\}$ is a proper subset of $\{1, 2, \cdots, K\}$ and $2 \leq z \leq K$ and w_{ijk} takes the value of the corresponding element with index ijk in parent matrix.
step 2: Reallocate commodity for the submatrix. The available amount of commodity a_i^w, the demands b_j^w, and the capability of conveyances e_k^w for the submatrix are determined as follows:

$$a_i^w = \sum_{j \in \{j_1, \cdots, j_y\}} \sum_{k \in \{k_1, \cdots, k_z\}} w_{ijk}; \quad i = i_1, \cdots, i_x \tag{11}$$

$$b_j^w = \sum_{i \in \{i_1, \cdots, i_x\}} \sum_{k \in \{k_1, \cdots, k_z\}} w_{ijk}; \quad j = j_1, \cdots, j_y \tag{12}$$

$$e_k^w = \sum_{i \in \{i_1, \cdots, i_x\}} \sum_{j \in \{j_1, \cdots, j_y\}} w_{ijk}; \quad k = k_1, \cdots, k_z \tag{13}$$

We can use the initialization procedure to assign new values to submatrix such that all constraints a_i^w, b_j^w and e_k^w are satisfied.
step 3: Replace appropriate elements of parent matrix by a new elements from the reallocated submatrix W.
crossover: Assume that a pair of parents are $X_1 = (x_{ijk}^1)$ and $X_2 = (x_{ijk}^2)$, the crossover is performed in following main three steps:
step 1: Create two temporary matrices $D = (d_{ijk})$ and $R = (r_{ijk})$ as follows:

$$d_{ijk} = \lfloor (x_{ijk}^1 + x_{ijk}^2)/2 \rfloor; \quad r_{ijk} = (x_{ijk}^1 + x_{ijk}^2) \bmod 2$$

The matrix D keeps rounded average values from both parents and matrix R keeps the track of whether any rounding was necessary.
step 2: Divide the matrix R into two matrices $R^1 = (r_{ijk}^1)$ and $R^2 = (r_{ijk}^2)$ such that:

$$R = R^1 + R^2; \tag{14}$$

$$\sum_{j=1}^{n} \sum_{k=1}^{K} r_{ijk}^1 = \sum_{j=1}^{n} \sum_{k=1}^{K} r_{ijk}^2 = \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{K} r_{ijk}; \quad i = 1, 2, \cdots, m \tag{15}$$

$$\sum_{i=1}^{m} \sum_{k=1}^{K} r_{ijk}^1 = \sum_{i=1}^{m} \sum_{k=1}^{K} r_{ijk}^2 = \frac{1}{2} \sum_{i=1}^{m} \sum_{k=1}^{K} r_{ijk}; \quad j = 1, 2, \cdots, n \tag{16}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} r_{ijk}^1 = \sum_{i=1}^{m} \sum_{j=1}^{n} r_{ijk}^2 = \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} r_{ijk}; \quad k = 1, 2, \cdots, K \tag{17}$$

step 3: Then we can produce two offspring of X_1' and X_2' as follows:

$$X_1' = D + R^1; \quad X_2' = D + R^2;$$

So, after finishing crossover operation *c_size* offspring are generated.

4.3 Selection

In many kind of special problems, GAs use different scaling methods and different selection schemes (e.g. propotional selection, ranking, tournament, elitist selection) to strike a balance between two factors: population diversity and selection pressure [4]-[10]. To find more Pareto optimal solutions, we use the registered Pareto optimal solution strategy in this stage and attempt to get the set of Pareto optimal solutions approximately (near-optimal solutions), and the elitist selection method is used in each generation.

Module for Pareto optimal solutions:
begin
  for generation index $t = 0$ **to** *max_gen*;
    count number of individuals;
      $ind\_size \Leftarrow pop\_size + m\_size + c\_size$;
    for $s = 1$ **to** *ind_size*;
      for $q = 1$ **to** Q;
        calculate each objective function value;
          $\tilde{z}_q(X_s) \Leftarrow \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\tilde{c}_{ijk}^{q}x_{ijk}^{s}$;
      endfor
      obtain the solution vector $\tilde{z}_s$;
        $\tilde{\mathbf{z}}_s \Leftarrow [\tilde{z}_1(X_s), \tilde{z}_2(X_s), \cdots, \tilde{z}_Q(X_s)]$;
      rank s solution vectors with intergal value;
      register Pareto optimal solutions and
        delete non-Pareto optimal solutions;
    endfor
  endfor
end

As the fitness function for survival, the weighted sums method is used to construct the fitness function at hand.

Handling for fitness function:

(1) At t-th generation, choose the solution points which contain the minimum $I_{\rm T}^{\alpha}(\tilde{z}_q^{min})$ (or $I_{\rm T}^{\alpha}(\tilde{z}_q^{max})$) that corresponding to each objective function with integral value, then compare with the stored solution points at the previous generation and select the best points to store again.

$$I_{\rm T}^{\alpha}(\tilde{z}_q^{min(t)}) = \min_s\{I_{\rm T}^{\alpha}(\tilde{z}_q^{min(t-1)}), \quad I_{\rm T}^{\alpha}(\tilde{z}_q(X_s)) \mid s = 1,2,\cdots,ind\_size\}, \tag{18}$$

$$I_{\rm T}^{\alpha}(\tilde{z}_q^{max(t)}) = \min_s\{I_{\rm T}^{\alpha}(\tilde{z}_q^{max(t-1)}), \quad I_{\rm T}^{\alpha}(\tilde{z}_q(X_s)) \mid s = 1,2,\cdots,ind\_size\}, \quad q = 1,2,\cdots,Q \tag{19}$$

where $\tilde{z}_q^{max(t)}(\tilde{z}_q^{min(t)})$ is the maximum (minimum) value of objective function q at generation t.

(2) Solve the following equations to get weights for evaluation function:

$$\tilde{\delta}_q = \tilde{z}_q^{max(t)} - \tilde{z}_q^{min(t)}, \quad q = 1,2,\cdots,Q$$

$$\beta_q = \frac{I_{\rm T}^{\alpha}(\tilde{\delta}_q)}{\sum_{q=1}^{Q} I_{\rm T}^{\alpha}(\tilde{\delta}_q)}, \quad q = 1,2,\cdots,Q$$

(3) Calculate the fitness value for each chromosome as follows:

$$eval(X_s) = \sum_{q=1}^{Q}\beta_q I_T^{\alpha}(\tilde{z}_q(X_s)), \quad s = 1,2,\cdots,ind\_size \tag{20}$$

4.4 The best compromise solution

To provide the optimal solution for the decision maker, the method of Technique for Order Preference by Similarity to Ideal Solution (called TOPSIS) was combined into GA implementation of solving mSTP which is based upon the principle that the chosen alternative should have the shortest distance from reference point as the positive ideal solution and the farthest other point as the negative ideal solution.

We define positive ideal solution(PIS) and negative ideal solution(NIS) from eqns. (18) and (19) denoted as PIS$=I_T^{\alpha}(\tilde{z}_q^*) = I_T^{\alpha}(\tilde{z}_q^{min(max_gen)})$ and NIS$=I_T^{\alpha}(\tilde{z}_q^-) = I_T^{\alpha}(\tilde{z}_q^{max(max_gen)})$ respectively.

Now, the computation of the weighted Euclidean distance can be carry out from the following steps:

step 1: Construct the normalized values:

$$h_q^k = \frac{I_T^{\alpha}(\tilde{z}_q^k)}{\sqrt{\sum_{p=1}^{v}(I_T^{\alpha}(\tilde{z}_q^p))^2 + (I_T^{\alpha}(\tilde{z}_q^*))^2 + (I_T^{\alpha}(\tilde{z}_q^-))^2}} \tag{21}$$

where p is the index for the set of Pareto optimal solutions and $\tilde{z}_q^p$ is the q-th objective function value with fuzzy numbers which corresponding to the p-th Pareto optimal solution.

step 2: Calculate the separation measures:

$$s_k^* = \sqrt{\sum_{q=1}^{Q} w_q^2(h_q^* - h_q^k)^2}, \quad k = 1,2,\cdots,v \tag{22}$$

$$s_k^- = \sqrt{\sum_{q=1}^{Q} w_q^2(h_q^k - h_q^-)^2}, \quad k = 1,2,\cdots,v \tag{23}$$

where h_q^*, h_q^- are the normalizations of $\tilde{z}_q^*$, $\tilde{z}_q^-$, respectively and the relative importance (weights) of objectives $w_q \in [0,1]$, $q = 1,2,\cdots,Q$ satisfies the equation $\sum_{q=1}^{Q} w_q = 1$.

step 3: Calculate the Euclidean distance d_k:

$$d_k = \frac{s_k^-}{s_k^* + s_k^-}, \quad k = 1,2,\cdots,v. \tag{24}$$

where the solution d_k closest to 1 is selected.

Table 1. Fuzzy coefficients in numerical example

q	1			2			3		
k	1	2	3	1	2	3	1	2	3
$\tilde{c}_{11k}^{q}$	(8, 9, 10)	(10, 12, 14)	(7, 9, 11)	(3, 6, 9)	(8, 9, 10)	(5, 7, 9)	(2, 3, 4)	(6, 7, 8)	(5, 7, 9)
$\tilde{c}_{12k}^{q}$	(4, 5, 6)	(5, 6, 7)	(3, 5, 7)	(7, 9, 11)	(8, 11, 14)	(1, 3, 5)	(5, 6, 7)	(6, 8, 10)	(5, 6, 7)
$\tilde{c}_{13k}^{q}$	(1, 2, 3)	(1, 2, 3)	(1, 1, 1)	(1, 2, 3)	(6, 7, 8)	(6, 7, 8)	(1, 1, 1)	(8, 9, 10)	(1, 3, 5)
$\tilde{c}_{21k}^{q}$	(1, 2, 3)	(8, 9, 10)	(6, 8, 10)	(1, 1, 1)	(2, 4, 6)	(1, 1, 1)	(7, 9, 11)	(7, 9, 11)	(4, 5, 6)
$\tilde{c}_{22k}^{q}$	(1, 2, 3)	(7, 8, 9)	(1, 1, 1)	(3, 4, 5)	(3, 5, 7)	(1, 2, 3)	(6, 8, 10)	(5, 6, 7)	(8, 9, 10)
$\tilde{c}_{23k}^{q}$	(4, 5, 6)	(1, 2, 3)	(5, 7, 9)	(6, 8, 10)	(8, 9, 10)	(6, 7, 8)	(3, 5, 7)	(1, 2, 3)	(3, 5, 7)
$\tilde{c}_{31k}^{q}$	(1, 2, 3)	(2, 4, 6)	(5, 6, 7)	(2, 3, 4)	(4, 6, 8)	(3, 4, 5)	(6, 8, 10)	(2, 4, 6)	(7, 9, 11)
$\tilde{c}_{32k}^{q}$	(1, 2, 3)	(3, 5, 7)	(1, 3, 5)	(3, 5, 7)	(4, 6, 8)	(4, 6, 8)	(7, 9, 11)	(4, 6, 8)	(1, 3, 5)
$\tilde{c}_{33k}^{q}$	(1, 1, 1)	(8, 9, 10)	(1, 1, 1)	(7, 8, 9)	(2, 3, 4)	(7, 9, 11)	(3, 5, 7)	(5, 7, 9)	(10, 11, 12)

5. Numerical example

We use the following example to show the effectiveness of the proposed method. There are three origins, three destinations and three conveyances. The supplies, demands, and capacities of transportation are given as $a_1 = 8$, $a_2 = 9$, $a_3 = 5$; $b_1 = 7$, $b_2 = 6$, $b_3 = 9$; $e_1 = 10$, $e_2 = 5$, $e_3 = 7$, respectively. Table 1 shows the fuzzy coefficients $\tilde{c}_{ijk}^{q}(q = 1, 2, 3)$.

In our experiment, we set the parameters as $max\_gen = 2000$, $pop\_size = 20$, $p_m = 0.2$, $p_c = 0.4$, $w_1 = 0.5$, $w_2 = 0.3$, w_3=0.2 and our program have ran 10 times with different values of $\alpha = 1,\ 0.5,\ 0$, respectively.

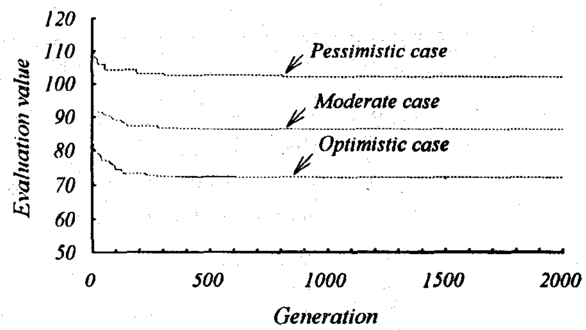

Fig. 2. Convergence process for $\alpha = 1,\ 0.5,\ 0$

Figure 2 shows the convergence process for $\alpha = 1,\ 0.5,\ 0$. Figure 2 indicates that when the decision maker determine this transportation project based on the optimistic degree($\alpha = 0$) and the objective function value is the smallest, otherwise, it is the most biggest when based on the pessimistic degree($\alpha = 1$). Hence, the decision maker can obtain the range of objective value that he/she expected under imprecise conditions by our proposed method. Table 2 shows Pareto optimal solutions obtained in our example, where the $\bar{\mathbf{z}}_q$ represents crisp data with integral value for TFN $\tilde{z}_q$. And the determined compromise solution is marked with notation $*$. According to results which the degree is $\alpha = 0$ and $\alpha = 0.5$, the solutions are as follows:

$$x_{121} = 6, x_{331} = 4,\ x_{132} = 2,$$
$$x_{232} = 2,\ x_{332} = 1, x_{213} = 7,$$
and
$$x_{121} = 5, x_{331} = 5, x_{122} = 1,$$
$$x_{132} = 2, x_{232} = 2, x_{213} = 7$$

respectively, with Euclidean measures $d_0 = 0.8383$ and $d_{0.5} = 0.8344$.

From Table 2 the determined compromise solutions are different with different degree. In moderate case, the set of Pareto optimal solutions involved the best compromise solution of optimistic case, but was not selected as the best determined compromise solution. It shows that the evolutionary program is efficient in searching good solutions and the obtained Pareto optimal solutions set is practical for decision support systems, resulted from its flexibility with degree of optimism.

6. Conclusions

In this paper, we presented an evolutionary program to solve the fuzzy mSTP in which the coefficients of objective function are presented as fuzzy numbers. Particularly, the ranking fuzzy numbers with integral value is used in the evaluation and selection. And the registered Pareto optimal solution technique was considered in our evolutionary implementation. Therefore the proposed evolutionary program more suitable to multicriteria optimization problem that the algorithm conductive to find out the set of Pareto optimal solutions approximately in nondominated hyper-plane of the criteria space with degree of optimism as simulation results shown. Furthermore, the evolutionary program have a characteristic which can use other kind of method instead of TOPSIS to determine the satisfactory nondominated solution.

Table 2. Obtained Pareto optimal solutions

No.	$\tilde{z}_1$	$\tilde{z}_2$	$\tilde{z}_3$	$\bar{\mathbf{z}} = (\bar{z}_1, \bar{z}_2, \bar{z}_3)$
		optimistic case ($\alpha = 0$)		
1	(67,100,133)	(97,123,149)	(82,120,158)	(83.5,110.0,101.0)
2	(75,98,121)	(87,112,137)	(70,107,139)	(86.5,99.5,88.5)
3	(71,103,135)	(78,106,134)	(70,107,144)	(87.0,92.0,91.0)
4*	(75,114,153)	(50,65,80)	(48,86,124)	(94.5,57.5,67.0)
5	(89,129,169)	(95,113,131)	(52,78,104)	(109.0,104.0,65.0)
6	(89,130,171)	(48,63,78)	(90,126,162)	(109.5,55.50,108.0)
7	(103,141,179)	(43,59,75)	(83,118,153)	(122.0, 51.0, 100.5)
8	(109,139,169)	(115,143,171)	(47,78,109)	(124.0,129.0, 62.5)
9	(106,143,180)	(42,56,70)	(61,94,127)	(124.5,49.0,77.5)
10	(117,152,187)	(38,55,72)	(76,110,144)	(134.5,46.5,93.0)
11	(134,158,182)	(116,139,162)	(40,71,102)	(146.0,127.5,55.5)
12	(136,159,182)	(114,138,162)	(47,78,109)	(147.5,126.0,62.5)
	$\bar{\mathbf{z}}^* = (83.5,\ 46.5,\ 55.5)$		$\bar{\mathbf{z}}^- = (155,\ 155.5,\ 142.5)$	
		moderate case ($\alpha = 0.5$)		
No.	$\tilde{z}_1$	$\tilde{z}_2$	$\tilde{z}_3$	$\bar{\mathbf{z}} = (\bar{z}_1, \bar{z}_2, \bar{z}_3)$
1	(73,78,123)	(132,162,192)	(105,147,189)	(98,162,147)
2	(74,103,132)	(100,130,160)	(86,125,164)	(103,130,125)
3	(71,104,137)	(94,121,148)	(80,118,156)	(104,121,118)
4*	(73,109,145)	(53,71,89)	(48,87,126)	(109,71,87)
5	(75,114,153)	(50,65,80)	(48,86,124)	(114,65,86)
6	(81,122,163)	(46,59,72)	(60,98,136)	(122,59,98)
7	(128,153,178)	(125,149,173)	(41,74,107)	(153,149,74)
8	(130,156,182)	(100,125,150)	(36,69,102)	(156,125,69)
	$\bar{\mathbf{z}}^* = (98,\ 59,\ 69)$		$\bar{\mathbf{z}}^- = (183,176,161)$	

ACKNOWLEDGMENT

This work was supported in part by a research grant from the Ministry of Education, Science and Culture, Japanese Government: Grant-in-Aid for Scientific Research, the International Scientific Research Program (No. 07045032).

REFERENCES

[1] Kaufmann, A. and Gupta, M.M., *Fuzzy Mathematical Models in Engineering and Management Science*, Van Nostrand Reinhold, 1988

[2] Liou, T. and M. J. Wang, "Ranking Fuzzy Numbers with Integral Value", *Fuzzy Sets and Systems 50*, pp.247–255, 1992

[3] Lai, Y.- J. and C.- L. Hwang, *Fuzzy Multiple Objective Decision Making: Methods and Applications*, Springer-Verlag, 1994

[4] Horn,J., N.Nafpliotis and D.E.Goldberg, "A Niched Pareto Genetic Algorithm for Multiobjective Optimization", *Proc. of IEEE Inter. Conf. on Evolutionary Computing*, pp.82–87, 1994

[5] Shaffer, J.d, "Multiple objective optimization with vector evaluated genetic algorithm", *Proc. of 1st Inter. Conf. on GAs and Their Applications*, pp.93–100, 1985

[6] Yang, X. and M. Gen: "Evolutionary Program for Bicriteria Transportation Problem", *Proc. of 16th Inter. Conf. Compt. & Indust. Engg.*, Ashikaga, Japan, pp.451-454, 1994.

[7] Gen, M., K. Ida, and Y.Z. Li, "Solving Multiobjective Solid Transportation Problem by Genetic Algorithm", *Jour. of Japan Indus. Manag. Associ.*, vol.46, No.5, pp.446–454, 1995(in Japanese).

[8] Michalewicz, Z., *Genetic Algorithms + Data structure = Evolution Programs*, 2nd ed. Springer Verlag, 1994

[9] Goldberg, D.E, *Genetic Algorithms in Search, Optimization and Machine Learning*, Reading, MA: Addison Wesley, 1989

[10] Gen, M. and R. Cheng, *Genetic Algorithms and Engineering Design*, John Wiley & Sons, New York, 1996