Genetic Algorithm for Supply Planning Optimization under Uncertain Demand

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Abstract. Supply planning optimization is one of the most important issues for manufacturers and distributors. Supply is planned to meet the future demand. Under the uncertainty involved in demand forecasting, profit is maximized and risk is minimized. In order to simulate the uncertainty and evaluate the profit and risk, we introduced Monte Carlo simulation. The fitness function of GA used the statistics of the simulation. The supply planning problems are multi-objective, thus there are several Pareto optimal solutions from high-risk and high-profit to low-risk and low-profit. Those solutions are very helpful as alternatives for decision-makers. For the purpose of providing such alternatives, a multi-objective genetic algorithm was employed. In practice, it is important to obtain good enough solutions in an acceptable time. So as to search the solutions in a short time, we propose Boundary Initialization which initializes population on the boundary of constrained space. The initialization makes the search efficient. The approach was tested on the supply planning data of an electric appliances manufacturer, and has achieved a remarkable result.

1 Introduction

Manufacturers and distributors deal with a number of products. Supply planning problems are to decide the quantity, the type, and the due time of each product to supply as a replenishment. The supply plan is decided depending on the demand forecast of the products, and the forecasted demand involves uncertainty. If the demand exceeds the supply, opportunity losses occur while excess supply increases inventory level, and may result in dead stocks. In order to supply products, several resources are consumed to produce, and deliver the products. Materials, production machines, and transportation are the resources. The availability of the resources is limited, thus the supply quantity of the products also has a limit. Under the resource constraints and the uncertainty of demands, the supply plan is made to maximize the profit.

Traditionally, inventory management approach has been used to create supply plans[1]. This approach decides the supply plan to minimize the stockout rate, or opportunity loss rate. However, in practice, this approach has a problem. This approach does not take account of the relationship between profit and

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risk. For example, under the resource constraints, we can expect more profit by reducing the opportunity loss of the product with high gross margin than by doing so with low gross margin. However, the demand of the high gross margin product is often uncertain, while the demand of regular products with low gross margin is relatively steady and the risk is low. Thus the best mixture of high risk and low risk products is important to achieve a certain amount of profit with minimizing risk. In other words, portfolio management needs to be introduced in supply planning problems. It is desired to optimize profit and risk of supply planning problems under uncertain demand.

There is also an operational problem. Supply planning problems are multi-objective. Though the maximization of profit and the minimization of risk are required simultaneously, those objectives are in trade-off relationship. Therefore, the problem has some Pareto optimal solutions from high-risk and high-profit to low-risk and low-profit. For decision-makers, such alternative solutions are very helpful because what they should do is to simply select one solution, which fits their strategy, from the alternatives. Many existing supply planning methods, however, create only one solution while most decision-makers do not like the black box system which proposes only one solution.

To produce an efficient supply plan, we introduced a Multi-objective Genetic Algorithm (GA) and Monte Carlo Simulation into the supply planning. GAs are considered as efficient ways of optimizing multi-objective problems[2,3,4]. In GAs, a number of individuals promote optimization in parallel and this characteristic is expedient to find Pareto optimal solutions all at once. We introduced Monte Carlo simulation[5] into the evaluation process of GA. Monte Carlo simulation simulates the uncertainty of demand, then profit and risk are evaluated from the simulation result. In order to search the solutions in an acceptable time, we proposed boundary initialization which initializes population on the boundary of constrained space and makes the search efficient.

We briefly describe the supply planning problems in section 2, and propose a GA with Monte Carlo simulation-based evaluation for supply planning and efficient population initialization method in section 3. Section 4 shows the result of computational experiments. Then we conclude in section 5.

2 Demand and Supply Planning

2.1 Supply Planning and Resource Constraints

Manufacturers and distributors deal with many kinds of products. A firm deals with I kinds of products and it wants to make a supply plan of a certain period consisting of T terms. d_{ti}, p_{ti}, and q_{ti} are the demand, the supply quantity, and the initial inventory quantity of product i in term t, respectively. Sales quantity s_{ti} and opportunity loss quantity l_{ti} are obtained by the following equations.

$$s_{ti} = \min\left(d_{ti}, p_{ti} + q_{ti}\right) \tag{1}$$

$$l_{ti} = d_{ti} - s_{ti} \tag{2}$$

The initial inventory quantity of the next term is

$$q_{(t+1)i} = p_{ti} + q_{ti} - s_{ti} \quad . \tag{3}$$

The firm sells the product i for unit price u_{ti} in term t, the unit supply cost is v_{ti} and the unit inventory cost is w_{ti}. Then the gross profit of the firm through the planning period is obtained by equation 4.

$$G = \sum_{t=1}^{T} \sum_{i=1}^{I} (s_{ti}u_{ti} - p_{ti}v_{ti} - q_{ti}w_{ti}) \tag{4}$$

The first term represents the sales amount, the second and the third terms are the supply cost and inventory cost. Opportunity loss amount L is the sales amount which could be gained if the firm had enough supply and inventory, so it is defined as the following equation.

$$L = \sum_{t=1}^{T} \sum_{i=1}^{I} l_{ti}u_{ti} \tag{5}$$

Demand forecast d_{ti} is an uncertain estimate, that is a stochastic variable while supply quantity p_{ti} is a decision variable. Since d_{ti} is stochastic, as its dependent variables, gross profit G and opportunity loss L are naturally stochastic. The distributions of G and L depend on the decision variable p_{ti}. Thus, the objective of the supply planning problem is to decide the supply quantity so that the distributions are optimal.

Resources are consumed as products are supplied. For example, the machines to produce products and the trucks to transport them are the resources. The firm has J kinds of resources. r_{ij} of resource j is consumed to supply the unit quantity of product i. a_{tj} is the available quantity of resource j in term t. The amount of the consumption of the resource must not exceed the available quantity. On the other hand, supply quantity should not be negative. Therefore, the constraints of supply planning problems are defined as the following equations.

$$\sum_{i=1}^{I} r_{ij}p_{ti} \leq a_{tj} \quad (t = 1, ..., T, j = 1, ..., J) \tag{6}$$

$$p_{ti} \geq 0 \quad (t = 1, ..., T, i = 1, ..., I) \tag{7}$$

The constraints space is convex.

2.2 Optimization Criteria

Supply planning problems are multi-objective. They have a number of criteria of profit and risk. Statistics of the distributions of the gross profit and opportunity loss are used as the optimization criteria. Figure 1 summarizes the statistics. X axis indicates gross profit G or opportunity loss L, and y axis indicates the probability of it.

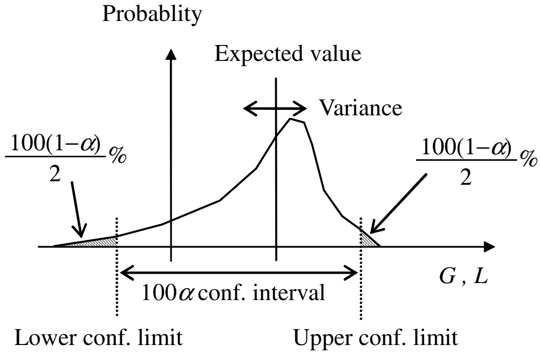


Fig. 1. Optimization Criterion

One of the most important objectives of business is maximizing profit. Thus, expected gross profit is naturally used as a criterion of profit, and it should be maximized. Variance of the gross profit reflects the volatility of outcome. Less volatility is preferable, thus variance is used as a risk criterion, and it should be minimized. Expected opportunity loss is also used as a risk criterion, and it should be minimized. The lower confidence limit of the 100α percent confidence interval is the lower $100(1-\alpha)/2$ percentile. For example, a lower confidence limit of 95 percent confidence interval of the gross profit is the lower 2.5 percentile of the profit. That means the probability of the gross profit falling below the limit is only 2.5 percent. Lower confidence limit is an inverse criterion of risk since it indicates the worst case of the profit, and it should be maximized. Likewise, upper confidence limit of the opportunity loss is a risk criterion since it indicates the worst case of the loss, and it should be minimized.

2.3 Demand Forecasting

Future demand can be forecasted according to the history of demand. There are several forecasting methods such as Moving Average Method, Exponential Smoothing Method, and Box-Jenkins Method [6].

In this paper, an existing commercial software was used to forecast future demand. The software analyzes the history of demand and automatically chooses the forecasting method which best fits to the historical data, then forecasts the expected value and the variance of the future demand. The demand forecast is supposed to follow Normal distribution though its left tail is truncated at zero.

3 Genetic Algorithms Approach

3.1 Genetic Representation

In order to optimize supply planning problems, we employed a real-coded genetic algorithm. Each individual is a vector of real value, and the vector

$\boldsymbol{x} = (x_1, ..., x_{T\times I})$ corresponds to the set of supply quantities. Its element $x_{(t-1)\times I+i}$ corresponds to supply quantity p_{ti}, i.e.

$$\boldsymbol{x} = (p_{11}, p_{12}, ..., p_{1I}, p_{21}, p_{22}, ..., p_{2I}, ..., p_{T1}, p_{T2}, ..., p_{TI}) \ . \tag{8}$$

3.2 Fitness Function Using Monte Carlo Simulation

In order to evaluate the criteria of supply plan problems, we introduced Monte Carlo Simulation in the fitness function of GA. Monte Carlo simulation is a simulation method in which a large quantity of random numbers are used to calculate statistics. It calculates multiple scenarios of the gross profit and the opportunity loss by sampling demand quantities from the random number following their probability distributions.

Figure 2 describes the detail of the Monte Carlo simulation to evaluate supply planning. The future demand of each product is forecasted as the expected value and the variance. μ_{ti} and σ_{ti} denote the expected demand and its standard deviation of product i in term t. Demand is considered to follow normal distribution.

```
begin evaluation
    m := 1
    repeat                     *** Repeat simulation for an evaluation ***
        G_m :=0,   L_m :=0     *** Initialize gross profit and opp. loss of m-th simulation ***
        i :=1
        repeat
            q_1i :=0
            t :=1
            repeat
                *** Calculate the profit and the loss from product i in term t ***
                *** and sum up them into n-th profit and loss                  ***
                d_ti := Random number following N( μ_ti, σ_ti )
                s_ti := min( d_ti, p_ti + q_ti )
                l_ti := d_ti − s_ti
                G_m := G_m + ( s_ti u_ti − p_ti v_ti − q_ti w_ti )
                L_m := L_m + l_ti u_ti
                q_(t+1)i := p_ti + q_ti − s_ti
                t :=t +1
            until t <= T
            i := i +1
        until i <= I
        m := m +1
    until m <= M
    Calculate optimization criteria
End evaluation
```

Fig. 2. Monte Carlo Simulation to Evaluate Supply Plan

The evaluation of one individual, or one supply plan, consists of M simulations of the gross profit and the opportunity loss. The block from line 4 to 22 in the figure corresponds to one simulation. In each simulation, the demand of each product in each term is simulated by the random number following normal distribution $N(\mu, \sigma)$(line 12 in the figure). After the iterations, we obtain M samples of gross profit G_m and opportunity loss L_m $(m = 1, ..., M)$. Statistics as the optimization criteria are calculated from the samples. For example, the expected gross profit and its variance are estimated by equation 9 and 10.

$$\hat{G} = \frac{1}{M} \sum_{m=1}^{M} G_m \tag{9}$$

$$U(G) = \frac{1}{M-1} \sum_{m=1}^{M} \left(G_m - \hat{G} \right)^2 \tag{10}$$

$\hat{G}$ should be maximized as a matter of course and $U(G)$ should be minimized since less volatility is preferable.

3.3 Genetic Operators and Selection

Supply planning problems are multi-objective. The outcome of multi-objective optimization is not a single solution, but a set of solutions known as Pareto optimal solutions. Among the solutions, each objective cannot be improved without the other objectives being degenerated. A vector $\boldsymbol{u} = (u_1, ..., u_n)$ is superior to $\boldsymbol{v} = (v_1, ..., v_n)$ when $\boldsymbol{u}$ is partially greater than $\boldsymbol{v}$, i.e.,

$$\forall i, u_i \geq v_i \wedge \exists i, u_i > v_i \ . \tag{11}$$

Any solution to which no other solution is superior is considered as optimal. Since supply plan optimization is multi-objective, it has several Pareto optimal solutions.

The supply plan optimization problem has convex constraints. The genetic operators proposed by Michalewicz [7][8] can handle convex constraints effectively. Thus we employed the operators and modified for multi-objective problems. The GA has two mutation operators, uniform and boundary, and two cross-over operators, arithmetic and heuristic.

The mutation operator selects one locus k from individual $\boldsymbol{x}$ randomly and changes the value of x_k in the range satisfying constraints. Uniform mutation changes x_k to a random number following uniform distribution, and boundary mutation changes x_k to the boundary of constrained space.

$\boldsymbol{x}$ and $\boldsymbol{y}$ denote parents, and $\boldsymbol{z}$ denotes offspring. Arithmetic crossover reproduces offspring as $\boldsymbol{z} = \lambda \boldsymbol{x} + (1 - \lambda) \boldsymbol{y}$ where λ is a random value following uniform distribution $[0, 1]$. Since the constrained solution space is convex, whenever both $\boldsymbol{x}$ and $\boldsymbol{y}$ satisfy the constraints, arithmetic crossover guarantees the feasibility of $\boldsymbol{z}$.

Heuristic crossover uses the evaluation of two parents to determine the search direction. The offspring reproduced by heuristic crossover is $\boldsymbol{z} = \boldsymbol{x} + \lambda (\boldsymbol{x} - \boldsymbol{y})$ where the evaluation of individual $\boldsymbol{x}$ is superior to $\boldsymbol{y}$ and λ is a random number. The range of λ is $[0, 1]$, and in the case when the offspring is not feasible, the operator makes attempts to generate a feasible one.

The supply plan problems we deal with are multi-objective. Thus, in order to determine the search direction of heuristic crossover, we introduced a comparison procedure consisting of three steps. $E_i(\boldsymbol{x}), (i = 1, ..., K)$ denotes the evaluation of i-th criterion of individual $\boldsymbol{x}$. K is the number of criteria. In case of maximization, the procedure is as follows:

- Step 1
  Compare two parents according to Pareto optimality. Individual $\boldsymbol{x}$ is superior to $\boldsymbol{y}$ when
  $$\forall i, E_i(\boldsymbol{x}) \geq E_i(\boldsymbol{y}) \wedge \exists i, E_i(\boldsymbol{x}) > E_i(\boldsymbol{y}) \quad . \tag{12}$$
  Otherwise, go to step 2.
- Step 2
  Compare the number of individuals superior to each parent. The parent dominated by the smaller number of other individuals in the current population is considered to be superior. If the same number of individuals dominate both $\boldsymbol{x}$ and $\boldsymbol{y}$, go to step 3.
- Step 3
  Randomly Select either of two parents as a superior.

We employed tournament selection [9]. It selects k individuals at random and selects the best one among these k individuals. k is a parameter called tournament size which determines selective pressure. Pareto optimality is also applied to determine which individual wins the tournament. Superior individuals have more chances to be selected. Consequently, Pareto optimal solution set is explored.

3.4 Boundary Initialization

In the GA we introduced, all the individuals in the initial population must satisfy the constraints. Thus, the population is generally initialized randomly in the constrained space.

In practice, it is more important to find good enough solutions in acceptable time than to find genuine optimal solutions. Thus, we propose a new initialization method, Boundary Initialization. Practically and empirically, most of optimal solutions of the real-world constraints problems are on the boundary of constraints. The method initializes the population randomly on the boundary of constrained space, and makes the search of solutions efficient. In figure 3, black dots are the individuals produced by Boundary Initialization, and white dots are produced by Random Initialization.

It is said that the diversity of initial population is very important because it ensures the exploration through the whole search space. Therefore, there is a

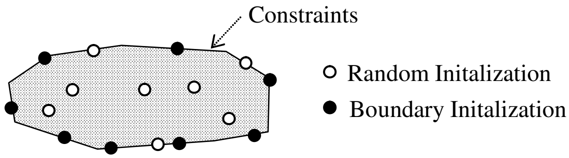


Fig. 3. Boundary Initialization and Random Initialization

possibility that search fails to reach optimal solutions when the initial population is biased on the boundary. However, we expect that the bias boosts the search efficiency in solving practical problems, and that Boundary Initialization should work well.

4 Computational Experiments

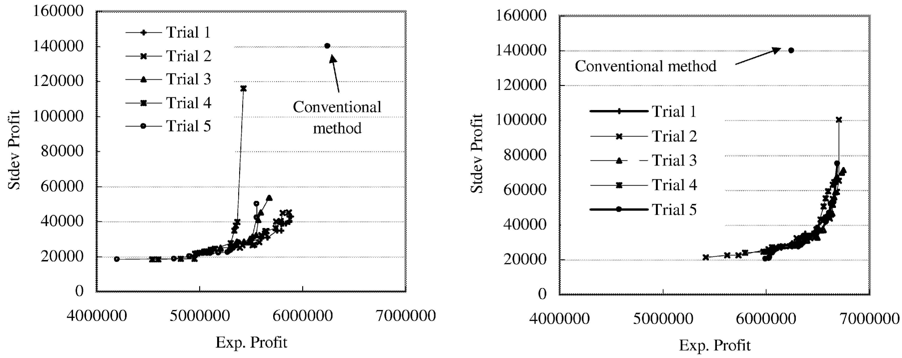
The supply planning method with GA was tested on the data provided by an electric appliances manufacturer. The data consist of ten product groups and four key resources. In our experiments, each product group is treated as a product.

In our experiments, population size was set to 100, termination was 50 generations, and tournament size was 4. We adopted the elitist policy [10] and elite size was 5. The iteration of simulations in one evaluation was 1000. That means gross profit and opportunity loss were calculated 5,000,000 times in one optimization run. It took about 255 seconds to run one optimization on Windows 2000 PC with Pentium IV 2.2GHz and 1GBytes RAM. The test program was implemented in C++. We carried out two experiments. In the first experiment, the objectives were maximizing expected gross profit and minimizing the standard deviation of the profit. The standard deviation was minimized since smaller volatility was preferable. In the second experiment, the objectives were maximizing expected gross profit and minimizing expected opportunity loss. Standard deviation of the profit and opportunity loss are the criteria of risk. We tested Random and Boundary Initialization in both experiments.

Figure 4 and Figure 5 show the Pareto optimal individuals, i.e. solutions at the last generation of five trials. Each marker corresponds to one solution and each line corresponds to the efficiency frontier of each trial. (a) is the result with Random Initialization and (b) is with Boundary Initialization. For comparison, we also tested conventional inventory management method.

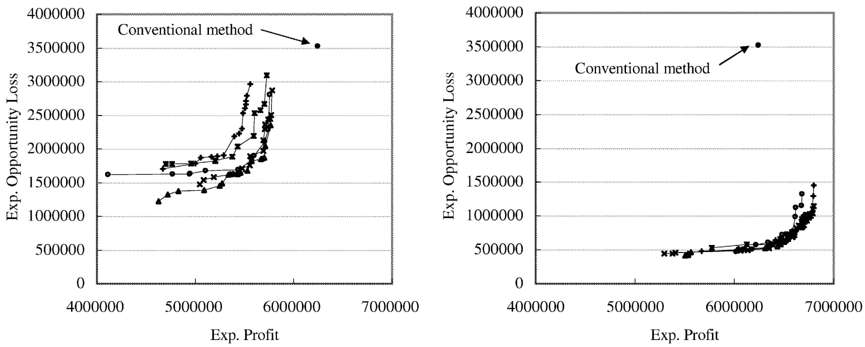
Figure 4 is the result of the first experiment maximizing expected gross profit and minimizing the standard deviation. Obviously, the optimization with Boundary Initialization is better than that with Random Initialization. The standard deviation was minimized with both methods. However, Random Initialization could not obtain a better solution even than the conventional method.

Figure 5 is the result of the second experiment maximizing expected gross profit and minimizing opportunity loss. The optimization with Boundary Initialization is much better than that with Random Initialization.



(a) Random Initialization (b) Boundary Initialization

Fig. 4. Maximization of expected profit and minimization of standard deviation of profit



(a) Random Initialization (b) Boundary Initialization

Fig. 5. Maximization of expected profit and minimization of opportunity loss

We can observe the trade-off between profit and risk from the figures. The solutions are appropriate as reasonable alternatives for decision-makers. We found that the supply planning GA with Boundary Initialization obtained extremely good solutions. The proposed approach could also provide several pareto-optimal solutions as alternatives, from high-risk and high-profit to low-risk and low-profit.

5 Conclusion

In this paper, we proposed a supply planning method employing a multi objective GA. The method uses Monte Carlo Simulation in the fitness function of GA. Monte Carlo simulation simulates uncertain demand, then profit and risk as fitness values are calculated from the simulation result. The GA searches a number of Pareto optimal solutions in one optimization run. We also proposed Boundary Initalization which initializes population on the boundary of constraints.

We tested our approach on the actual data from an electric appliances manufacturer. The proposed approach successfully optimized the supply planning problem. We also found that Boundary Initialization is more effective than Random Initialization.

The GA provided a number of Pareto optimal solutions covering from high-risk and high-profit to low-risk and low-profit. We believe this feature is very helpful to decision-makers. Since the Pareto optimal solutions can be the alternative choices, decision-makers can select one preferable solution from the alternatives according to their risk appetite and business strategies.

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