Reliability Assessment and Optimization under Uncertainty in the Dempster-Shafer Framework

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Abstract

Epistemic uncertainty has been a subject heavily discussed in recent years. Where uncertainty of estimations for long time has been disregarded, it is now viewed as an inherent property of the system model. This work demonstrates how different data types like expert estimations and manufacturer's data with various degrees of epistemic uncertainty are acquired and aggregated in a coherent framework. Using this model we show how to improve reliability using multi-objective optimization. A Pareto-based evolutionary algorithm is applied to find a choice of nondominated solutions. A new strategy for biasing the search to desired objectives without loosing diversity is presented. The user can select a posteriori between solutions covering a wide range of the objective space but clustering in the specified area.

Keywords: Dempster-Shafer theory of evidence, epistemic uncertainty, evolutionary algorithms, multi-objective optimization, nondominated repository

1 Introduction

Early product development phases are characterized by fundamental uncertainties in the reliability modeling process. Such uncertainties may have several reasons: uncertain or incomplete component data, uncertainty about the influencing factors, vague estimations of failure functions and coarse-grained system models. On the other hand, just this phase allows design changes without the loss of a substantial amount of time and money. Methods that help to calculate system reliability from sparse and uncertain data therefore can be a great support for product designers.

At least two types of uncertainty have to be distinguished because of their difference in origin, modeling and effects: Aleatory and epistemic uncertainty. Oberkampf et al. define aleatory uncertainty as the *"…inherent variation associated with the physical system or the environment under consideration"* [1]. Aleatory uncertainty of a quantity may be expressed by its characterization as a random value with known distribution. The exact value will change but is expected to follow the distribution.

On the contrary, epistemic uncertainty describes not uncertainty about the outcome of some random event due to system variance but the uncertainty of the outcome due to *"…any lack of knowledge or information in any phase or activity of the modeling process"* [1]. This shows the important difference between both types of uncertainty. Epistemic uncertainty is not an inherent property of the system. A gain of information about the system or environmental factors can lead to a reduction of epistemic uncertainty. But before this information is received, the analyst has to live with this uncertainty.

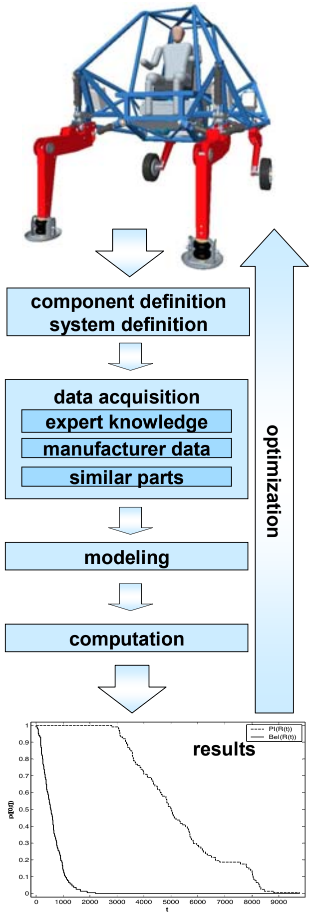

Figure 1. Process diagram of the Alduro analysis.

Using the example of an ALDURO walking machine this work shows step by step how reliability modeling and design optimization may be carried out in early development stages (Figure 1). The probabilistic framework chosen is the Dempster-Shafer theory of evidence allowing representation of aleatory and epistemic uncertainties and flexible methods to handle them.

In section 2 we introduce the mechatronic system analyzed and its representation as a reliability block diagram. Section 3 illustrates the different forms of component data available and their representation. Section 4 introduces the Dempster-Shafer theory, different ways of data aggregation and the system reliability calculation. Section 5 provides a method to optimize the design for reliability using a multi-objective evolutionary algorithm (MOEA) with priorization possibility.

2 System and component definition

The system investigated is the hydraulically driven four-legged walking machine ALDURO (Anthropomorphically Legged and Wheeled Duisburg Robot) [2]. Its operational areas are heavily unstructured or steep terrain or terrains where wheeled/tracked vehicles would cause too much damage to the soil. An important feature of the two tonnes prototype currently being built at the Mechatronics Laboratory is its anthropomorphic leg geometry (Figure 3). The topology and workspace are similar to a human leg and allow high mobility. The spherical hip joint with its three degrees of freedom is actuated with three hydraulic cylinders, the one d.o.f. revolute joint of the knee with a fourth cylinder. Each cylinder is equipped with a linear sensor and two pressure sensors. The coordination of the movement of all 16 actuators is done by the central computer which translates the operator commands to cylinder motions.

In cooperation with the engineers of the ALDURO team it was decided to separate the components into three different groups: Electronics, mechanics and hydraulics. As each group differs in its failure characteristics, reliability templates were defined that are the base for the used failure functions. All measurements are denoted in operating hours.

2.1 The concept of failure templates

There are two general data types to derive a failure function F(t) of a component: Raw data (accelerated lifetime testing, field failures,…) and processed data (manufacturer's data, expert estimations). For this study, only preprocessed data was available. This data type has some condensed form that is either preprocessed from raw data or estimated and often expressed as some statistical property (e.g. MTTF). We have to define a model or "failure template" which needs only the given property as a parameter to produce a failure function.

As failure templates, we chose functions that map preprocessed date to uncertain failure distributions expressed as discrete Dempster-Shafer structure. Templates are created through modeling experience or similarities to building parts with known failure functions. All manufacturing data available had MTTF form which was also what the engineers desired to estimate. The following templates were used:

2.1.1 <u>Electronics</u>

Electronic failures were modeled by exponential distributions which are applicable for components that don't suffer from degradation and burn-in failures [3].

2.1.2 <u>Mechanics</u>

Mechanical failures were modeled by a Weibull-shaped template. Weibull distributions are commonly used for mechanic components and allow the modeling of wear-out failures. The mechanics template fixes the shape parameter β=[1,4] which represents an uncertainty in wear-out characteristics.

2.1.3 <u>Hydraulics</u>

Hydraulics, like mechanical parts may suffer from wear-out which led to the choice of the Weibull distribution with β=[1,3]. Furthermore, the hydraulic component may not work longer than five years without inspection. This is modeled by truncating the template after the estimated operating hours in five years. This estimate was given as [50000h,100000h,10;50000h,700000h,6;60000h,90000h,5] (see section 4).

2.2 System definition

A two-state reliability block diagram without component interaction is used to model the impact of a component failure onto the system. In collaboration with the ALDURO engineers, we decided to separate the system into 19 non-redundant subsystems (Figure 2). Inside the subsystem, redundancy occurs only in the hip. The hip movements are controlled by three hydraulic complexes. One complex is essential, the other complexes are redundant (displayed as a parallel system structure) .In the further work, this subsystem will be the subject of the demonstrated methodology

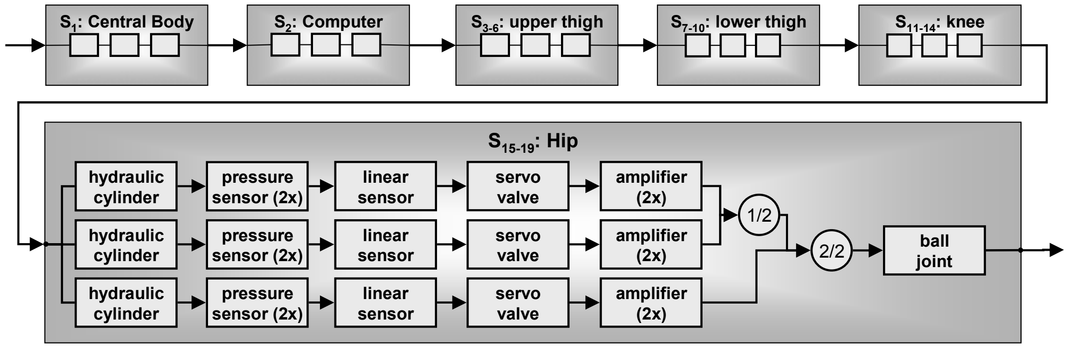

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Figure 2. ALDURO reliability block system

S_1: Central Body (welded framework, steel profiles, combustion engine, hydraulic supply, interface to operator)
S_2: Computer (CPU, I/O cards, peripheral parts)
S_{3-6}: Upper thigh (welded structure)
S_{7-10}: Lower thigh: welded structure, foot)
S_{11-14}: Knee (Hydraulic cylinder, servo valve, linear sensor, pressure sensors, amplifiers, revolute joint)
S_{15-19}: Hip (Hydraulic cylinders, pressure sensors, linear sensors, servo valves, amplifiers, ball joints)

3 Data acquisition

Some parts used to build ALDURO are commercially available. Their failure data (MTTF) was obtained either directly from the documentation or by contacting the manufacturer. The majority of the components has been created or heavily modified by the ALDURO engineers and thus no failure data is available. We had to rely on expert estimations and on the failure behaviour of similar parts. Two types of estimations were allowed: MTTF and failure times. This parameters are denoted easier to estimate than failure distribution parameters like Weibull characteristic lifetime and shape [4]. An expert may give an arbitrary number of estimations $[\underline{a},\overline{a}], r$ where she expects the measurement to be inside interval $[\underline{a},\overline{a}]$ with certainty r. r is expressed on a scale from 1 (almost impossible) to 10 (almost certain). If the expert provided only one estimate, r was neglected. On the same scale the expert or his team leader denotes his competence and knowledge of the component. Three engineers participated in the reliability analysis of the hip subsystem and provided es-

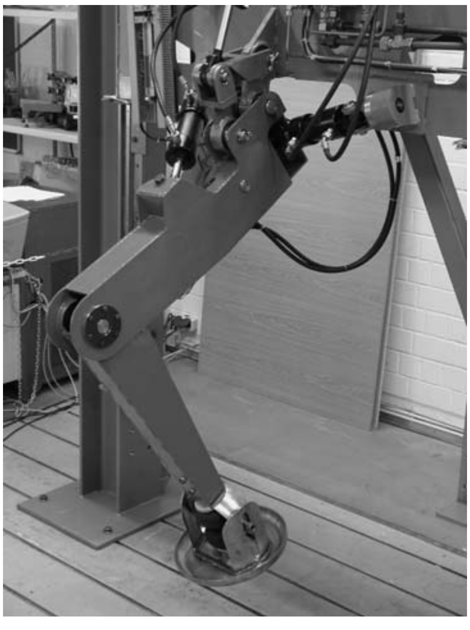

Figure 3. The ALDURO leg.

Table I: Failure data from various sources.

Component	Data source	Competence	Estimation (in hours)	Template
Hydraulic cylinder	Expert 1	7	MTTF=[2000,8000]	Hydraulics
	Expert 2	2	Failure time=[0,10],5;[10,1000],1;[1000,50000],7	
	Expert 3	5	MTTF=[5000,9000]	
Pressure sensor	Expert 1	1	MTTF=[1000,50000]	Electronics
	Similar Part	3	MTTF=[100000,100000]	
	Similar Part	4	MTTF=[105120,105120]	
Linear sensor	Manufacturer	10	MTTF=[400000,800000]	Electronics
Servo valve	Expert 1	6	MTTF=[3000,10000]	Hydraulics
	Expert 2	2	MTTF=[500,500]	
	Expert 3	2	MTTF=[2000,5000]	
	Manufacturer	10	MTTF=[24000,24000]	
Amplifier	Expert 1	1	MTTF=[1000,10000]	Electronics
	Expert 2	1	MTTF=[7000,20000]	
Ball joint	Expert 1	3	MTTF=[3000,5000]	Mechanics
	Expert 3	3	MTTF=[2000,5000]	

timation of the component MTTF and failure times as given in Table I. Where the manufacturer provided data it was rated as a very competent source (Competence 10). For the pressure sensor, no manufacturers data and only one expert estimate was available. Thus, the data of two similar products was used and rated with low certainty.

4 Modeling and Calculations

After defining the system model and collecting the data, it is necessary to choose an appropriate statistical framework. Which characteristics should be regarded?

- Uncertainty is high, so the framework must include ways to specify it.
- Different data types have to be combined in the same model.
- Expert estimations may be conflicting, controversial and even simply false.
- The expert's competence must impact on his estimate's importance.

Several probabilistic frameworks have been proposed that model and preserve uncertainty and disagreement in reliability analysis, e. g. fuzzy sets [5] and the Dempster-Shafer theory of evidence. The latter one will be applied in this work. The huge difference between the presented fusion methods and Bayes modelling is the requirement of precise prior distributions. Walley points out that it is part of the Bayesian principle that both prior and likelihood are precisely known [6]. Unknown priors are modelled as uniform distributions without taking into account that reality may be anywhere else. The presented methods incorporate epistemic uncertainties where exactly this is not necessary.

4.1 The Dempster-Shafer theory of evidence

The Dempster-Shafer theory [7] has proven to be a well-suited framework for representing both epistemic and aleatory uncertainty. It has found application in various fields [8, 9] and is an ideal tool for reliability prediction. In the classical discrete probability calculus, a probability mass m(a) is defined for each possible value of a random variable X and p(X = a) = m(a). Real-valued Dempster-Shafer structures are similar to normal probability distributions with one important difference. The probability mass function is not a mapping $\mathbb{R} \to [0,1]$ but instead a mapping from $2^{\mathbb{R}} \to [0,1]$, where probability masses are assigned to sets instead of discrete values. A Dempster-Shafer structure can be described by its basic probability assignment (bpa) or by a set of focal elements with associated mass.

Definition 1: A basic probability assignment (bpa) m over the real line is a mapping $m: 2^{\mathbb{R}} \to [0,1]$ provided

$$m(\varnothing) = 0 \tag{1}$$

$$\sum_{B \subseteq \mathbb{R}} m(B) = 1 \tag{2}$$

Definition 2: A focal element A is an interval with a nonzero mass m(A)>0.

Because of the uncertainty modelled it is not possible to give an exact probability p(X∈B) for a value or interval B, yet upper and lower bounds can be calculated. Associated with each bpa are two functions $\mathrm{Bel}, \mathrm{Pl}: 2^{\mathbb{R}} \to [0,1]$ referred to as belief and plausibility.

Definition 3: Belief and plausibility of an interval $B \subseteq \mathbb{R}$ are defined as

$$\mathrm{Bel}(B) = \sum_{A \subseteq B} m(A) \tag{3}$$

$$\mathrm{Pl}(B) = \sum_{A \cap B \neq \varnothing} m(A) \tag{4}$$

It is obvious that Bel(B) ≤ Pl(B) because $A \neq \varnothing, A \subseteq B \Rightarrow A \cap B \neq \varnothing$. Bel(B) and Pl(B) can be interpreted as bounds on the probability p(X∈B). Informally, the belief function represents the maximal value that we despite all epistemic uncertainty “believe” to be smaller than p(X∈B), the plausibility function represents the highest “plausible” value of p(X∈B). Belief and plausibility will be used to display a Dempster-Shafer structure. For our use it is adequate (but not necessary) to restrict focal elements to intervals $A = [\underline{a}, \overline{a}]$ rather than more complicated sets.

4.2 Data representation

Expert estimations can be transformed to a bpa by mapping the ratings to mass values. The intervals estimated by the user are interpreted as focal elements. Let n be the number of intervals $A_{1\ldots n}$. A function $\phi: \{1,...,10\} \to [0,1]$ assigns to each rating $r_i, i \in 1\ldots n$ of an expert estimate a value $\phi(r_i)$ as defined in Table II. The mass of a focal element A_i is then defined as:

Table II: Ratings and their respective Φ-values.

Rating r	Φ(r)
10	0.99
9	0.9
8	0.8
7	0.7
6	0.58
5	0.42
4	0.3
3	0.2
2	0.1
1	0.01

$$m(A_i) = \frac{\phi(r_i)}{\sum_{j=1}^{n} \phi(r_j)} \tag{5}$$

In case of estimating failure times, the estimate now already describes a failure function. If templates are used, then we deal with an uncertain statistical property that has to be propagated through the template function.

4.3 Data aggregation using the Dempster-Shafer theory

There are many ways of combining different sources of evidence to a joint bpa [10]. Probably the most famous is Dempster's rule of combination which combines evidence assuming all sources include the correct value. According to Dempster's rule, the aggregated bpa is given through:

$$m_{DP}(A) = \frac{\sum_{A_1 \cap A_2 = A} m_1(A_1) \cdot m_2(A_2)}{1 - K} \tag{6}$$

$$K = \sum_{A_1 \cap A_2 = \varnothing} m_1(A_1) \cdot m_2(A_2) \tag{7}$$

Dempster's rule has the big disadvantage that only pieces of evidence are regarded which are agreed by all sources. If at least one source is faulty and/or there is no intersection between both sources, Dempster's rule is not reasonable or even applicable. In addition there is no possibility of weighting the importance of expert estimations. Not fulfilling two of the given requirements, this renders Dempster's rule unapplicable for our case. Therefore we propose the weighted mixture method [10] which is an extension of the linear opinion pooling [11] for Dempster-Shafer structures. All focal elements are combined into one structure and their mass is weighted by the weights $w_{1...n}$ which were determined from the competence values using the Φ-mapping (Table II).

$$m_{mix}(A) = \frac{\sum_{i=1}^{n} w_i m_i(A)}{\sum_{i=1}^{n} w_i} \tag{8}$$

For the mixing of data from different sources and component types, a conversion to the same format is necessary. The templates produced Dempster-Shafer structures by the outer discretization method [9] with a minimal mass resolution of $m(A) \geq 0.01$. Higher resolutions are not feasible because of the huge amount of computation time and the low data accuracy.

Statistical characteristics of the mixed failure distributions of the ALDURO hip components (belief and plausibility) are given in Table III, the failure cumulative distribution function of a servo valve is shown in Figure 4.

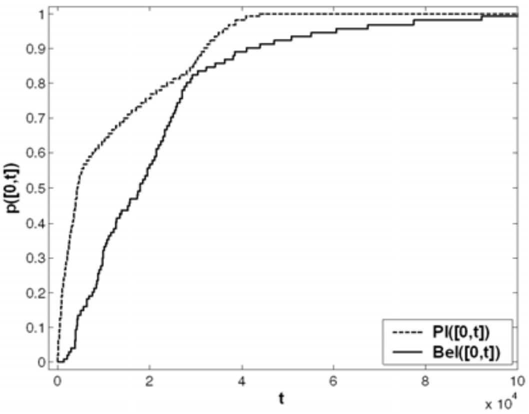

Figure 4. Failure cdf over time (in hours), servo valve.

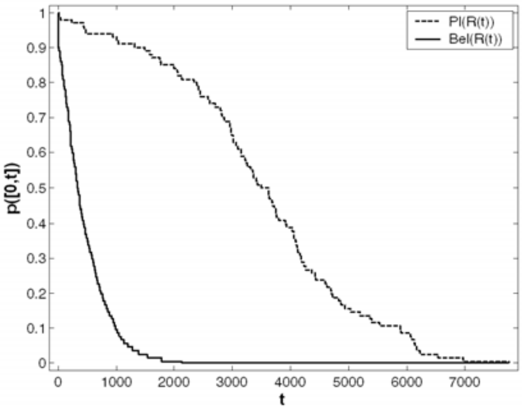

Figure 5. Reliability over time (in hours), ALDURO hip.

Table III: Reliability statistics of hip components and hip subsystem.

	MTTF	Median TTF	95% conf.	75% conf.	25% conf.	5% conf.
Cylinder	[2224h, 12464h]	[1568h, 8543h]	[18h, 2362h]	[643h, 5938h]	[2856h, 13656h]	[6664h, 40981h]
Pressure sensor	[97771h, 108000h]	[64574h, 66514h]	[1903h, 9361h]	[24450h, 26779h]	[131684h, 135443h]	[276023h, 292554h]
Linear sensor	[386943h, 847569h]	[261533h, 538598h]	[12182h, 32654h]	[104533h, 219524h]	[523625h, 1077431h]	[1063172h, 2249476h]
Servo valve	[11020h, 21400h]	[4164h, 17456h]	[116h, 2913h]	[1374h, 8655h]	[18108h, 26097h]	[34985h, 55243h]
Amplifier	[3375h, 10583h]	[3002h, 8078h]	[142h, 4083h]	[1199h, 6407h]	[5029h, 11591h]	[7427h, 24036h]
Ball joints	[1816h, 6614h]	[1589h, 5031h]	[71h, 2552h]	[630h, 3983h]	[2649h, 7457h]	[3937h, 13674h]
ALDURO Hip subsystem	[441h, 3505h]	[319h, 3430h]	[<10h, 429h]	[125h, 2443h]	[635h, 4370h]	[1129h, 6131h]

4.4 Calculation of the system function

The system reliability calculation is performed by the minimal cut set method [12]. The failure mass function m_Θ of a minimal cut set $\Theta = \{C_1,...,C_n\}$ is calculated from the component mass functions $m_1...m_n$ as:

$$\begin{aligned}&\forall[\underline{a}_1,\overline{a}_1]: m_1([\underline{a}_1,\overline{a}_1]) > 0,\ldots,[\underline{a}_n,\overline{a}_n]: m_n([\underline{a}_n,\overline{a}_n]) > 0:\\ &m_\Theta([\max(\underline{a}_{1\ldots n}),\max(\overline{a}_{1\ldots n})]) = \prod_{i=1}^{n} m_i([\underline{a}_i,\overline{a}_i])\end{aligned} \tag{9}$$

The system failure mass function m_S then results from the minimal cut set masses $m_{\Theta_{1\ldots n}}$ as:

$$\begin{aligned}&\forall[\underline{a}_{\Theta_1},\overline{a}_{\Theta_1}]: m_{\Theta_1}([\underline{a}_{\Theta_1},\overline{a}_{\Theta_1}]) > 0,\ldots,[\underline{a}_{\Theta_n},\overline{a}_{\Theta_n}]: m_{\Theta_n}([\underline{a}_{\Theta_n},\overline{a}_{\Theta_n}]) > 0:\\ &m_S([\min(\underline{a}_{\Theta_{1\ldots n}}),\min(\overline{a}_{\Theta_{1\ldots n}})]) = \prod_{i=1}^{n} m_{\Theta_i}([\underline{a}_{\Theta_i},\overline{a}_{\Theta_i}])\end{aligned} \tag{10}$$

Belief and plausibility of the system reliability are given in Figure 5. As the component failure distributions were highly uncertain, it would be window-dressing to expect certain system reliability values. This is reflected by the broad difference between belief and plausibility. MTTF, Median TTF and some confidence rates are listed in Table III.

5 Design optimization

The design optimization for reliability is modeled as a multi-objective optimization problem by defining a mapping from the space of all possible (or interesting) designs $\mathbf{x} \in X$ to the objective space $\mathbf{y} \in Y$ defining several objective values that describe the properties of the system like cost and reliability.

$$f : X \to Y \tag{11}$$

$$f(\mathbf{x}) = \mathbf{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \tag{12}$$

We choose the optimization of the ALDURO hip as a fictive example. The goal is to maximize the system Mean Time To Failure MTTF($\mathbf{x}$) while minimizing the system costs CS($\mathbf{x}$) which are computed as the sum of all component costs. This is expressed as a maximization problem. 20000 – CS($\mathbf{x}$) is used to map the system costs to a maximization objective.

$$f(\mathbf{x}) = \begin{pmatrix} \mathrm{MTTF}(\mathbf{x}) \\ 20000 - \mathrm{CS}(\mathbf{x}) \end{pmatrix} \tag{13}$$

The main difference to standard multi-objective optimization problems is the property of the objective values to be uncertain themselves. In this approach, the quantity MTTF($\mathbf{x}$) is uncertain and can only be described via belief and plausibility values.

Two general multi-objective optimization approaches should be separated:

- The aggregation approach (a priori decision) where a function is defined that maps the objective space Y to a total ordered space (most common the real line): $u : Y \to \mathbb{R}$. u has to be defined before the start of the optimization process and thus some extra problem knowledge is needed.
- The Pareto (or a posteriori) approach where u is not defined. The only criterion used is the vector dominance which is a partial order and allows a whole set of so-called Pareto optimal solutions. Decisions between this optimal solutions are afterwards left to the user.

Definition 4: A vector $\mathbf{y} \in \mathbb{R}^n$ Pareto dominates another vector $\mathbf{y}' \in \mathbb{R}^n$ ($\mathbf{y} \succ_p \mathbf{y}'$) if:

$$\forall i \in 1...n : y_i \geq y'_i \tag{14}$$

$$\exists i \in 1...n : y_i > y'_i \tag{15}$$

The Pareto relation relies on the total order inside the dimensions of the element vector. But what if the results of f are uncertain values, represented as intervals. In our case, the objective function changes to:

$$\mathrm{f}(\mathbf{x}) = \begin{pmatrix} [\mathrm{Bel}(\mathrm{MTTF}(\mathbf{x})), \mathrm{Pl}(\mathrm{MTTF}(\mathbf{x}))] \\ 20000 - \mathrm{CS}(\mathbf{x}) \end{pmatrix} \tag{16}$$

One of our premises has been that we do not know which distribution describes the epistemic uncertainty. Therefore it is difficult to define an aggregation function which maps the interval [Bel(MTTF($\mathbf{x}$)), Pl(MTTF($\mathbf{x}$))] to a total order. We can diversify between three cases:

1. MTTF($\mathbf{x}$)=[10000h,15000h] and MTTF($\mathbf{x}'$)=[6000h,8000h]: $\mathbf{x}$ is certainly more preferable than $\mathbf{x}'$.
2. MTTF($\mathbf{x}$)=[10000h,15000h] and MTTF($\mathbf{x}'$)=[8000h,12000h]: $\mathbf{x}$ could be more preferable than $\mathbf{x}'$. Either we could leave the choice to the user or infer that $\mathbf{x}$ is more preferable than $\mathbf{x}'$ because its belief and plausibility values are higher.
3. MTTF($\mathbf{x}$)=[10000h,15000h] and MTTF($\mathbf{x}'$)=[12000h,14000h]: There is no obvious way to decide if $\mathbf{x}$ or $\mathbf{x}'$ is better and we should leave the choice to the expert afterwards.

The first example is the ideal case where we can make a certain decision. In the third case it is difficult to make any decision without additional knowledge about MTTF($\mathbf{x}$). The second case is the critical one. Both belief and plausibility values are higher, but we can not decide without the danger of an error. But for any monote aggregation function $\mathrm{f}(\mathrm{Bel}(\mathrm{MTTF}(\mathbf{x}), \mathrm{Pl}(\mathrm{MTTF}(\mathbf{x})) : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, $\mathbf{x}$ would be superior. Therefore, we stated that the optimization algorithm should make a decision in favor of $\mathbf{x}$.

The straightforward way to map this ideas to a standard multi-objective optimization criterion is to interpret [Bel(MTTF($\mathbf{x}$)), Pl(MTTF($\mathbf{x}$))] as two independent objective functions and use Pareto-based optimization:

$$\mathrm{f}(\mathbf{x}) = \begin{pmatrix} \mathrm{Bel}(\mathrm{MTTF}(\mathbf{x})) \\ \mathrm{Pl}(\mathrm{MTTF}(\mathbf{x})) \\ 20000 - \mathrm{CS}(\mathbf{x}) \end{pmatrix} \tag{17}$$

If we apply Pareto-based optimization methods like the popular algorithms SPEA2 or MOPSO [13, 14], we yield a choice of nondominated solutions where the user can select his favourite design. All algorithms maintain a bounded repository where the currently found nondominated solutions are kept. Problems arise if there are more nondominated solutions than the user could handle. Solutions in the repository also influence the optimization progress. Therefore it is of importance to keep the repository as small as possible without omitting interesting solutions. There may be thousands of nondominated solutions at a time. Thus, the algorithm has to select which are the best to keep. Each of the listed algorithms encloses a different strategy to delete solutions if their number exceeds the repository bounds while preserving diversity of the solutions kept in the repository.

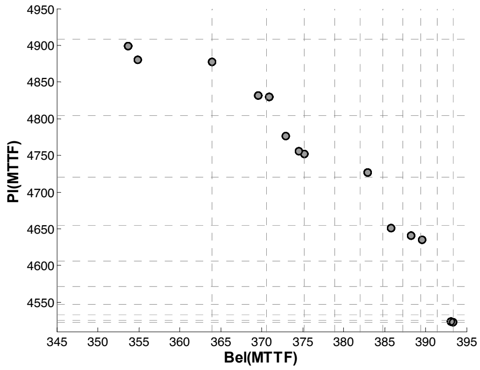

Figure 6. Optimization results, priority on Bel(MTTF).

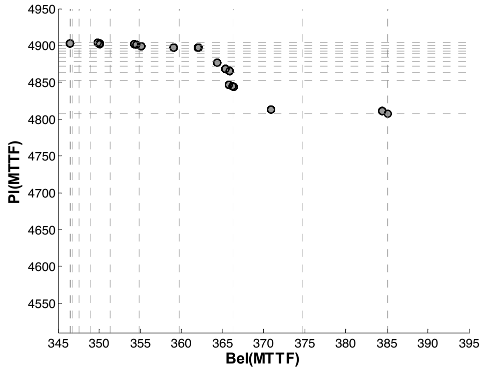

Figure 7. Optimization results, priority on Pl(MTTF).

But maximal diversity may not be the goal of the a posteriori decision maker. She may be much more interested in a high density of solutions in some regions of the objective space while not omitting that other regions may also include interesting designs. We present a method to set a priori preferences for certain objective functions that bias the optimization process to the desired objective space regions extending the adaptive hypercube repository [15].

The adaptive hypercube strategy for real objective values is based on the histogram technique. The objective space occupied by individuals in the repository is separated in hypercubes $h_{i_1,\dots,i_n} \subseteq \mathbb{R}^n$. The covered space of a cube is:

$$h_{j_1,\dots,j_n} = \begin{pmatrix} \left[\min_1 + \frac{j_1 - 1}{c|rep|}(\max_1 - \min_1), \min_1 + \frac{j_1}{c|rep|}(\max_1 - \min_1)\right] \\ \vdots \\ \left[\min_n + \frac{j_n - 1}{c|rep|}(\max_n - \min_n), \min_n + \frac{j_n}{c|rep|}(\max_n - \min_n)\right] \end{pmatrix} \quad (18)$$

where $\min_i$ and $\max_i$ are the minimal/maximal values of objective i regarding all solutions in the repository with size constraint |rep|. The constant $c \in [0,1]$ controls the number of cubes. It represents a selection pressure in favour of solutions in regions with low density. Each solution **y** receives a deletion fitness which is computed as the number of solutions sharing the same hypercube. The cubes are updated every algorithmic iteration (generation).

Our extension uses adaptive priority cubes, hypercubes that are not of uniform length. Desired regions of the objective space are covered with a high cube density while uninteresting regions contain much fewer cubes. Objective priorities are determined by raw priority values $P_{1\dots n} \in \mathbb{R}$ which the user defines to describe the proportion of an objective's importance. She chooses an arbitrary value P_j for one of the objectives y_j and the rest by mapping expressions like "Objective y_i is r times (double/half/…) as important as y_j" to $P_i = r \cdot P_j$. From this raw priority values, priority indices $p_{1\dots n} \in \mathbb{R}$ are generated to create the hypercubes.

$$p_j = \frac{p_j^{\,2}}{\prod\limits_{k=1\ldots n} p_k^{2/n}} \tag{19}$$

$$h_{j_1,\ldots,j_n} = \begin{pmatrix} \left[\min_1 + \left(\frac{j_1 - 1}{c|rep|} \right)^{p_1} (\max_1 - \min_1), \min_1 + \left(\frac{j_1}{c|rep|} \right)^{p_1} (\max_1 - \min_1) \right] \\ \vdots \\ \left[\min_n + \left(\frac{j_n - 1}{c|rep|} \right)^{p_n} (\max_n - \min_n), \min_n + \left(\frac{j_n}{c|rep|} \right)^{p_n} (\max_n - \min_n) \right] \end{pmatrix} \tag{20}$$

The resulting cube lattice is warped (Figure 6 and Figure 7). Therefore solutions with high values of the preferred objective may cluster much denser without raising their deletion probability.

Five different choices were possible for each component. Beside the original part with its real costs, four fictive variants with different characteristics were defined. They vary both in their MTTF and costs. Due to the combinatorial nature of the problem, the overall number of possible designs exceeds 10^{14}. Two exemplary optimization runs with the component choice listed in Table IV were carried out. The first optimization aims on maximizing belief and plausibility of the system MTTF while the second run introduces a third dimension (cost). A multi-objective evolutionary algorithm with a population size of 20 optimized the system over 100 generations (2000 tested systems). Crossover probability was set to 0.9, mutation probability to 0.1 and selection pressure c to 0.25.

Figure 6 and Figure 7 show the nondominated set of solutions with a repository size of 20. In Figure 6, Bel(MTTF) was priorized higher, in Figure 7 Pl(MTTF). It could be seen how most of the solutions cluster in regions with high values of the priorized objective. Nevertheless the nondominated set covers a large region of the objective space leaving a broad choice of systems to choose from. Figure 8 shows the optimization results in the three-objective case with priority on the system costs. The repository size was set to 50 as the number of optimal solutions increases.

Table IV: Component variants.

Component	Variant	MTTF (Bel/Pl)	Template	Costs
Cylinder	Original	[2224,12464]	Aggregated	2000
	2	[4000,7000]	Hydraulics	2200
	3	[2000,12000]	Hydraulics	1500
	4	[5000,5500]	Hydraulics	1800
	5	[3000,10000]	Hydraulics	1600
Pressure sensor	Original	[97771,108100]	Aggregated	570
	2	[70000,120000]	Electronics	600
	3	[90000,110000]	Electronics	480
	4	[80000,115000]	Electronics	520
	5	[75000,118000]	Electronics	650
Linear sensor	Original	[386943,847569]	Aggregated	250
	2	[8000,25000]	Hydraulics	280
	3	[14000,19000]	Hydraulics	240
	4	[10000,21000]	Hydraulics	260
	5	[11000,20000]	Hydraulics	300
Servo valve	Original	[11020,21400]	Aggregated	1000
	2	[8000,25000]	Hydraulics	1200
	3	[14000,19000]	Hydraulics	890
	4	[10000,21000]	Hydraulics	950
	5	[11000,20000]	Hydraulics	1050
Amplifier	Original	[6495,20685]	Aggregated	210
	2	[8000,15000]	Electronics	240
	3	[10000,12000]	Electronics	220
	4	[9000,12500]	Electronics	190
	5	[8000,14000]	Electronics	200
Ball joints	Original	[1816,6614]	Aggregated	230
	2	[8000,15000]	Electronics	250
	3	[10000,12000]	Electronics	210
	4	[9000,12500]	Electronics	240
	5	[8000,14000]	Electronics	235

Both 20 and 50 candidate systems are a number The diversity of the solutions from systems with low costs to systems with high costs and high MTTF allows to select between cheap and high-quality solutions. Due to the clearness of the plot it was not possible to draw the hypercube grid, but nevertheless it could be seen that the solutions cluster in regions with high cost values. At this point, the engineer could select which of the design she would prefer. From a huge amount of alternatives, a choice of 20-50 designs are left, most of them with the characteristics she claimed as important. Other solutions with different characteristics that she didn't thought before are also proposed, which would never be possible in an a priori approach.

6 Conclusion and outlook

This work shows that the Dempster-Shafer theory is a well-suited framework for reliablity analysis in early design stages. It was demonstrated how data uncertainty can be preserved and propagated during the whole analytical process. The design optimization algorithm enables the user to define his objective priorities and afterwards comes up with a set of solutions meeting his demands. From over 10^{14} system variants, 20 are preselected reflect a wide range of characteristics. Our further research will include ways to specify, store and aggregate uncertain failure characteristics in early design stages received from experts, previous experiences and other sources. This knowledge should then be used to influence the design process as outlined in our work.

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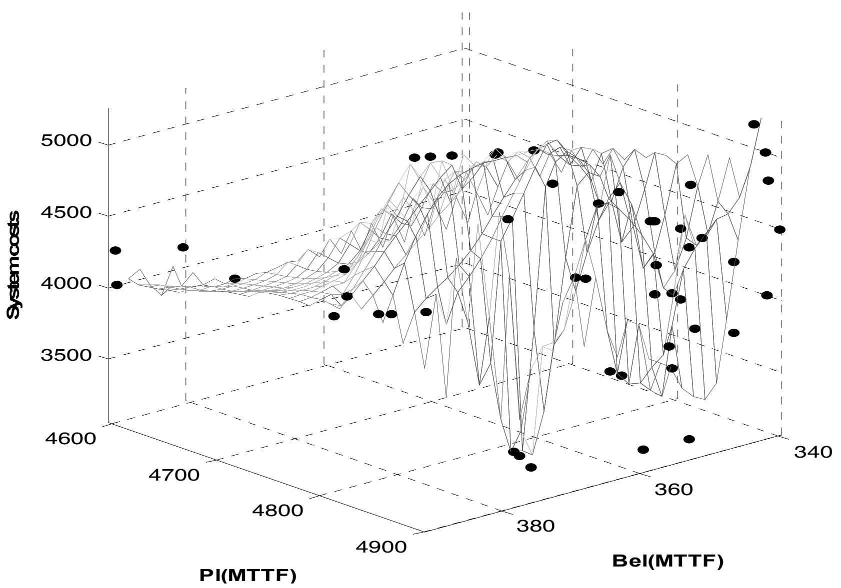

Figure 8. Three-objective optimization of cost, Bel(MTTF) and Pl(MTTF).

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