# Multicriteria Optimization and Decision Engineering of an Extrusion Process Aided by a Diploid Genetic Algorithm

**Silvère Massebeuf**
Laboratoire des Sciences du Génie Chimique – UPR CNRS 6811
ENSIC, 1 rue Grandville, BP 451,
F-54001 Nancy Cedex, France
Massebeu@ensic.u-nancy.fr

**Christian Fonteix**
Laboratoire des Sciences du Génie Chimique – UPR CNRS 6811
ENSAIA, 2 avenue de la forêt de Haye, BP 172,
F-54505 Vandœuvre-lès-Nancy Cedex, France
Fonteix@ensaia.u-nancy.fr

**Laszlo N. Kiss**
Université Laval – Faculté des Sciences de l'Administration
Bureau 3202A/PAP,
Sainte-Foy G1K 7P4, Québec, Canada
Laszlo.kiss@fsa.ulaval.ca

**Ivan Marc**
Laboratoire des Sciences du Génie Chimique – UPR CNRS 6811
13 rue du bois de la Champelle,
F-54500 Vandœuvre-lès-Nancy, France
Imarc@ensic.u-nancy.fr

**Fernand Pla**
Laboratoire des Sciences du Génie Chimique – UPR CNRS 6811
ENSIC, 1 rue Grandville, BP 451,
F-54001 Nancy Cedex, France
Pla@ensic.u-nancy.fr

**Kazimierz Zaras**
Département des Sciences de la Gestion, Université du Québec en Abitibi-Témiscamingue,
445 boulevard de l'Université,
Rouyn-Noranda J9X 6N9, Canada
Kazimierz.Zaras@uqat.uquebec.ca

**Abstract- In many, if not most, optimization problems, industrialists are often confronted with multiobjective decision problems. For example, in manufacturing processes, it may be necessary to optimize several criteria to take into account all the market constraints. So, the purpose is to choose the best tradeoffs among all the defined and conflicting objectives. In multicriteria optimization, after the decision maker has chosen all his objectives, he has to determine the multicriteria optimal zone by using the concept of domination criterion called the Pareto domination. Two points, in the research domain, are compared. If one is better for all attributes, it is a non dominating solution. All the non dominating points form the Pareto's region. In this paper, several multiobjective optimization algorithms are used to obtain this zone. These methods are based on a diploid genetic algorithm and are compared to an industrial application: food's granulation. In the optimal zone, the decision maker has to choose the best solution after he has made a ranking with all potential solutions. A partition is made and the decision maker has more information on the process. Finally, a Decision Support System shell is developed in order to classify all solutions.**

## 1 Introduction

In a lot of domains, processing or product formulation depends on several objectives to take into account their different features. Extrusion processes for food are a good illustration for this case. For example, it may be necessary to optimize different parameters such as texture, flavour, and so on, in order to formulate a new product and to maximize yields, production and product quality with minimal investment whereas these criteria are not optimal for the same working conditions. In the engineering processes domain, multiple objectives have usually been combined, often through a linear combination, to form a scalar objective function (Courcoux *et al.* 1995) or a geometric average of these ones (Derringer and Suich 1980). Another classical method consists in optimizing one chosen answer and in turning into constraints the others (Logothetis and Haigh 1988). These techniques depend on the user's choice at the begining, so preferences can bias the results.

Methods incorporating a domination criterion are often more interesting because they are more general, more accurate and without *a priori* knowledge. The object is to find a non dominated zone in which a decision maker will be able to choose the best solution. This region, called the Pareto domain, is the set of all non dominated points. For the Pareto's domination criterion, a point dominates another if it is better for one criterion, and better or equal for the others. The aim of the study is not to find immediately the preferred solution but to exclude all conditions which are not interesting. This information allows to choose the industrial decision maker's preferences, because it restricts all possible choices, mistakes and bias.

An extension of the traditional Genetic Algorithm (GA) is described (Schaffer 1985) which allows the searching of the parameter space where multiple objectives are to be optimized. His VEGA (Vector Evaluated GA) gives a selection preference to the non dominated members of a population. But, only extreme points on the Pareto front seem to be found with VEGA, so a niching pressure (Horn *et al.* 1994) is applied to spread its population out along the Pareto optimal tradeoff surface. In the same way, the non domination ranking (Fonseca and Fleming 1993) is used for selection to move a population towards the Pareto front as well as a niching mechanism, such as sharing (Goldberg 1989), to keep the GA from converging to a single point. Moreover, a direct intervention of an external Decision Maker (DM) gives interactive information in the

multiobjective optimization loop. So, a satisfactory solution of the problem is found as soon as the knowledge is acquired (Fonseca and Fleming 1993).

In this study, the Pareto domain is obtained, at first, without external influences. Several methods are presented and compared in the next section, using an adapted GA. These algorithms are then applied to an industrial problem (the example of an extrusion process) and a first partition is made to try to separate the Pareto's region into zones of similar criteria values. On the last step, the DM influences are taken into account and notions of decision engineering are required. So, the global multicriterion analysis leads to the best tradeoff.

## 2 Theoretical algorithmic background to the multiobjective optimization

### 2.1 Pareto's domination criterion on a global stochastic research

The easiest method to determine the Pareto set is to take a lot of random points in the research area and to compare them all two by two. All the non dominating points can form a view of the Pareto's zone. But, for a n-dimensionnal optimization problem (n>3) with a large area, the non dominating solutions can be out of the researched region. Moreover, if the applications have a small zone, the attainment of the multicriteria optimal region may be impossible, or with so few points, it is imposible to obtain it with a lot of precision.

As an illustration, a didactical example is presented: minimize two functions (n=2):

$$f_1(x,y) = 200(1+3x-2y)^2 + 4(1+4x-4y)^2$$

optimum (-0.5,-0.25)

$$f_2(x,y) = 200(1-2x)^2 + 7(1+4y)^2$$

optimum (0.5,-0.25)

In this case, it is easy to show that the Pareto domain is the curve:

$$y = (5925 - 7538x - (14485361 - 105968388x + 196619844x^2)^{1/2})/4424$$

The representations are drawn in Figure 1 with the method of global stochastic research (left) and compared with the Pareto's region found with the method described in 2.4 (right).

The obtained Pareto's region with the simple method is not very precise in this example. 1000 points randomly created in the begining give 60 non dominated points to form a set Pareto view. In the second graph, the Pareto's region is represented by 1000 points and is very close to the real Pareto curve. So, to have a more precise and real zone, other methods are based on a genetic algorithm and an improved one is presented with the concept of diploidy and the control of the dominance.

### 2.2 An improved Diploid Genetic Algorithm (DGA)

The multiobjective optimization methods developed use a DGA whose principles were elaborated by (Perrin *et al.* 1997). The diploid version is kept because its performances were found to be better compared with a haploid one (Fonteix *et al.* 1995). Each individual (which can be a possible solution of the problem) is described by a four-tuple ($a_j$, $a_j'$, $D_j$, $x_j$). $a_j$ and $a_j'$ represent the two alleles of the j gene, $D_j$ is the dominance of one allele over the other chosen in {0,1} values and $x_j$ represents the phenotype which is the result of the combination of the respective alleles, $a_j$ and $a_j'$:

$$x_j = D_j.a_j + (1-D_j).a_j'$$

An initial population is created by generating a set of m points from the search area. Each point is tested and evaluated. If this population is not the solution, then genetic operators are used to make it evolve. Only the better individuals will survive (elitist selection) and participate in the creation of a new generation. The reproduction of the individuals in the diploid model consists of a multi-crossover of the two chromosomes of each parent, a mutation and a homozygosity. Mutation consists, for the selected individual, of a randomly draw for all the genes of the two chromosomes and all the dominances. Homozygosity allows to modify a child by copying the phenotype x of the two chromosomes a and a' ($x_j = a_j = a_j'$). All the genetic operators and their usefulness are detailed in (Perrin *et al.* 1997). If the generated child is worse than the worst parent, he is not adapted. He is eliminated and another is created. So dominance may be controlled to decrease the child death rate.

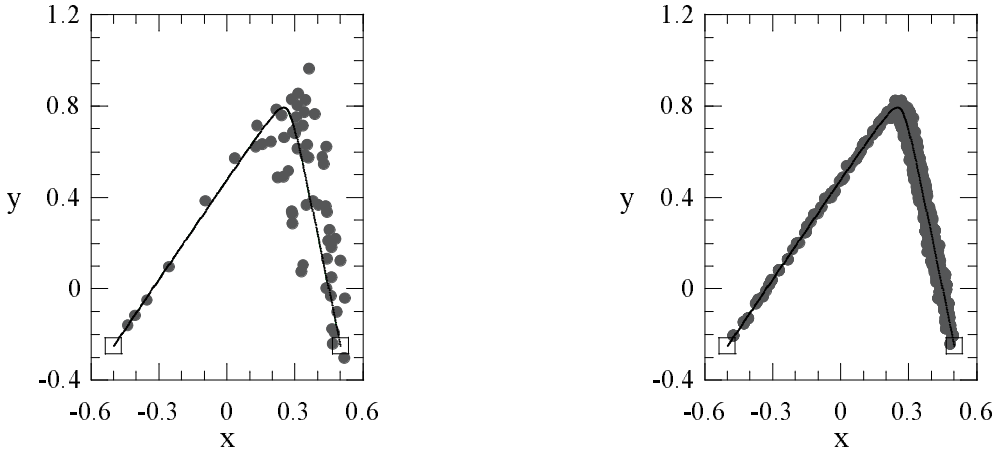



Figure 1 : *Pareto's zone of the example with two methods*

●: *solutions of Pareto's zone*, □: *optimum of each function*, -: *real Pareto curve*

| | | Dominance of $a_j$ in the first parent | |
|---|---|---|---|
| | | Recessive | Dominant |
| Dominance of $a_j$' in the second parent | Recessive | Random | $D_j = 1$ |
| | Dominant | $D_j = 0$ | Function of the best parent |

Table 1: *choice of the value of the dominance* $D_j \in \{0,1\}$

The dominance may be randomly chosen in {0,1} values. But, to keep dominance features, table 1 shows how to control $D_j$. Statistically, children have more chances of being better than the parents with the control of the dominance, so the convergence of the algorithm speeds up and less points are tested.

The population of each generation is evaluated until it satisfies the stop criterion: $f_{max} - f_{min} < \varepsilon$ where $f_{min}$ and $f_{max}$ are respectively the minimal and the maximal objective function values in the current population and $\varepsilon$ is the given precision for solution estimation.

**2.3 Algorithm with intermediate populations based on a DGA**

The approach developed uses the multicriteria optimization algorithm previously elaborated (Viennet *et al.* 1995). The first step is to find the optimal solution for each objective separately with the DGA described in 2.2. For each optimum, the value of the function is determined for all the objectives and the worst is kept. A population is then built in the second step for each criterion in such a way that the individuals are better than the worst value determined before. Populations are completed by a birth phase to have enough individuals. Then, all populations are gathered into a single one. The Pareto's zone is included in the proposed solutions, so the domination criterion is applied to compare all the points two by two and to give the final population, called optimal zone (Viennet *et al.* 1996).

This algorithm allows to restrict the search area and to define a more precise region. But the number of points in the Pareto's zone is not controlled. After the domination criterion has been applied, the number of points is not known and may be low, depending on the problems. So, a new method is developed to control the number of points. This parameter is very important for the decision aid methods as the objective is to find the best solution. Another crucial point is to decrease the calculus time because the algorithm summons up the DGA for each criterion. So it may be interesting to save of time by using the DGA just once.

**2.4 Use of one function to define the set Pareto**

The aim of this section is to define one function to make a monocriterion optimization with the DGA. A set of m given points is randomly generated. All individuals are evaluated by the calculation of each objective $f_i$ (i = 1,…,n). Then, for each point $x_j$, a value is associated, $F(x_j)$, the number of times that it is dominated by all the others in the current generation:

$$F(x_j) = \sum_{p=1}^{m} h_{jp} \qquad \text{where} \quad h_{jp} = 1 \text{ if } x_p \text{ dominate } x_j$$

$$h_{jp} = 0 \text{ else}$$

Let be denoted by n the number of criteria. For two points $x_j$ and $x_p$ in the same population, and for all criteria i (i = 1,…,n):

if $f_i(x_j)$ WORSE THAN $f_i(x_p)$ $c_{ijp} = 0$
if $f_i(x_j)$ BETTER THAN $f_i(x_p)$ $c_{ijp} = n + 1$
if $f_i(x_j)$ EQUAL TO $f_i(x_p)$ $c_{ijp} = 1$

where $c_{ijp}$ is an intermediate variable.
In the case of minimization WORSE THAN is equivalent to <, and BETTER THAN to >.

if $\sum_{i=1}^{n} c_{ijp} < n$ then $h_{jp}=1$ and $h_{pj}=0$ because $x_j$ is dominated by $x_p$.

if $\sum_{i=1}^{n} c_{ijp} \geq n$ and $\sum_{i=1}^{n} c_{ipj} \geq n$ then $h_{jp}=h_{pj}=0$ ($h_{jj}= 0$).

So, for each point, a value F corresponds by applying the Pareto's domination criterion. Then, all the points may be classified. The value of the function for the better individuals in the current generation is equal to zero. The classified individuals and their function value are presented in Table 2 in the $k^{th}$ generation. The s better individuals are non dominated in the current population (k) and are the chosen parents for the next (k+1). The (m-s) worse individuals are eliminated for the next generation but are used for the children evaluation. Each new individual is compared with all m points in the previous population. Then, a threshold t is defined to keep the better children:

$t = s + E[0.3*(m-s)]$ where E[x] is the entire part of x and 0.3 is empirically chosen.

So, a child death rate is applied to the convergence when the new individual $x_j$ is worse than the $x_t$ point, *i.e.* $F(x_j) > F(x_t)$. When the new population is reformed, all the m points are evaluated with the new values of the function F. The population of each generation is evaluated until the stop criterion is satisfied: all the points are non dominated, that is to say that F is equal to zero for the m points.

| **X** | $x_1$ | $x_2$ | $x_3$ | ..... | $x_s$ | $x_{s+1}$ | ..... | $x_t$ | $x_{t+1}$ | ..... | $x_m$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **F** | 0 | 0 | 0 | ..... | 0 | $F(x_{s+1})$ | ..... | $F(x_t)$ | $F(x_{t+1})$ | ..... | $F(x_m)$ |

Table 2: *classified individuals in the $k^{th}$ generation*

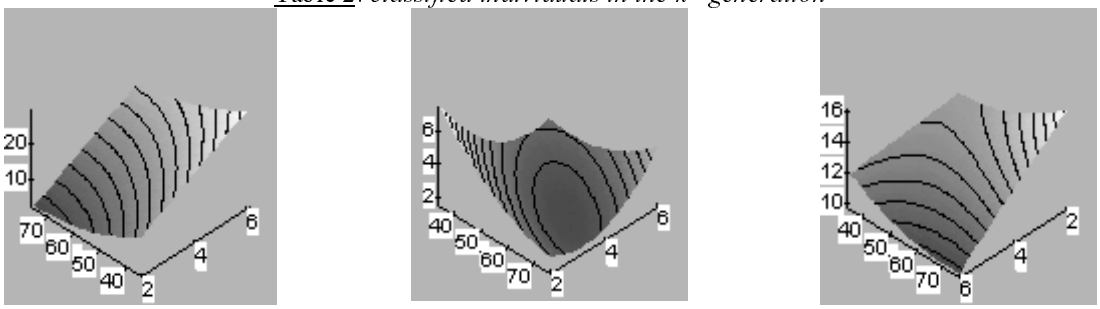



Figure 2: *The evolutions of energetic consumption (left), friability index (middle) and moisture (right) of granules vs the temperature and drawplate profile, (Courcoux et al. 1995)*

The DGA is used only once and the algorithm converges rather rapidly (3 to 5 generations, depending on the problem). The most interesting thing is that the number of points in the Pareto's zone is controlled whatever the problem studied. If 5000 points are required to clearly draw the Pareto's area, it is represented by all these points with a lot of accuracy and without an increasing calculus time.

## 3 Application to an extrusion process

### 3.1 Description of the process

Food granulation for cattle is presented as an industrial application. The goal is the optimization of the working conditions of an extrusion process (Courcoux *et al.* 1995). A pulverulent product is converted into granules due to the conjugated effects of heat, moisture and pressure. The industrialist would like to simultaneously minimize moisture and friability of his product and the energetic consumption of the process. The modelling of these criteria needs an experimental design (Courcoux *et al.* 1995). Factors having an influence on this process are the feeding rate, speed of rotation for mixing, flour temperature, speed of rotation and drawplate profile. For this study, only two factors are taken into account, the flour temperature T, situated between 35 and 75 °C, and the drawplate profile D, between 2 and 6 cm (figure 2). Courcoux *et al.* obtained the supposed « optimal working conditions » by minimizing a linear combination of criteria (white square in Figure 3).

### 3.2 Pareto domain determination

The optimal Pareto's zone is represented in figure 3 by black points with the two methods described in 2.3 (left) and 2.4 (right). Note that the supposed « optimal working conditions » obtained by Courcoux *et al.*, is placed on the bordline of the Pareto's region. Consequently, any negative variation in the flour temperature could entail a degradation of the quality of the product. This solution is not robust. The objective in 4.3 is to search for a more robust solution (the point must stay in the Pareto domain in spite of a medium temperature variation).

The two methods give satisfactory results for the Pareto's zone. The first method begins with 2000 points for each of the three criteria (*i.e.* 6000 points) and 2500 points remain at the end of the algorithm. The second method keeps its 5000 points for the representation. It allows to have a more precise area with a controlled

number of points. The calculus time is equivalent for the two methods but the result is better with the use of the domination function. So, the decision maker will be able to choose in this zone the best point.

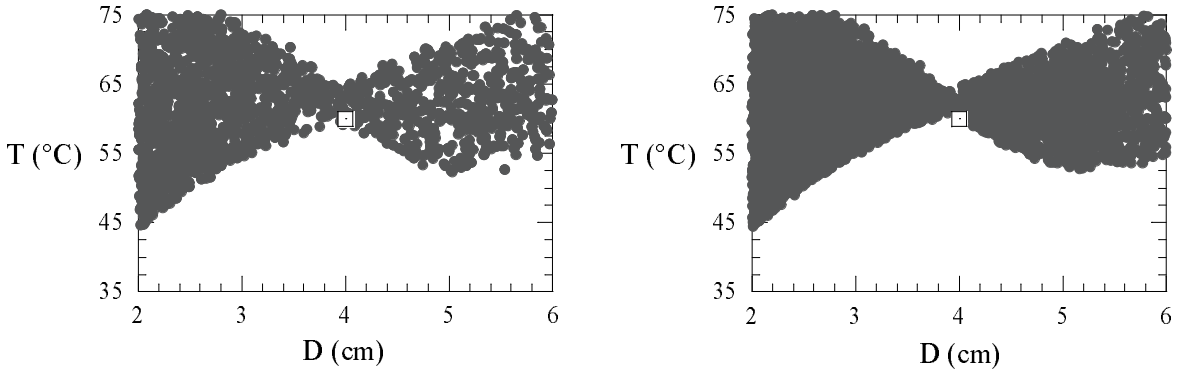



Figure 3: *representation of the optimal Pareto's zone and a supposed « optimal point » (white square, by Courcoux et al.) for the example of an extrusion process*

### 3.3 Pareto domain partition with DGA

A future ranking of the points in the Pareto's zone will be studied by rough sets (Zaras 1998). But, this technique needs a classification of some points by the DM. The problem of the points choice is not solved. So, in this paper, we propose to separate the Pareto domain by DGA in zones of similar criteria values. It is not considered as a ranking because the DM does not intervene. It is an aid before the DM preferences choice. In order to separate the Pareto domain in to several similar zones, an algorithm is elaborated. All the points in the same class have a comparable influence on the process. All the criteria values are normed between 0 and 1 for all the Pareto efficient points and q centers $C_k$ (k=1,…,q) are found in the objectives normed space. Then, each point is associated with the closest center. The centers are found minimizing the largest average quadratic radius and using the DGA described in 2.2:

$$\text{MIN}\left\{ \max_{(k)} \frac{\sum_{j \in J_k} (C_k - z_j)^2}{\sum_{j \in J_k} 1} \right\}$$

where $j \in J_k$ if $v(j) = k$ ; $v(j) = \text{Arg}\left( \min_{(k)} (C_k - z_j)^2 \right)$

and where $z_j$ is a normed criteria vector (each value in [0,1]) for all points j in the set Pareto (j=1,…,m). Note that $J_k \cap J_p = \varnothing \ \forall \ k \neq p$.

In the extrusion process example, 5 centers are chosen to separate the zone (q=5). Figure 4 shows the result of the Pareto domain partition in the action variables space after the use of DGA.

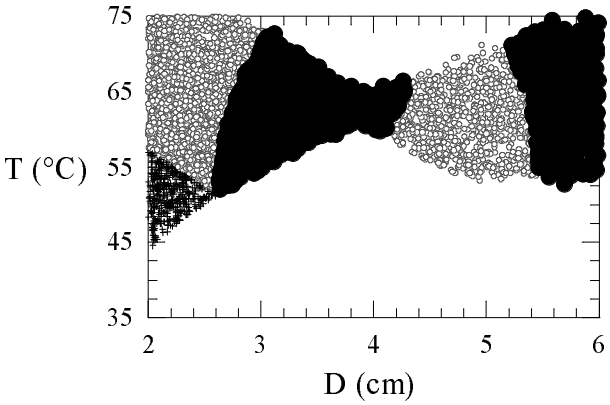



Figure 4: *Pareto domain partition in 5 classes*
*+: zone 1, ○: zone 2, ●: zone 3, ○: zone 4, ●: zone 5*

5 influence zones are determined without *a priori* decisions about a ranking to know which is the best. To classify all the Pareto efficient points, some decision making notions are required in order to point out the best working conditions.

## 4 Multicriteria analysis framework

### 4.1 An original approach

Many real problems can be modeled as multicriteria problems but the decision engineering knowledge intervention in a specific chemical engineering competence field as the cattle-food granulation process is a very interesting new research. We dare to believe that even if our results are inscribed in a relatively narrow axis and in spite of their limits, they will contribute to the development of the knowledge in the chemical process engineering and in the decision making sciences.

This process control problem can obviously be most appropriately structured around an ELECTRE type Outranking Synthesis Approach (OSA) (Roy *et al.* 1993). The main reasons justifying our choice are, in fact, the realistic way in which this OSA links the preferences; this is somewhat similar to the "democratic" principle much more natural when faced with a decision having product quality and product profitability impacts.

**4.2 Algorithmic design**

Let us consider the *input parameters' set of the granulation process* containing the discretized *temperature* (between 35 and 75 °C) and *drawplate profile* D (between 2 and 6 cm) values on the one hand and *the output parameters' set of the granulation process* containing three criteria, namely the energetic consumption, the friability index and the moisture of the granules on the other hand. The criteria role has been attributed to the output parameters' set of the granulation process, *i.e.* energetic consumption, friability index and moisture of granules (to minimize the 3), even when the alternatives role has been attributed to the input parameters' set of the granulation process, *i.e.* temperature and drawplate profile.

Our multicriterion analysis algorithm is essentially a comparison process where the decision maker's preferences are expressed by means of some constraint-like parameters: indifference, preference, veto thresholds and the weights of the criteria. In order to operationally run the Multicriterion Analysis Process (MAP), it is necessary to introduce the MAP's input set $\Psi$ with the following synoptic structure (Derot et al. 1997):

$$\Psi = \Gamma_{\S}\,[m,n,M_{[m\,x\,n]},K_{[n]},\Phi_{[n]},$$
$$R_{[n]},Q_{[n]},P_{[n]},V_{[n]},W_{[n]}]$$

where

- $\Gamma_{\S}$ - MAP's algorithm
- m - number of alternatives
- n - number of criteria
- $M_{[m\,x\,n]}$ - real mxn cardinal matrix containing the alternatives' evaluations in relation to the criteria and $\overline{m}_{ij} = f_j(x_i) \in M_{[m\,x\,n]}$; i = 1,…,m; j = 1,…,n
- $K_{[n]}$ - binary n cardinal vector indicating the nature of the criteria evaluation, with

$$\kappa_j = \begin{cases} 0 \text{ if the evaluation in relation to the } j^{th} \text{ criterion is an ordinal type} \\ 1 \text{ if the evaluation in relation to the } j^{th} \text{ criterion is a cardinal type} \end{cases}$$

and $\kappa_j \in K_{[n]}$; j = 1,…,n (in this paper all the criteria are cardinal type)

- $\Phi_{[n]}$ - binary n cardinal vector indicating the nature of the orientation of the criteria, with

$$\phi_j = \begin{cases} 0 \text{ if the } j^{th} \text{ criterion is to be minimized} \\ 1 \text{ if the } j^{th} \text{ criterion is to be maximized} \end{cases}$$

and $\phi_j \in \Phi_{[n]}$; j = 1,…,n (0 for all criteria in this paper)

- $R_{[n]}$ - real n cardinal vector containing the range for every criteria and $\rho_j \in R_{[n]}$; j = 1,…,n
- $Q_{[n]}$ - real n cardinal vector containing the indifference thresholds and $q_j \in Q_{[n]}$; j = 1,…,n
- $P_{[n]}$ - real n cardinal vector containing the preference thresholds and $p_j \in P_{[n]}$; j = 1,…,n
- $V_{[n]}$ - real n cardinal vector containing the veto thresholds and $v_j \in V_{[n]}$; j = 1,…,n
- $W_{[n]}$ - real n cardinal vector containing the criteria's relative importance coefficients and $w_j \in W_{[n]}$; j = 1,…,n.

The following relations must be verified between the thresholds:

$$0 \le q_j \le p_j \le v_j \le \rho_j;\ j = 1,\ldots,n$$

The relative importance coefficients are normalized, *i.e.* $\sum_{j=1}^{n} w_j = 1$.

This input set $\Psi$ is processed using the MAP algorithm $\Gamma_{\S}$ to determine the ranking from the best to the worst (*i.e.* "nadir") alternative with possible *ex aequo*.

For each pair of alternatives $[i, \bar{i}]$; i = 1,…,m; $\bar{i}$ =1,…,m; $i \neq \bar{i}$, a global concordance index $C\left|[i; \bar{i}]\right.$ is calculated as follows:

$$C\left|[i; \bar{i}]\right. = w_j \cdot \delta_j\left|[i, \bar{i}]\right.$$

i = 1,…,m; $\bar{i}$ =1,…,m; $i \neq \bar{i}$; j = 1,…,n.

where $\delta_j\left|[i, \bar{i}]\right. = \begin{cases} 1 & \text{if } -\Delta_j \le q_j \\ \dfrac{\Delta_j + p_j}{p_j - q_j} & \text{if } q_j < -\Delta_j \le p_j \\ 0 & \text{if } p_j < -\Delta_j \end{cases}$

and $\Delta_j = \overline{m}_{ji} - \overline{m}_{j\bar{i}} \in M_{[m\,x\,n]}$; j = 1,…,n; i = 1,…,m; $\bar{i}$ =1,…,m; $i \neq \bar{i}$. If some criteria call for maximization while others require minimization, we have introduced two temporary *sentries* (*i.e.* guards) variables k and $\bar{k}$ in our calculations of $\Delta_j$. If $\phi_j$ = 0, meaning that this criterion is to be minimized, then k = $\bar{i}$ and $\bar{k}$ = i, else if $\phi_j$ = 1 (criterion to be maximize) k = i and $\bar{k}$ = $\bar{i}$; this way $\Delta_j\left|[i, \bar{i}]\right. \equiv \Delta_j = \overline{m}_{jk} - \overline{m}_{j\bar{k}}$; $\forall\, \phi_j$; j = 1,…,n; k = 1,…,m; $\bar{k}$ = 1,…,m; $k \neq \bar{k}$. A discordance

index $\mathbf{D}_j\left|\left[i;\bar{i}\right]\right.$ is also calculated for every pair of alternatives at each criterion level as with ELECTRE III:

$$\mathbf{D}_j\left|\left[i;\bar{i}\right]\right. = \begin{cases} 0 & \text{if } -\Delta_j \le p_j \\ \dfrac{-\Delta_j - p_j}{v_j - p_j} & \text{if } q_j < -\Delta_j \le v_j \\ 1 & \text{if } v_j < -\Delta_j \end{cases}$$

for $j = 1,\ldots,n$; $i = 1,\ldots,m$; $\bar{i} = 1,\ldots,m$; $i \neq \bar{i}$ .

Using the concordance and discordance indexes, we generate outranking degrees $\sigma\left|\left[i;\bar{i}\right]\right.$ for every pairs of alternatives. These outranking degrees are obtained using the following formula (Rousseau and Martel 1994):

$$\sigma\left|\left[i;\bar{i}\right]\right. = C\left|\left[i;\bar{i}\right]\right. \prod_{j=1}^{n}\left[1 - \left(\mathbf{D}_j\left|\left[i;\bar{i}\right]\right.\right)^3\right]$$

The obtained outranking relation sets may be represented as an outranking matrix. Working with outranking relations implies synthetizing that matrix in such a way as to provide a process control recommendation in the form of an alternative ranking.

We make that synthesis following the notion of *outgoing flow* and *incoming flow* from PROMETHEE (Brans *et al.* 1984).
For calculations, we use:

$$\sigma_i^+ = \begin{cases} \sum_{i=1}^{m} \sigma\left|\left[i;\bar{i}\right]\right. & \text{if } i \neq \bar{i} \\ 1 & \text{if } i = \bar{i} \end{cases}$$

$$\text{and } \sigma_i^- = \begin{cases} \sum_{\bar{i}=1}^{m} \sigma\left|\left[\bar{i};i\right]\right. & \text{if } i \neq \bar{i} \\ 1 & \text{if } i = \bar{i} \end{cases}$$

for $i = 1,\ldots,m$; $\bar{i} = 1,\ldots,m$; $i \neq \bar{i}$, and we get the total ranking of the alternatives from the *net flow* as in PROMETHEE II:

$$\sigma_i = \sigma_i^+ - \sigma_i^- \quad \text{for } i = 1,\ldots,m.$$

**4.3 Results**

In order to illustrate the proposed treatment, we present the results, using the earlier described $\Gamma_{\S}$ algorithm. We have developed a Decision Support System (DSS) shell based on our multicriterion analysis conception with the tresholds indicate in table 3.

| | $q_j$ | $p_j$ | $v_j$ |
|---|---|---|---|
| Friability index | 0.2 | 0.5 | 0.8 |
| Moisture | 0.5 | 1.5 | 3 |
| Energetic consumption | 1 | 3 | 6 |

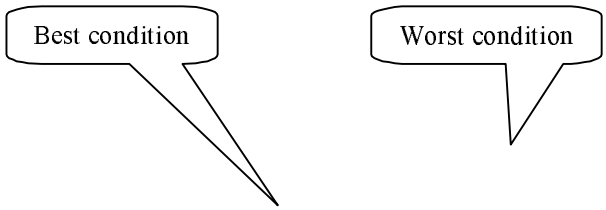
Table 3: *indifference, preference and veto tresholds for each criterion*

| **Best** | | **Nadir** | |
|---|---|---|---|
| D (cm) | T (°C) | D (cm) | T (°C) |
| 3.32 | 64.9 | 5.29 | 72.7 |

Table 4: best and worst parameters alternatives

Table 4 gives the best and the worst alternatives in the Pareto domain, after the total ranking has been made.

This DSS shell produces very encouraging final results as process control recommandation for the chemical engineering. To illustrate the robustness of the results, we present *quintile* by *quintile* the input parameters of the granulation process as well as the positions of the best and the worst input parameters (Figure 5). Each *quintile* contains 500 points to have the ranking of about 2500 points.



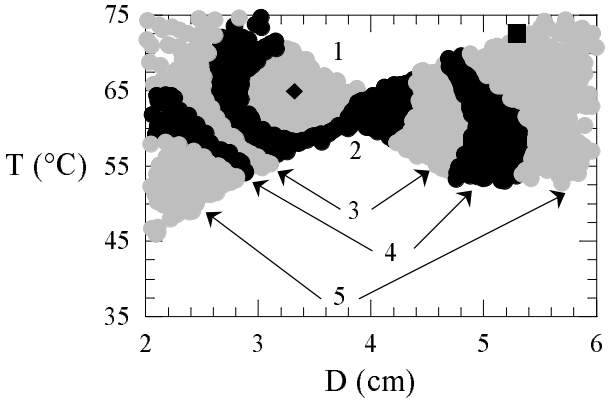




Figure 5: *best (◆) and worst (■) conditions giving by DSS shell*

We can notice that the best point is a robust one and the little variations of the working conditions do not alter much the product quality, the point stay in the Pareto domain. On the contrary, the worst point is on the borderline of the zone and so is not robust. The *quintiles* are represented by concentric zones (first zone: ●, second

zone: ●, third zone: ●, *etc*…). Courcoux's supposed « optimal working conditions » (white square in figure 3) are not only unrobust, but also in the second *quantile*.

If we want to compare the total ranking of the alternatives by DSS shell with the partition made by DGA in 3.3, Table 5 presents the characteristics of the found centers. In the normed criteria space, (0 0 0) corresponds to the best theoretical solution and (1 1 1) to the worst theoretical solution. So, square distances between the centers and the Ideal $D_I^2$, and between the centers and the theoretical Nadir $D_N^2$ are determined. The real criteria value of each center is noted too. Class number does not have a decisional value. We can notice that the best alternative found by DSS shell is in class 3 which is certainly the best class (its center is closest to the ideality and farest to the theoretical nadir). On the contrary, the found nadir is in class 5 which is certainly the worst class (its center is farest to the ideality and closest to the theoretical nadir). So the Pareto domain partition gives a first idea before a final ranking with decision maker influences. It is an aid for the DM to choose his preferences, *i.e.* parameters needed by the DSS shell algorithm or by a rough sets method.

| | Distances | | Criteria | | |
|---|---|---|---|---|---|
| center | $D_I^2$ | $D_N^2$ | friability index | moisture | energetic consumption |
| 1 | 0.85 | 1.11 | 3.05 | 10.86 | 18.11 |
| 2 | 0.58 | 1.31 | 2.68 | 14.10 | 5.81 |
| 3 | 0.39 | 1.39 | 2.26 | 13.00 | 8.74 |
| 4 | 0.59 | 1.26 | 2.46 | 11.38 | 15.71 |
| 5 | 0.95 | 0.74 | 4.37 | 13.29 | 8.56 |

Table 5: *characteristics of the centers*

## 5 Concluding remarks and further considerations

In this work, a multiobjective decision problem is treated in order to explain an extrusion process. The main objective is the determination of the extrusion processs' operating conditions which correspond not only to the traditional optimization problems, but also to the choice of the operating conditions according to the desired technological, chemical engineering priorities or preferences of the decision maker. So, the proposed steps (at first, the determination of the multiobjective optimal zone and then the multicriteria analysis) give a lot of data to choose the best tradeoff of the problem. This type of analysis design concerns principally the decisional engineering competence, but requires also a good knowledge of the chemical phenomena as well as the definition of a sufficiently precise model.

A Decision Support System shell gives the complete ranking of the Pareto domain. We notice the robustness of the best operating conditions, *i.e.* it can stay in the same *quintile* even with small variations of temperature and profile drawplate. It is an important result for the decision making science. The good agreement between the *quintiles* presented in Figure 5 and the Pareto domain partition with a Diploid Genetic Algorithm allows us to think that the Decision Maker has useful information to define his preferences. With this partition, he has more knowledge to decide.

The results of our analysis can be registered in the chapter of the innovating contributions of the art of chemical engineering and opens promising perspectives towards the exploration for the decision robustness problem concerning the effective control of the chemical processes. For example, one of the close perspectives is to apply the analysis to polymerization processes which are more complex for modelling (Massebeuf *et al.* 1998). On the other hand, decision engineering applied to chemical engineering leads us to define decision parameters ($w_j$, $p_j$, $q_j$, $v_j$, $\forall$ j) in the form of intervals or with fuzzy membership functions. In the case of a complex process, the natural difficulty to define simple preferences leads to adapt decision engineering techniques. The objective is to search for the decision capable of staying the best (or among the bests) in spite of small parameter variations. So, adaptation techniques and new projections in the decision aid science will be needed to solve the problem of decision robustness.

It seems that this paper proves even the natural *raison d'être* of the symbiosis between scientific disciplines apparently very different, such as the science of decision making and the chemical science in the area of controling various processes. Now, chemical engineering has the possibility to solve its multiobjective decision problems and decision engineering can adapt their techniques to chemical industrial problems.

## Bibliography


Brans J.P., Marechal B. and Vincke Ph. (1984) "PROMETHEE: A new family of outranking methods in MCDM", *IFORS'84*, North Holland, pp.477- 490.

Courcoux P., Qannari E. M., Melcion J. P. and Morat J. (1995) "Optimisation Multiréponse : Application à un Procédé de Granulation d'Aliments". *Récents Progrès en Génie des Procédés : Stratégie Expérimentale et Procédés Biotechnologiques*, 36, 9, pp. 41-47.

Derot B, Gareau J., Kiss L.N. and Martel J.-M. (1997) "The solver of Volvox Multicriterion Table", *in Multicriteria Analysis*, J. Clìmaco(Ed.), pp. 113-126, Springer-Verlag.

Derringer G. and Suich R. (1980) "Simultaneous optimization of several response variables", *Journal of Quality Technology*, 12, pp. 214-219.
Fonseca C.M. and Fleming P.J. (1993) "Genetic algorithms for multiobjective optimization: formulation, discussion and generalization", *Proc. of the fifth Int. Conf. On Genetic Algorithms*, Morgan-Kauffman, pp. 416-423.
Fonteix C., Bicking F., Perrin E. and Marc I. (1995) "Haploid and Diploid Algorithms, a New Approach for Global Optimization: Compared Performances". *Int. J. of Systems Science*, 26, 10, pp. 1919-1933.
Goldberg D.E. (1989) "Genetic algorithms in search, optimization, and machine learning", Addison-Wesley.
Horn J., Nafpliotis N. and Goldberg D.E. (1994) "A niched Pareto genetic algorithm for multiobjective optimization", *Proc. of the first Conference on Evolutionary Computation, IEEE World Congress on Computational Intelligence*, vol. 1, pp. 82-87.
Logothetis N. and Haigh A. (1988) "Characterizing and optimizing multi-response processes by the Taguchi method", *Quality and Reliability Engineering International*, 4, pp.159-169.
Massebeuf S., Fonteix C. and Pla F. (1998) "Modélisation et optimisation multicritère du procédé de polymérisation en émulsion du styrène", Pla-Schrauwen ed., *17èmes Journées Scientifiques du Club Emulsion*.
Perrin E., Mandrille A., Oumoun M., Fonteix C. and Marc I. (1997) "Optimisation Globale par Stratégie d'Evolution : Technique Utilisant la Génétique des Individus Diploïdes". *RAIRO-Recherche Opérationnelle*, 31, 2, pp. 161-201.
Rousseau A. and Martel J.-M. (1994) "Environnemental Assessment of on Electric Transmission Line Project: A MCDM method", *in Applying Multiple Criteria Aid for Decision to Environmental Management*, Paruccini (Ed.), Kluwer Academic Publishers, pp. 163-165.
Roy B. and Bouyssou D. (1993) "Aide multicritère à la décision: méthodes et cas", *Economica*, Paris.
Schaffer J.D. (1985) "Multiple objective optimization with vector evaluated genetic algorithms", In Grefenstette J.J. editor, *Proc. Int. Conf. On Genetic Algorithms and their Applications*, pp. 93-100.
Viennet R., Fonteix C. and Marc I. (1995) "New Multicriteria Optimization Method Based on the Use of a Diploid Genetic Algorithm: Example of an Industrial Problem". *Lecture Notes on Computer Science. Artificial Evolution: European Conference, AE 95*, Brest, France, Springer-Verlag Ed. 1063, pp. 120-127.
Viennet R., Fonteix C. and Marc I. (1996) "Multicriteria Optimization Using a Genetic Algorithm for Determining a Set Pareto". *Int. J. of Systems Science*, 27, 2, pp.255-260.
Zaras K. (1998) "A Rough approximation of a preference relation by a multi-attribute stochastic dominance for reduced number of attributes". *Proceedings of ASAC98 Conference*, Sakatoon (Saskatchewan), 19, pp. 82-89.