A Two-Level Evolutionary Approach to Multi-Criterion Optimization of Water Supply Systems

Matteo Nicolini

University of Udine, Faculty of Engineering
Dipartimento di Georisorse e Territorio, via Cotonificio 114, 33100 Udine (Italy)
nicolini@dgt.uniud.it

Abstract. Purpose of the paper is to introduce a methodology for a parameter-free multi-criterion optimization of water distribution networks. It is based on a two-level approach, with a population of inner multi-objective genetic algorithms (MOGAs) and an outer simple GA (without crossover). The inner MOGAs represent the network optimizers, while the outer GA – the *meta* GA – is a supervisor process adapting mutation and crossover probabilities of the inner MOGAs. The hypervolume metric has been adopted as fitness for the individuals at the meta-level. The methodology has been applied to a small system often studied in the literature, for which an exhaustive search of the entire decision space has allowed the determination of all Pareto-optimal solutions of interest: the choice of this simple system was done in order to compare the hypervolume metric to two performance measures (a convergence and a sparsity index) introduced on purpose. Simulations carried out show how the proposed procedure proves robust, giving better results than a MOGA alone, thus allowing a considerable ease in the network optimization process.

1 Introduction

The problem of choosing the optimal combination of pipe diameters, in order to minimize the overall cost of a looped water distribution system (given a finite set of commercial available sizes), is proven to be NP-hard [1]. In the last decades, many authors have proposed several approaches based on different optimization techniques, mainly linear programming [2], [3], [4], [5], [6], [7] and non-linear programming [8], [9].

More recently, several researchers have applied genetic algorithms (GAs) to single-objective optimization of water supply systems, introducing some improvements with respect to the simple GA, [10], [11], [12], [13]. [14] applied GAs to optimal location of control valves, while [15] and [16] to leak detection and calibration problems; [17] used GAs for optimal scheduling of pipe replacement.

[18] have shown that networks designed taking into account only cost minimization (and in the case of just one loading condition) tend to branched configurations, as also pointed out by [19]. In a recent editorial, [20] stressed the need of adopting a multi-objective approach for the design of water supply systems.

These last years have seen an increasing number of applications of multi-objective optimization algorithms: generally, only two-objective problems have been considered, the first criterion being the total cost of the system and the second representing a measure of the network performance: [21] adopted for the first time a multi-objective algorithm for water network rehabilitation, minimizing cost and maximizing benefits; [22] considered the minimization of cost and of the maximum pressure deficit at nodes; [23] took into account the maximization of entropy or demand supply ratio, while [24] and [25] the maximization of the reliability of the system.

A multi-objective evolutionary algorithm (MOEA) has two main goals [26]: firstly, to find a set of solutions as close as possible to the Pareto optimal front; secondly, to find a set of solutions as diverse as possible. However, the performance of the algorithm is quite affected by crossover and mutation type and probability: as a result, many runs with different starting populations and parameter sets are usually performed in order to find a good population of non-dominated solutions.

In this paper, a different approach is proposed, consisting of a population of MOGAs at the inner level, and an outer single-objective GA (meta GA) controlling the MOGAs crossover and mutation probabilities. The fitness of each individual of the meta GA is given by the hypervolume (that is, the amount of the objective space dominated by the obtained non-dominated front, [27], [28]) obtained by the inner MOGA it represents.

This methodology reconsiders some ideas of [29] and [30], and is *non-self-adaptive* [31], thus basically different from the *self-adaptive* mechanism based on the inclusion of operators and control parameters within the individual representation, [32], [33].

In order to asses the validity of the hypervolume metric, it has been compared to two performance measures, namely a convergence and a sparsity index [34], which quantify the exploitation and exploration issues of the inner MOGAs.

The paper is organized as follows: in Section 2, the mathematical formulation of the problem is presented, together with the test problem adopted for the numerical analyses; Section 3 describes the performance metrics, while Section 4 the two-level approach; Section 5 presents the results obtained and Section 6 some concluding remarks.

2 Two-objective water supply system optimization

2.1 Mathematical formulation

The problem is formulated as the minimization of the total cost of the network and the maximization of the minimum pressure level at nodes: for pressure level, we mean the deviation from the required pressure (see Figure 1 for an explanation), and hence both negative and positive values are allowed; however, in this work, the attention is focused only on negative values, indicating situations of pressure deficit (the maximum bound on the pressure level is then zero). The problem is constrained by continuity of mass at every node and energy conservation along every path in the system, giving:

$$\min\ f_1(d_1,\dots,d_{N_p}) = \sum_{i\in D}\sum_{j=1}^{N_p} c(d_i)L_{ij} \tag{1}$$

$$\max\ f_2(d_1,\dots,d_{N_p}) = \min_{k=1,\dots,N_n}\left[\min(H_k - H_k^{req},0)\right] \tag{2}$$

$$\text{s. to:}\qquad \sum_{i\in n_k} Q_i - \sum_{j\in m_k} Q_j = Q_{e,k} \tag{3}$$

$$\sum_{i\in p_j} h_{f,i} = \Delta E_j \tag{4}$$

$$H_k \geq H_k^{req} \tag{5}$$

$$d_{\min} \leq d_i \leq d_{\max} \tag{6}$$

in which

- $d_1,\dots,d_{N_p}$ are the N_p (number of pipes in the network) decision variables;
- $c(d_i)$ and L_{ij} are, respectively, the cost per unit length and the total length of pipe j whose diameter is d_i;
- D is the set of the N_D available commercial diameters (whose minimum and maximum sizes are $d_{\min}$ and $d_{\max}$, respectively);
- N_n is the number of supply nodes in the system;
- H_k is the actual piezometric head at node k;
- H_k^{req} is the required pressure at node k;
- $Q_{i,k}$ and $Q_{j,k}$ respectively represent the n_k and m_k flows entering or leaving node k;
- $Q_{e,k}$ is the erogated flow at node k;
- p_j is the number of links belonging to path j;
- $h_{f,i}$ represents the energy loss in link i of path j;
- ΔE_j is the total energy loss along path j: for a closed loop, $\Delta E_j = 0$.

Equations (3) and (4) are guaranteed by the hydraulic simulator (in this work EPANET 2 [35]), to which the optimizer has been coupled. The following expression for the energy loss, h_f, has been adopted:

$$h_f = 10.668\,\frac{Q^{1.852}L}{C_{HW}^{1.852}d^{4.871}} \tag{7}$$

in which Q is the discharge in the pipe (m^3/s), L the length (m), d the diameter, and C_{HW} is the Hazen–Williams (adimensional) pipe roughness coefficient.

2.2 Test problem adopted for the analyses

The two-loop network illustrated in Figure 2 has been considered, [2]: all links are 1000 m long, with a Hazen-Williams coefficient $C_{HW} = 130$. Nodal characteristics are also shown on the figure, while the available commercial diameters

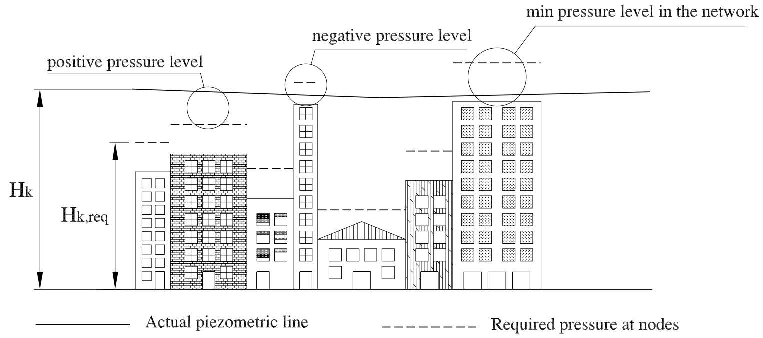
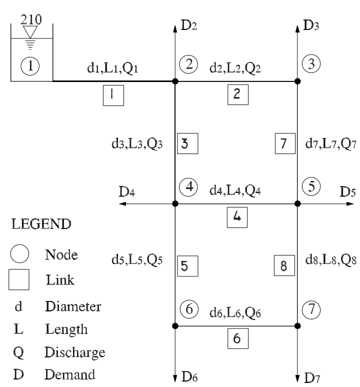


Fig. 1. Actual piezometric line and required pressures in a water supply system.

(inches) are: 1, 2, 3, 4, 6, 8, 10, 12, 14, 16, 18, 10, 22, 24, and their respective costs per unit length ($/m): 2, 5, 8, 11, 16, 23, 32, 50, 60, 90, 130, 170, 300, 550.

The decision space consists of 14^8 configurations, and has been totally explored by exhaustive search (requiring nearly 50 hours of CPU time on a Pentium III 1GHz). In particular, the subsequent analyses have been focused only on the region of interest (ROI) of the objective space characterized by configurations having minimum pressure level at nodes not below -30 meters (Figure 3): this resulted in 647691 solutions inside the ROI, of which only 38 are Pareto-optimal, and are reported on Table 1.



Node	Demand (m^3/h)	Elevation (m)	Pressure required (m)
1	-1120	210	--
2	100	150	30
3	100	160	30
4	120	155	30
5	270	150	30
6	330	165	30
7	200	160	30

Fig. 2. Two-loop network adopted for the numerical analyses.

A problem which arises when considering such multi-objective problem is the huge dimension of the Pareto front: there are actually non-dominated individuals characterized by not realistic pressure levels (extremely negative numbers). In

Table 1. Pareto-optimal solutions belonging to the region of interest for the two-loop network optimization problem.

n	f_1 (m)	f_2 (m)	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	n	f_1 (m)	f_2 (m)	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8
1	419000	0.000	18	10	16	4	16	10	10	1	20	336000	-9.851	16	8	16	8	14	10	6	1
2	415000	-0.387	18	10	16	6	16	10	8	1	21	331000	-10.449	16	8	16	8	14	10	4	1
3	414000	-1.155	18	14	14	1	12	1	14	12	22	330000	-10.964	16	6	16	10	14	10	1	3
4	413000	-1.453	18	10	16	2	16	10	10	1	23	327000	-11.019	16	6	16	10	14	10	1	2
5	408000	-1.718	18	14	14	3	12	3	14	10	24	324000	-11.043	16	6	16	10	14	10	1	1
6	407000	-1.844	18	12	16	1	14	10	10	4	25	310000	-11.159	16	10	14	1	14	10	10	1
7	404000	-1.853	18	12	16	1	14	10	10	3	26	309000	-14.456	16	8	14	6	14	10	8	2
8	401000	-1.863	18	12	16	1	14	10	10	2	27	306000	-14.738	16	8	14	6	14	10	8	1
9	398000	-1.879	18	12	16	1	14	10	10	1	28	301000	-17.340	16	8	14	8	14	10	4	1
10	380000	-1.909	18	10	16	1	14	10	10	1	29	300000	-17.704	16	10	14	1	12	10	10	1
11	379000	-4.615	18	8	16	8	14	10	6	2	30	294000	-17.847	16	6	14	8	14	10	4	1
12	376000	-4.619	18	8	16	8	14	10	6	1	31	291000	-19.740	16	6	14	8	14	10	3	1
13	371000	-5.216	18	8	16	8	14	10	4	1	32	288000	-21.656	16	6	14	8	14	10	2	1
14	370000	-5.731	18	6	16	10	14	10	1	3	33	280000	-22.143	14	10	14	1	14	10	10	1
15	367000	-5.786	18	6	16	10	14	10	1	2	34	278000	-25.095	16	6	14	8	12	10	2	1
16	364000	-5.810	18	6	16	10	14	10	1	1	35	275000	-25.273	16	6	14	8	12	10	1	1
17	350000	-5.926	18	10	14	1	14	10	10	1	36	271000	-28.324	14	8	14	8	14	10	4	1
18	340000	-7.141	16	10	16	1	14	10	10	1	37	270000	-28.688	14	10	14	1	12	10	10	1
19	339000	-9.848	16	8	16	8	14	10	6	2	38	264000	-28.831	14	6	14	8	14	10	4	1

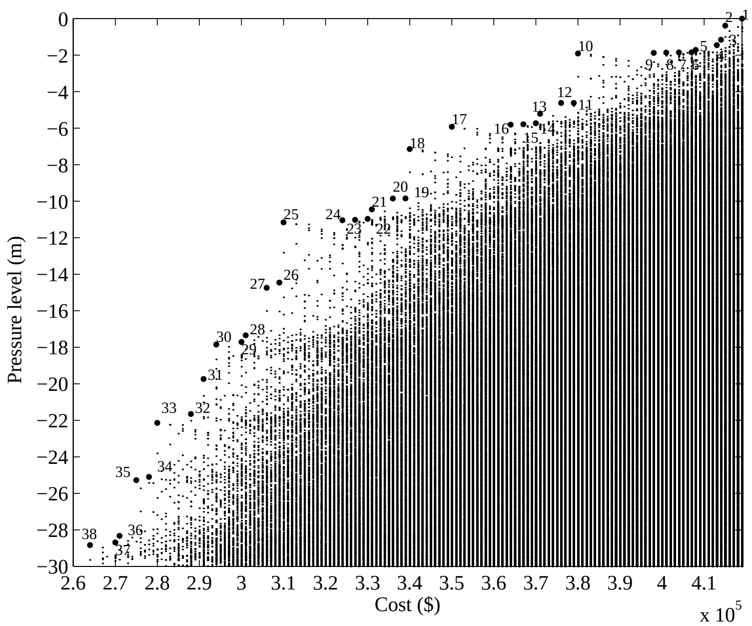


Fig. 3. Region of interest of the objective space with Pareto-optimal solutions evidentiated.

order to avoid such (useless) configurations, the search has been biased towards the solutions inside the ROI through a bending of the Pareto front as indicated in Figure 4, thus transforming all Pareto-optimal solutions outside the ROI into dominated individuals: mathematically, this has been achieved through a slight change in the first objective function, namely:

$$f_1(d_1,\dots,d_{N_p}) = \begin{cases} \sum_{i\in D}\sum_{j=1}^{N_p} c(d_i)L_{ij} & \text{if} \quad f_2(d_1,\dots,d_{N_p}) \ge -30.0; \\ \sum_{i\in D}\sum_{j=1}^{N_p} c(d_i)L_{ij} + p\big[-f_2(d_1,\dots,d_{N_p}) - 30.0\big] & \text{otherwise.} \end{cases} \tag{8}$$

in which $p > 0$ is a penalty factor.

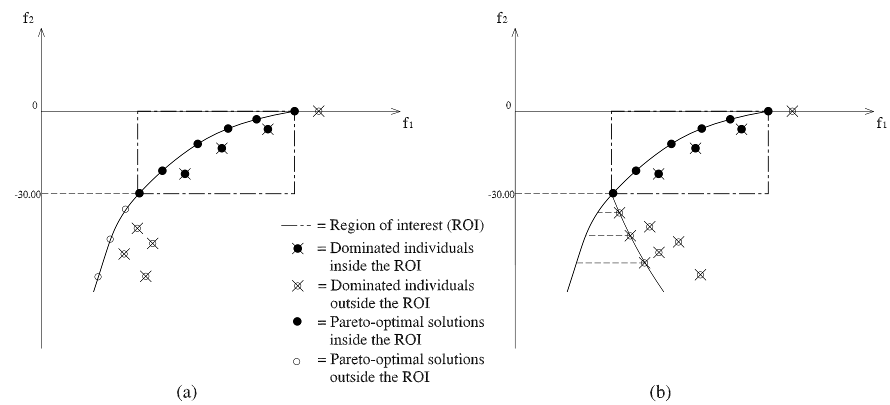


Fig. 4. Bending of the Pareto front in order to bias the search only on configurations of interest: (a), Pareto front with non-dominated solutions evidentiated; (b), preferred region of the frontier, with Pareto-optimal solutions outside the ROI being now dominated.

3 Performance metrics

Two kinds of performance measures have been considered:

1. the hypervolume, $\mathcal{HV}$, which quantifies with only one scalar the amount of criterion space dominated by the current non-dominated front, [26], [27], [28]. Since we knew the Pareto-optimal solutions (POS), it was decided to adimentionalize the metric with respect to its maximum value.
2. Two indices representing, respectively, the convergence towards the non-dominated solutions and the distribution of the individuals in the generic population along the front [34].

In the following, the convergence and sparsity indices are described in more detail.

3.1 Convergence index

The convergence index, $\mathcal{CI}$, is expressed as:

$$\mathcal{CI} = \frac{N_f^{POS}}{N^{POS}}(1 - \overline{\delta}) \tag{9}$$

where N_f^{POS} represent the number of POS inside the ROI found by the algorithm, $N^{POS} = 38$ (the size of P^*, P^* being the set of Pareto-optimal solutions reported on Table 1), and $\overline{\delta}$ is the average of the adimentionalized Euclidean distance values of all individuals inside the ROI from their nearest solution in P^*, given by:

$$\overline{\delta} = \frac{1}{N^{ROI}} \sum_i \delta_i = \frac{1}{N^{ROI}} \sum_i \sqrt{\left(\frac{f_{1,i}^{POS} - f_{1,i}}{f_{1,\max} - f_{1,\min}}\right)^2 + \left(\frac{f_{2,i}^{POS} - f_{2,i}}{f_{2,\max} - f_{2,\min}}\right)^2} \tag{10}$$

where N^{ROI} is the number of individuals inside the ROI, $f_{1,i}$ and $f_{2,i}$ are respectively the cost and pressure level of the i-th individual, $f_{1,i}^{POS}$ and $f_{2,i}^{POS}$ the same quantities referred to the Pareto-optimal solution closest to the i-th individual, $f_{1,\max} = 419000$, $f_{1,\min} = 264000$, $f_{2,\max} = 0$, $f_{2,\min} = -30$ (these last values delimiting the ROI). The symbols are also represented in Figure 5.

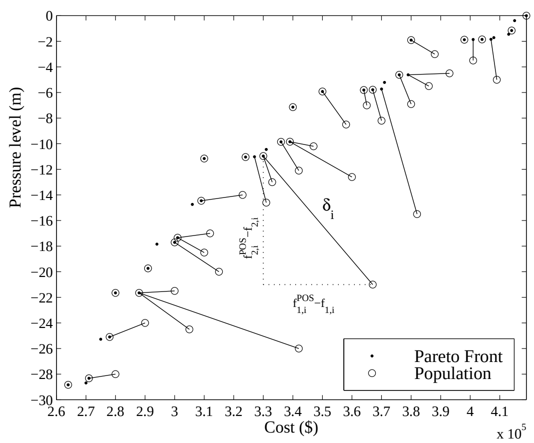


Fig. 5. Example of a generic population of 50 individuals (49 of which are inside the ROI here illustrated) and the Pareto front: solid lines connect each individual to its closest non-dominated solution, and represent the distance δ_i in equation (10).

3.2 Sparsity index

The sparsity index, $\mathcal{SI}$, is expressed as:

$$\mathcal{SI} = \frac{N_r^{POS}}{N^{POS}} \left(1 - \frac{z_{\max}}{N^{POS}}\right)(1 - \sigma_{adim}) \tag{11}$$

where N_r^{POS} represents the number of POS in P^* which have been approached (*reached*) by at least one individual in the population (of course, every POS found is also reached, but the contrary is not necessarily true), $z_{\max}$ is the maximum number of consecutive (adjacent) POS in P^* not reached, and σ_{adim} takes into account the actual distribution of individuals around the POS reached.

To understand the meaning of $z_{\max}$ and σ_{adim}, consider the example of before, in which 49 individuals are inside the ROI, and 29 POS have been reached: the actual distribution may be deduced from Figure 5 (counting the individuals around each POS), and is schematized in Figure 6 (a). Actually, the individuals inside the ROI may be distributed in many different ways around the POS reached: in particular, there will be (best) distributions characterized by the minimum standard deviation, as in Figure 6 (b), and (worst) distributions with the maximum standard deviation, as in Figure 6 (c). σ_{adim} is the adimentionalized standard deviation between these two extreme situations.

n_a	1	1	1	1	1	1	2	2	2	1	2	2	2	1	3	2	3	1	1	1	2	3	2	1	4	1	2	2	1
n	1	3	6	7	8	9	10	11	12	14	15	16	17	18	19	20	22	23	24	25	26	28	29	31	32	33	34	36	38

(a)

n_a	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1
n	1	3	6	7	8	9	10	11	12	14	15	16	17	18	19	20	22	23	24	25	26	28	29	31	32	33	34	36	38

(b)

n_a	21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
n	1	3	6	7	8	9	10	11	12	14	15	16	17	18	19	20	22	23	24	25	26	28	29	31	32	33	34	36	38

(c)

Fig. 6. Schematic representation of the distribution of the 49 individuals of the population around the $N_r^{POS} = 29$ reached. In (a), the actual distribution is reported (cfr. Figure 5), while in (b) and in (c), two examples of distributions characterized, respectively, by the minimum standard deviation (best distribution) and the maximum standard deviation (worst distribution). n_a is the number of individuals that have approached the n-th POS, whose progressive number is shown below (according to the numeration given on Table 1 and represented in Figure 3).

4 The two-level approach

The methodology consists of a population of inner multi-objective GAs and an outer simple GA, adapting crossover and mutation probabilities of the inner level.

Figure 7 shows a schematic representation of the inner and outer populations. In the figure, N_{in} and N_{out} are the inner and outer population sizes, respectively, while p_m and p_c the mutation and crossover probabilities.

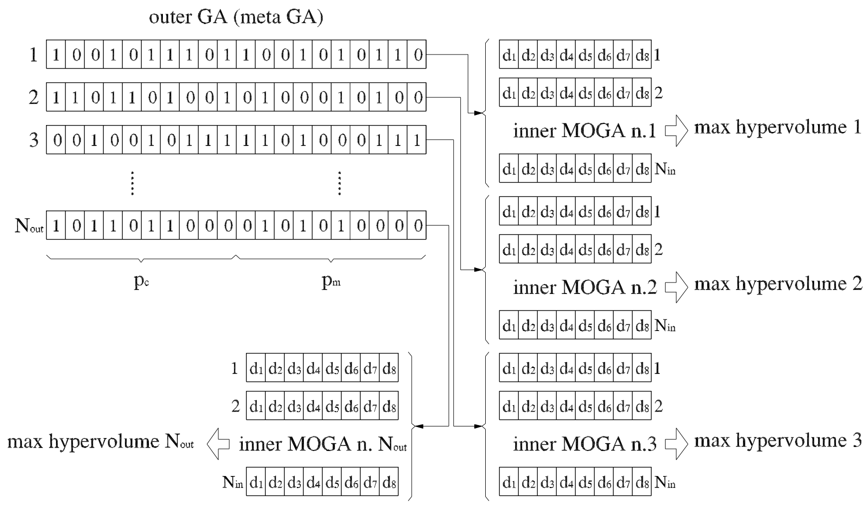


Fig. 7. Representation of inner and outer GAs for the two-loop network.

4.1 The inner GA

Two elitist multi-objective evolutionary algorithms have been implemented at the inner level with the aid of GAlib library [36], which has been modified in order to handle multi-criterion optimization problems:

1. the NSGA-II [37], a parameter-free NSGA based on the crowding distance of individuals; one problem of NSGA-II resides in the way elitism is performed, since it determines a deletion of solutions of non-elitist fronts and, as a result, the search process may suffer from stagnation or premature convergence.
2. The Controlled NSGA-II (CNSGA-II), introduced by [38], which, if compared to NSGA-II, performs elitism in a controlled manner, that is, instead of preserving all individuals of rank one, each front is allowed to have an exponentially decreasing number of solutions, thus forcing part of all non-dominated fronts to coexist in the new generation.

Every individual has a direct coding of commercial diameter values for each of the 8 decision variables, representing the 8 diameters to be assigned to the network. A population size of $N_{in} = 50$ individuals has been adopted, together with uniform crossover and adjacent mutation, allowing only changes to the nearest larger or smaller diameters.

4.2 The outer GA

The meta GA is a simple GA with elitism, no crossover operator, a population size of $N_{out} = 5$, and a binary (gray) coding, mapping both mutation and probabilities from 0 to 1 with 3-digit precision (thus requiring 10 bits each), for a total of 20 bits chromosome representation. For every individual at the outer level, an inner multi-objective evolutionary algorithm is performed, and its fitness is represented by the (maximum) hypervolume $\mathcal{HV}$ reached during the evolution process.

5 Results

Numerical experiments have been divided in two phases: in the first, an assessment of the hypervolume metric has been performed, through its comparison with the convergence and sparsity indices previously introduced; in the second, the two-level approach has been tested.

Tables 2-5 report the results obtained with the NSGA-II and CNSGA-II, indicated as mean and standard deviation over ten runs with different initial populations. In particular, Tables 2 and 4 refer to the last generations of the evolution process, while Tables 3 and 5 to the best results achieved during the runs. Some considerations follow:

1. there is a good agreement between the hypervolume metric and the convergence and sparsity indices, also evidentiated by the number of Pareto-optimal solutions found or reached.
2. There may be situations in which hypervolume is high, although convergence and sparsity indices are not; this is due to the discrete character of the problem and to the actual distribution of Pareto-optimal solutions: looking at the front in Figure 3, it may be noted that there are some clusters of solutions which only marginally contribute to the hypervolume (a closer look at Table 1 reveals also that neighbour solutions on the objective space are characterized by changes in only one or two diameters).
3. CNSGA-II actually outperforms NSGA-II with respect to all the metrics, having also lower values of standard deviations, especially for the sparsity index; looking at Figure 8, which shows an example of the best evolutions of the two MOGAs, it may be observed that CNSGA-II exhibits both higher rapidity in reaching final values of performance measures, both lower oscillating evolutions, as confirmed by the comparison between Tables 4 and 5; NSGA-II, on the contrary, shows higher oscillating patterns (cfr. Tables 2 and 3).
4. There is a dependance of the two MOGAs with respect to crossover probability. Although mutation probability has been kept constant, some runs performed confirmed the same behaviour, thus introducing the problem of the optimal choice of such values: this motivated the two-level approach.

Tables 6 and 7 report the results obtained with the two-level methodology, respectively adopting roulette wheel and tournament selection for the outer GA.

Table 2. Values of performance measures obtained with NSGA-II at last generation, for different crossover probabilities, p_c: μ is the mean and σ the standard deviation.

	$\mathcal{HV}$		$\mathcal{CI}$		$\mathcal{SI}$		N_f^{POS}		N_r^{POS}		$z_{\max}$	
p_c	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
0.00	0.9315	0.0836	0.3498	0.2812	0.6344	0.1918	13.40	10.65	26.70	5.95	1.90	0.70
0.25	0.9547	0.0694	0.4583	0.3028	0.6783	0.1683	17.50	11.47	28.20	5.04	1.80	0.60
0.50	0.9618	0.0673	0.5775	0.3232	0.7377	0.1615	22.00	12.22	29.80	4.64	1.30	0.46
0.75	0.9773	0.0472	0.6192	0.2950	0.7747	0.1399	23.60	11.16	31.30	3.98	1.40	0.49
1.00	0.9429	0.0718	0.4292	0.3200	0.5323	0.1528	16.40	12.13	29.80	5.56	1.50	0.50
Mean	0.9536	0.0679	0.4868	0.3044	0.6715	0.1629	18.58	11.52	29.16	5.03	1.58	0.55

Table 3. Best values of performance measures obtained with NSGA-II, for different crossover probabilities, p_c: μ is the mean and σ the standard deviation.

	$\mathcal{HV}$		$\mathcal{CI}$		$\mathcal{SI}$		N_f^{POS}		N_r^{POS}		$z_{\max}$	
p_c	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
0.00	0.9324	0.0824	0.3602	0.2900	0.6539	0.2008	13.80	10.99	27.40	6.48	1.90	0.70
0.25	0.9554	0.0681	0.4716	0.3239	0.6949	0.1824	18.00	12.29	28.80	5.47	1.70	0.64
0.50	0.9619	0.0671	0.6063	0.3345	0.7698	0.1694	23.10	12.65	31.10	4.93	1.30	0.46
0.75	0.9774	0.0470	0.6272	0.3021	0.7737	0.1440	23.90	11.42	31.40	4.08	1.40	0.49
1.00	0.9430	0.0717	0.4292	0.3200	0.5272	0.1474	16.40	12.13	29.80	5.56	1.50	0.50
Mean	0.9540	0.0673	0.4989	0.3141	0.6839	0.1688	19.04	11.89	29.70	5.31	1.56	0.56

Table 4. Values of performance measures obtained with CNSGA-II at last generation, for different crossover probabilities, p_c: μ is the mean and σ the standard deviation.

	$\mathcal{HV}$		$\mathcal{CI}$		$\mathcal{SI}$		N_f^{POS}		N_r^{POS}		$z_{\max}$	
p_c	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
0.00	0.9722	0.0534	0.5205	0.2552	0.6669	0.1179	19.90	9.66	28.60	3.72	1.80	0.40
0.25	0.9656	0.0622	0.4264	0.2369	0.6371	0.1325	16.30	8.98	27.20	4.56	1.80	0.60
0.50	0.9935	0.0082	0.6268	0.1635	0.7232	0.0317	23.90	6.16	30.30	1.01	1.40	0.49
0.75	0.9877	0.0096	0.5052	0.2122	0.7057	0.0702	19.30	8.00	29.70	2.05	1.60	0.49
1.00	0.9812	0.0110	0.4878	0.2102	0.6892	0.0814	18.70	7.94	31.00	2.05	1.90	0.54
Mean	0.9800	0.0289	0.5133	0.2156	0.6844	0.0868	19.62	8.15	29.36	2.68	1.70	0.50

Table 5. Best values of performance measures obtained with CNSGA-II, for different crossover probabilities, p_c: μ is the mean and σ the standard deviation.

	$\mathcal{HV}$		$\mathcal{CI}$		$\mathcal{SI}$		N_f^{POS}		N_r^{POS}		$z_{\max}$	
p_c	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
0.00	0.9734	0.0509	0.5235	0.2618	0.6892	0.1267	20.00	9.91	29.20	4.29	1.50	0.50
0.25	0.9667	0.0597	0.4395	0.2460	0.6656	0.1351	16.80	9.32	28.60	4.78	1.70	0.64
0.50	0.9940	0.0084	0.6614	0.1884	0.7517	0.0380	25.20	7.10	31.40	1.11	1.30	0.46
0.75	0.9879	0.0097	0.5338	0.2253	0.7235	0.0607	20.40	8.51	31.00	1.84	1.60	0.49
1.00	0.9812	0.0110	0.4904	0.2111	0.6852	0.0746	18.80	7.97	31.30	1.90	1.70	0.46
Mean	0.9807	0.0279	0.5297	0.2265	0.7030	0.0870	20.24	8.56	30.30	2.78	1.56	0.51

Numbers represent the hypervolume obtained after 10 and 20 outer generations (indicated in parentheses in the Tables). CPU times were about 30 seconds for each outer generation. It may be observed that:

1. fitness obtained in one run is much higher than that achieved with a MOGA alone, even when several runs with different parameters are performed.
2. Very often results after 20 generations determine only a slight improvement with respect to those after 10.
3. NSGA-II and CNSGA-II present nearly the same performances, as well the tournament and roulette wheel selections.

Table 6. Results obtained with the two-level approach and adopting roulette wheel selection at the outer level: numbers represent hypervolume, with μ the mean and σ the standard deviation.

Run	NSGA-II (10)	NSGA-II (20)	CNSGA-II (10)	CNSGA-II (20)
1	0.999622	0.999622	0.999667	0.999667
2	0.999514	0.999710	0.999656	0.999656
3	0.999190	0.999585	0.999434	0.999483
4	0.999633	0.999650	0.999658	0.999761
5	0.999673	0.999719	0.999382	0.999382
6	0.999634	0.999661	0.999687	0.999687
7	0.999673	0.999673	0.999652	0.999652
8	0.999122	0.999698	0.999064	0.999481
9	0.999504	0.999591	0.999183	0.999508
10	0.999630	0.999630	0.999532	0.999532
μ	0.999520	0.999654	0.999492	0.999581
σ	0.000190	0.000045	0.000211	0.000113

6 Concluding remarks

The paper has presented a two-level methodology for multi-criterion optimization of water distribution systems. Results show how such an approach, although requiring more computational effort than using a multi-objective genetic algorithm alone, is able to achieve very good performances in only one simulation run, thus proving its robustness and easing the network optimization process. In this work, a non-self adaptive procedure has been proposed: future research will be focused on the inclusion of mutation and crossover probabilities in the string representation, as in a fully self-adaptive mechanism.

References

1. Yates, D.F., Templeman, A.B., Boffey, T.B.: The Computational Complexity of the Problem of Determining Least Capital Cost Designs for Water Supply Networks. Engineering Optimization **7** (1984) 142-155

Table 7. Results obtained with the two-level approach and adopting tournament selection (with two competing individuals) at the outer level: numbers represent hypervolume, with μ the mean and σ the standard deviation.

Run	NSGA-II (10)	NSGA-II (20)	CNSGA-II (10)	CNSGA-II (20)
1	0.999646	0.999646	0.999760	0.999760
2	0.999570	0.999570	0.999506	0.999573
3	0.999603	0.999622	0.999553	0.999583
4	0.999141	0.999547	0.999569	0.999677
5	0.999463	0.999615	0.999393	0.999564
6	0.999641	0.999664	0.999687	0.999820
7	0.999560	0.999618	0.999652	0.999652
8	0.999660	0.999660	0.999526	0.999554
9	0.999305	0.999522	0.999238	0.999267
10	0.999800	0.999800	0.999756	0.999756
μ	0.999359	0.999446	0.999564	0.999621
σ	0.000480	0.000484	0.000154	0.000148

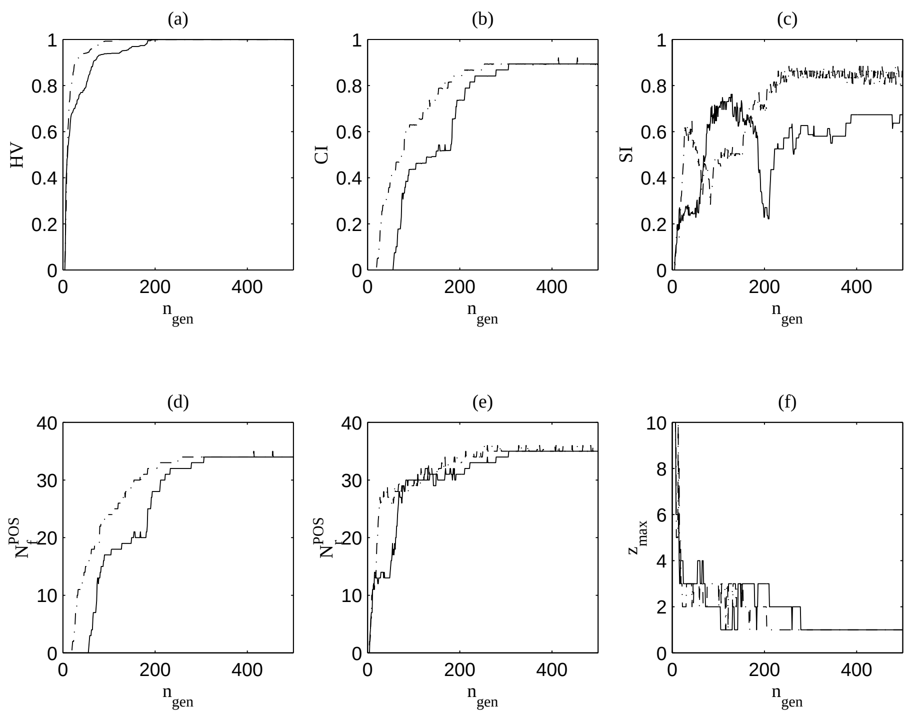


Fig. 8. Performance indices obtained with the best evolutions of NSGA-II (solid line) and Controlled NSGA-II (dash-dotted line): (a), hypervolume; (b), convergence index; (c), sparsity index; (d), Pareto solutions found; (e), Pareto solutions reached; (f) maximum number of consecutive POS not reached.

2. Alperovits, E., Shamir, U.: Design of Optimal Water Distribution Systems. Water Resources Research **13** (1977) 885-900
3. Quindry, G., Brill, E.D., Liebman, J.C.: Optimization of Looped Water Distribution Systems. Journal of Environmental Engineering **107** (1981) 665-679
4. Goulter, I.C., Lussier, B.M., Morgan, D.R.: Implications of Head Loss Path Choice in the Optimization of Water Distribution Networks. Water Resources Research **22** (1986) 819-822
5. Fujiwara, O., JenChaimahakoon, B., Edirisinghe, N.: A Modified Linear Programming Gradient Method for Optimal Design of Looped Water Distribution Networks. Water Resources Research **23** (1987) 977-982
6. Kessler, A., Shamir, U.: Analysis of the Linear Programming Gradient Method for Optimal Design of Water Supply Networks. Water Resources Research **25** (1989) 1469-1480
7. Bhave, P.R., Sonak, V.V.: A Critical Study of the Linear Programming Gradient Method for Optimal Design of Water Supply Networks. Water Resources Research **28** (1992) 1577-1584
8. El-Bahrawy, A., Smith, A.A.: Application of MINOS to Water Collection and Distribution Networks. Civil Engineering Systems **2** (1985) 38-49
9. Duan, N., Mays, L.W., Lansey, K.E.: Optimal Reliability-Based Design of Pumping and Distribution Systems. Journal of Hydraulic Engineering **116** (1990) 249-268
10. Simpson, A.R., Dandy, G.C., Murphy, L.J.: Genetic Algorithms Compared to other Techniques for Pipe Optimization. Journal of Water Resources Planning and Management **120** (1994) 423-443
11. Dandy, G.C., Simpson, A.R., Murphy, L.J.: An Improved Genetic Algorithm for Pipe Network optimization. Water Resources Research **32** (1996) 449-458
12. Savic, D.A., Walters, G.A.: Genetic Algorithms for Least-Cost Design of Water Distribution Networks. Journal of Water Resources Planning and Management **123** (1997) 67-77
13. Montesinos, P., Garcia-Guzman, A., Ayuso, J.L.: Water Distribution Network Optimization Using a Modified Genetic Algorithm. Water Resources Research **35** (1999) 3467-3473
14. Reis, L.F.R., Porto, R.M., Chaudry, F.H.: Optimal Location of Control Valves in Pipe Networks by Genetic Algorithm. Journal of Water Resources Planning and Management **123** (1997) 317-326
15. Vitkovsky, J.P., Simpson, A.R., Lambert, M.F.: Leak Detection and Calibration Using Transients and Genetic Algorithms. Computer Aided Civil and Infrastructure Engineering **15** (2000) 374-382
16. Meyer, R.W., Barkdoll, B.D.: Sampling Design for Network Model Calibration Using Genetic Algorithms. Journal of Water Resources Planning and Management **126** (2000) 245-250
17. Dandy, G.C., Engelhardt, M.: Optimal Scheduling of Water Pipe Replacement Using Genetic Algorithms. Journal of Water Resources Planning and Management **127** (2001) 214-223
18. Walters, G.A., Lohbeck, T.: Optimal Layout of Tree Networks Using Genetic Algorithms. Engineering Optimization **22** (1993) 27-48
19. Abebe, A.J., Solomatine, D.P.: Application of Global Optimization to the Design of Pipe Networks. In: Babovic and Larsen (Eds.), Hydroinformatics 1998, World Scientific, 989-996
20. Walski, T.M.: The Wrong Paradigm – Why Water Distribution Optimization Doesn't Work. Journal of Water Resources Planning and Management **127** (2001) 203-205

21. Halhal, D., Walters, G.A., Ouazar, D., Savic, D.A.: Water Network Rehabilitation with Structured Messy Genetic Algorithm. Journal of Water Resources Planning and Management **123** (1997) 137-146
22. Cheung, P.B., Reis, L.F.R., Formiga, K.T.M., Chaudry, F.H., Ticona, W.G.C.: Multiobjective Evolutionary Algorithms Applied to the Rehabilitation of a Water Distribution System: a Comparative Study. In: C.M. Fonseca et al. (Eds.), EMO 2003, LNCS 2632, 662-676 (2003) Springer
23. Formiga, K.T.M., Chaudry, F.H., Cheung, P.B., Reis, L.F.R.: Optimal Design of Water Distribution System by Multiobjective Evolutionary Methods. In: C.M. Fonseca et al. (Eds.), EMO 2003, LNCS 2632, 677-691 (2003) Springer
24. Tolson, B.A., Maier, H.R., Simpson, A.R., Lence, B.J.: Genetic Algorithms for Reliability-Based Optimization of Water Distribution Systems. Journal of Water Resources Planning and Management **130** (2004) 63-72
25. Prasad, T.D., Park, N.-S.: Multiobjective Genetic Algorithms for Design of Water Distribution Networks. Journal of Water Resources Planning and Management **130** (2004) 73-82
26. Deb, K.: Multi-Objective Optimization Using Evolutionary Algorithms. John Wiley & Sons (2001)
27. Knowles, J.D., Corne, D.W.: On Metrics for Comparing Nondominated Sets. Proceedings of the 2002 IEEE Congress on Evolutionary Computation (CEC 2002) 711-716
28. Fleischer, M.: The Measure of Pareto Optima In: C.M. Fonseca et al. (Eds.), EMO 2003, LNCS 2632, 519-533 (2003) Springer
29. Grefenstette, J.J.: Optimization of Control Parameters for Genetic Algorithms. IEEE Transactions Systems Man Cybernetics **16** (1986) 122-128
30. Bramlette, M.F.: Initialization, Mutation and Selection Methods in Genetic Algorithms for Function Optimization. In: R.K. Belew and L.B. Booker (Eds.), Proceedings of the Fourth International Conference on Genetic Algorithms, 100-107 (1991) Morgan Kaufmann
31. Spears, W.M.: Adapting Crossover in Evolutionary Algorithms. In: J.R. McDonnel et al. (Eds.), Proceedings of the Fourth Annual Conference on Evolutionary Programming, 367-384 (1995) The MIT Press
32. Back, T.: Self-Adaptation in Genetic Algorithms. In: F.J. Varela and P. Bourgine (Eds.), Proceedings of the First European Conference on Artificial Life, 263-271 (1992) The MIT Press
33. Srinivas, M., Patnaik, L.M.: Adaptive Probabilities of Crossover and Mutation in Genetic Algorithms. IEEE Transactions System Man Cybernetics **24** (1994) 656-666
34. Nicolini, M.: Evaluating Performance of Multi-Objective Genetic Algorithms for Water Distribution System Optimization. In: Liong et al. (Eds.), Hydroinformatics 2004, World Scientific, 850-857
35. Rossman, L.A.: EPANET 2 USERS MANUAL. U.S. Environmental Protection Agency, Cincinnati, Ohio (2000)
36. Wall, M.: GAlib: A C++ Library of Genetic Algorithm Components (v. 2.4.5). Mechanical Engineering Department, Massachussets Institute of Technology (2000)
37. Deb, K., Agrawal, S., Pratap, A., Meyarivan, T.: A Fast Elitist Non-dominated Sorting Genetic Algorithm for Multi-Objective Optimization: NSGA-II. In: Proceedings of the Parallel Problem Solving from Nature VI 849-858 (2000)
38. Deb, K., Goel, T.: Controlled Elitist Non-dominated Sorting Genetic Algorithms for Better Convergence. In: E. Zitzler et al. (Eds.), EMO 2001, LNCS 1993, 67-81 (2001) Springer