

## Aerodynamic Inverse Optimization with Genetic Algorithms

Shigeru Obayashi  
Department of Aeronautics and Space Engineering  
Tohoku University  
Sendai, 980-77 Japan  
obayashi@ad.mech.tohoku.ac.jp

**Abstract** – A multiobjective Genetic Algorithm based on Fonseca-Fleming's Pareto ranking method has been applied to optimize the three-dimensional target pressures for the aerodynamic inverse design of wing shape. The optimization problem was formulated to minimize the induced drag for wings as well as to minimize the viscous drag for airfoil sections. Performances of both the simple Genetic Algorithm and Vector Evaluated Genetic Algorithm were found unsatisfactory to the present optimization problem. The present design procedure was successfully applied to transonic wing design.

### I. INTRODUCTION

Development of aerodynamic shape optimization method is important for commercial aircraft industry to improve the design efficiency in today's competitive environment. With the aid of Computational Fluid Dynamics (CFD), various aerodynamic design techniques have been proposed. In [1], these aerodynamic optimization methods were categorized into two classes: direct and inverse numerical optimization methods.

The direct numerical optimization methods are formed by coupling aerodynamic analysis methods based on CFD with numerical optimization algorithms. They minimize (or maximize) a given aerodynamic objective function by iterating directly on the geometry. The geometry is represented by a general function, such as polynomial and cubic splines, by a linear combination of known airfoils, or by a basic shape plus a combination of typical geometry perturbations. Such procedures, however, become extremely expensive as the number of geometry parameters is increased. Unfortunately, flow fields are often very sensitive to the geometry, and we have to increase number of parameters to define geometry precisely. Thus, those procedures do not seem practical even with the aid of the current supercomputers.

The inverse numerical optimization methods deal with pressure distributions rather than the geometry, to minimize, for example, drag under given lift and pitching moment. Since pressure is the primary force acting on aerodynamic

objects, one can design desired aerodynamic characteristics by specifying pressure distributions. Once the target pressure distributions are optimized, corresponding geometry can be determined by the inverse methods. This approach avoids most of the limitations of the standard inverse methods while requiring considerably less computational effort than the direct numerical optimization approach. Therefore, this paper considers the inverse optimization of wing shape, since, among aircraft components, wing shape has the primary impact on the aircraft performance.

The design of wing usually proceeds in two steps. First, the midspan section of the wing called airfoil is designed. Since a typical wing for commercial aircraft has longer span than its chord, wing performance can be predicted by the sectional shape in the midspan. This reduces the three-dimensional design problem into the two-dimensional one. In [2], a Genetic Algorithm (GA) has been applied to optimize target pressure distributions around airfoils for inverse design methods. Pressure distributions are parameterized by B-spline polygons and the airfoil drag is minimized under constraints on lift, airfoil thickness and other design principles. Once target pressure distribution is obtained, corresponding airfoil geometry can be computed by an inverse design code by Takanashi [3] coupled with a Navier-Stokes solver. Successful design results were obtained for transonic cases with and without a shock wave.

As an extension of [2], optimization of target pressure distributions for the three-dimensional wing is considered here by using GAs. Once the airfoil shape is designed, the next step of the wing design is to determine the variation of the designed airfoil in the spanwise direction. The design principles for this step are essentially twofold. One is to preserve the two-dimensional performance as much as possible. This is easily achieved by the inverse method by specifying the same chordwise pressure distribution along the wing span. The resulting wing has the straight isobar pattern of pressure contours on the wing surface. The other is to minimize the induced drag. Since the induced drag becomes one half to two third of the total drag during climb, reduction of the induced drag is an important goal for the three-dimensional wing design.

The induced drag is essential to the three-dimensional wing. If the wing has lift, the average pressure over the bottom surface of the wing is greater than that over the top surface. Consequently, there is some tendency for the air to flow around the wingtips from the high- to low-pressure sides. This flow establishes wingtip vortices. These vortices induce a small downward component of air velocity in the neighborhood of the wing itself. Because the local relative wind is canted downward, the lift vector itself is tilted back, hence it contributes a certain component of force parallel to the freestream, that is, a drag force. The incompressible flow theory predicts that the minimum induced drag is achieved by an elliptical lift distribution (the lift per unit span varies elliptically along the span) [4]. Therefore, the elliptical lift distribution is the key design principle for the wing shape optimization.

The two design principles described above, however, contradict each other in general. Since the sectional lift is given by the chordwise pressure distribution, the elliptical lift distribution can be materialized by specifying the same chordwise pressure distribution along the wing span only if the wing has an elliptic planform. Because of the manufacturing cost, modern commercial aircraft usually uses a tapered wing instead of the elliptic wing.

In this paper, target pressure distributions will be optimized for wing shape to minimize the induced drag, that is, to achieve the elliptical lift distribution on a tapered wing as well as for airfoil sections of the wing to reduce the viscous drag using the previous two-dimensional approach. This naturally leads to the multiobjective optimization. As a multiobjective GA (MOGA), we have adapted the Pareto-based ranking method by Fonseca and Fleming [5]. The design result will be given for a typical transonic wing.

## II. OPTIMIZATION OF TARGET PRESSURES

### A. Pressure Distribution for Airfoil Section

In GA, design candidates are considered as individuals in the population. An individual is characterized by genes represented as a string of parameters. In this work, B-spline curve is used to represent chordwise pressure distribution in terms of pressure coefficient,  $C_p$ . Chordwise pressure distribution can be split into two curves, corresponding to the upper and lower surfaces of an airfoil. Seven points are used to define each B-spline polygon as shown in Fig. 1. Except for the leading- and trailing-edge points, total of 12 points are considered as genes representing design candidates. The real number coding is used with a crossover operator

defined by a randomized weighted average[2].

The two-dimensional optimization problem is then defined as

Minimize: Drag coefficient  $C_d$

Subject to: 1. Lift coefficient  $C_l$  = specified

2. Airfoil thickness to chord  $t/c$  = specified

3. Additional six constraints for chordwise pressure distribution

where  $C_d$ ,  $C_l$  and  $t/c$  can be evaluated from the pressure distribution. The specification of airfoil thickness can be done approximately in two dimensions, but not in three dimensions. Thus, it was dropped in the following three-dimensional optimization. Additional six constraints are required to guarantee a reasonable solution of the aerodynamic inverse problem (see [2] for details).

### B. Pressure Distribution for Wing

Target pressure distribution for the three-dimensional wing can be obtained by specifying the chordwise pressure distributions at several spanwise sections. Planform shape of wing is usually determined by other means and thus a typical wing planform of transonic transport aircraft is assumed here.

The present objective of the wing design is to minimize the induced drag. This is achieved by the elliptical lift distribution in the spanwise direction of the wing. The constraint in the total lift will specify an elliptical lift distribution uniquely. Thus, the objective function can be given by differences of the sectional lifts to the elliptic distribution at the several spanwise sections. The three-dimensional optimization problem is now defined as

Minimize: 1. Difference of the spanwise lift distribution to the elliptic distribution

2. Two-dimensional drag coefficient  $C_d$  at each spanwise section

Subject to: Additional constraints for chordwise pressure distribution at each spanwise section

We can further redefine the constrained problem to the unconstrained multiobjective optimization problem as

Minimize: 1. Difference of the spanwise lift distribution to the elliptic distribution

2. Two-dimensional drag coefficient  $C_d$  at each spanwise section

3. Penalty function for chordwise pressure

distribution at each spanwise section [2]

### C. MOGA

Before implementing the Pareto ranking approach for the present MOGA, we have tried a few other ways to construct a GA for the present multiobjective optimization. First, a simple GA was used by combining three objective functions into a single one. However, this approach not only failed to search Pareto-optimal solutions, but also produced premature convergence. Certain spanwise section had unacceptable chordwise pressure distribution for airfoil section. Next, the Vector Evaluated Genetic Algorithm (VEGA) [6] was adapted to the present problem. As pointed out in [7], however, the solution was extremely good for one objective but not for the others. These experiences led us to Fonseca-Fleming's Pareto ranking method [5].

In the present MOGA, the third objective for the penalty function is used to pool the top 30% individuals in the population. Then Fonseca-Fleming's Pareto ranking method is applied to these individuals by using the first and second objectives. Selection operator is defined by using the nonlinear function suggested in [8]. Crossover and mutation operators are defined similar to those in [2]. The elite strategy is also used to preserve the best individual for each objective. After 200 generations, the best solution among the Pareto-optimal set in terms of the first objective is selected as the optimal solution.

As mentioned in [2], random creation of initial population produces infeasible solutions due to the severe constraints. Thus, we ran the two-dimensional GA by using only the constraints to evolve a population of feasible solutions. Then we distributed the sectional pressure distribution to the six spanwise sections from the root to the 83.3% span so as to give the elliptical lift distribution approximately. To do this, we only changed the pressure on the lower surface of the airfoil. In this way, we were able to implicitly satisfy the first design principle for the wing mentioned in Introduction, that is, to maintain the two-dimensional performance. The straight isobar pattern of pressures on the upper surface of the wing is expected to produce the drag divergence at the same Mach number along the wing span and thus the resulting drag-divergence Mach number of the wing will be similar to that of the airfoil section. The population of 210 individuals was used as the initial population of the present MOGA.

### III. INVERSE DESIGN

Once the present MOGA finds an optimum target pressure

distribution, corresponding wing geometry can be obtained by an inverse design method. Here the inverse design code, WinDes [3], is used. WinDes uses the following iterative procedure. Suppose the initial geometry and surface pressure distribution obtained from any CFD code are given. First, pressure differences are calculated from the given initial and target pressure distributions. From these pressure differences, corresponding geometry corrections can be computed from the integral equations discretized at the panels on the initial geometry. Improved geometry is then obtained from the initial geometry and the computed geometry corrections. Finally, the CFD code is used again to check how close the resulting pressure distribution is to the target distribution. If the differences are still large, the process will be iterated. In practice, 15 design cycles are sufficient to obtain the final geometry.

The inverse design code, Navier-Stokes code, and algebraic grid generator constructs a nearly automated loop for the inverse design with reasonable computational requirements [2]. These codes were implemented on a NEC SX-4 supercomputer at Department of Aeronautics and Space Engineering, Tohoku University. The inverse design for one cycle required about 45 min of single CPU time (most of the time is used for the Navier-Stokes computation).

### IV. RESULTS

As a model wing for transonic transport aircraft, a simple, swept and tapered wing shown in the left-hand side of Fig. 2 is considered for the shape optimization. The wing has a sweep angle of 20.4 deg, an aspect ratio of 7.38 and a taper ratio of 0.3. It should be noted that the taper ratio is small because such a wing approximately has the elliptical lift distribution.

The elliptical lift distribution is monitored at six locations from the root to the 83.3% span as indicated. The inverse solver used the same spanwise locations for the geometry correction. For the Navier-Stokes grid, the modification of wing geometry was linearly interpolated between those sections. In the tip region, the same airfoil section was used outside of the 83.3% section, while the wing twist was linearly extrapolated. The tip region is usually designed by other means and thus the optimization of this region is not considered here.

The right-hand side of Fig. 2 shows the computed pressure contours on the upper surface of the wing designed by the inverse method based on the target pressure distribution optimized by the present MOGA. Flow conditions were the freestream Mach number of 0.75, the Reynolds number based on the root chord of  $10^7$  and an angle

of attack of 0 deg. The resulting straight isobar pattern satisfies the first design principle well and also indicates good performance at higher Mach numbers. On the other hand, it shows a minor oscillation near the leading edge toward the root section. Although the airfoil sections vary very much from the root to 16.7 % section, a linear interpolation is used to create a Navier-Stokes grid for the brevity. To treat the root region as well as the tip region more precisely, an elaborated procedure may be necessary.

Fig. 3 shows the computed lift distribution of the designed wing in comparison to the elliptic distribution. The result is found to satisfy the second design principle closely. Fig. 4 shows the target chordwise pressures obtained from the present MOGA, the resulting airfoil shape of the wing and the corresponding pressures computed by the Navier-Stokes solver at the 16.7%, 50.0% and 83.3% spanwise sections. It confirms that the inverse problem is solved satisfactory except at the leading edge near the root section. The discrepancy of the pressure profiles there corresponds to the oscillation found in Fig. 2.

Figure 5 summarizes the aerodynamic performance of the designed wing in terms of the lift-to-drag ratio. For comparison purpose, two other wings were designed by the inverse method. The design indicated as 'Alternate' was obtained by changing the upper surface pressures when distributing the two-dimensional pressure distributions to the six spanwise sections for the initial population. Then the same MOGA was run. This procedure allows a wide variation of the pressure distributions in the spanwise direction. The resulting wing satisfies the second design principle of the elliptic distribution better than the present design but not the first one. Thus, it performs better at the design point but worse at higher Mach numbers.

The other design indicated as 'Isobar' was obtained by specifying the straight isobar pattern on both upper and lower surfaces of the wing. The resulting wing satisfies the first design principle of the wing exactly but not the second one. However, this is the standard design procedure for transonic wings. The reduction of the induced drag is simply relied on the use of a proper taper ratio. In fact, due to the present taper ratio of 0.3, this wing gives a good performance similar to the present design. Since the geometries of the two are completely different, this result confirms the present design gives a Pareto optimal solution under the contradicting design principles for the wing.

## V. CONCLUSION

MOGA based on Fonseca-Fleming's Pareto ranking method has been developed to optimize the three-

dimensional target pressures for the aerodynamic inverse design of wing shape. The optimization problem was formulated to minimize the induced drag for wings as well as to minimize the viscous drag for airfoil sections. Performances of both the simple GA and VEGA were found unsatisfactory to the present optimization problem.

The resulting procedure was successfully applied to transonic wing design. The standard design procedure for transonic wings was previously focused on materializing the straight isobar pattern over the wing. Reduction of the induced drag was merely relied on the use of a proper taper ratio for the wing planform. The present design procedure allows the minimization of the induced drag for arbitrary wing planform with any taper ratio. This will provide more design opportunity for wing shape in terms of better aerodynamic performance, lighter structural weight, and less expensive manufacturing cost.

## VI. REFERENCES

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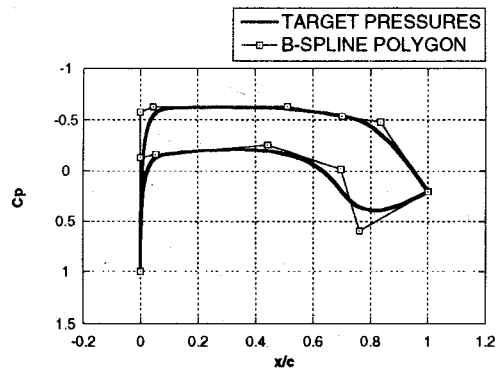


Fig. 1 B-spline polygon and corresponding pressure distribution.

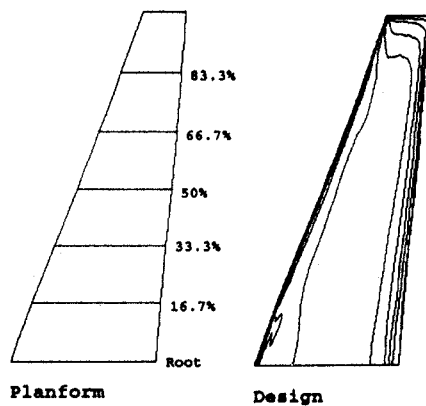


Fig. 2 Wing planform and computed pressure distribution on the designed wing.

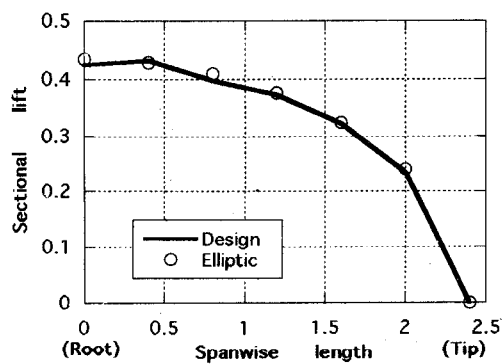


Fig. 3 Sectional lift distribution in the spanwise direction.

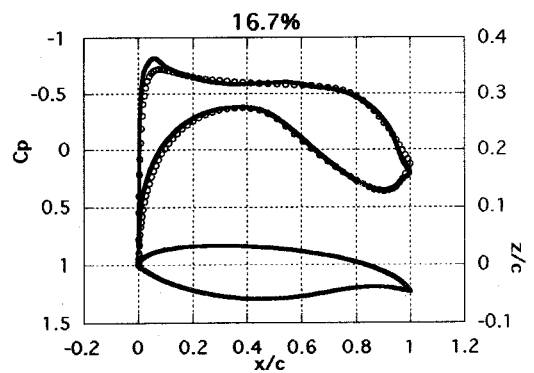
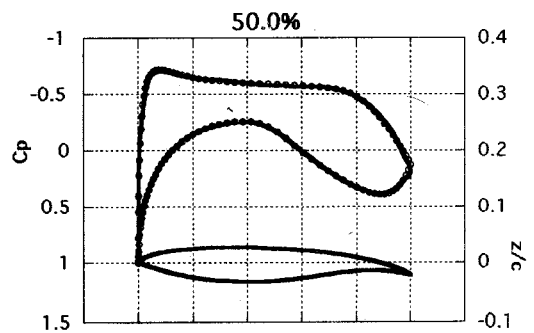
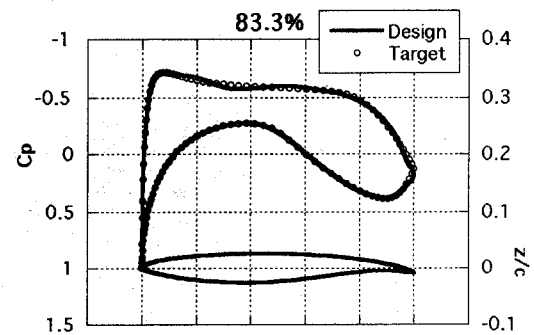


Fig. 4 Designed airfoil sections and corresponding chordwise pressure distributions.

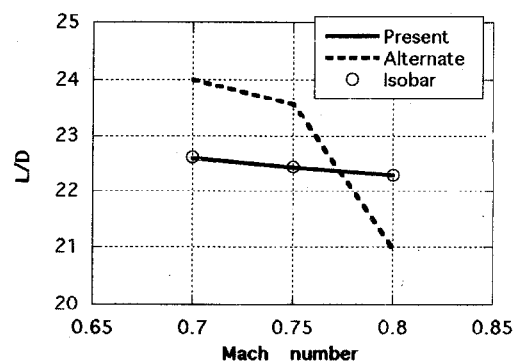


Fig. 5 Comparison of  $L/D$  performances.