Handling Integrated Quantitative and Qualitative Search Space in a Real World Optimisation Problem

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Abstract- Since information in engineering design problems can be both quantitative (Q^T) and qualitative (Q^L) in nature, combining both types of information can result in more realistic solution for real world optimisation problems. However, most of the approaches reported in literature are incapable of conducting optimisation search in such mixed environment. Therefore this paper proposes a mathematically proven methodology for handling integrated Q^T and Q^L search space in real world optimisation problems. The paper begins by presenting the definition of these optimisation problems an analysis of the challenges that they pose for existing optimisation strategies and related research. The paper then presents the proposed solution strategy and the mathmatical proof. Furthermore, a case study on rod rolling problem is presented to validate the effectiveness of the proposed metholodology. The paper concludes with a brief outline of limitations and future research activities.

1. Introduction

Information in real world engineering design problems can be both quantitative (Q^T) and qualitative (Q^L) in nature (Oduguwa *et al.,* 2003). Q^T models are very popular in real world design optimisation problems. Even though such models have been very useful in providing detailed information about the design problem, they can be ineffective in situations where the mathematical formulation of a design problem is not available or is partially defined. In such cases Q^L information can provide a valuable access to the design problem by taking advantage of human approximate reasoning to improve the complex design problem representation. Integrated Q^T and Q^L search space can therfore be defined as the combination of both types of information within a framework that enables an optimisation algorithm to facilitate a search towards a desirable goal. This tends to improve the efficient use of information and can result in more realistic solutions.

Such mixed forms of information within real world design optimisation problems can either complement, substitute or contradict each other. This paper focuses on the forms that contradit each other. Here the mixed type of information are conflicting in nature. There are various approaches reported in the literature for dealing with such mixed information engineering design problems. When used with design search scenarios, most of these approaches do not explore the trade-off relationships that exist between Q^T and Q^L search space. This can bias the search toward sub-optimal regions and can result in unrealistic solutions.

This paper proposes a methodology to deal with the challenges posed by integrated Q^T and Q^L search space optimisation problems. The mathematical proof of the solution strategy is also presented. Furthermore, a case study on rod rolling problem is presented to validate the effectiveness of the proposed metholodology. The paper concludes with a brief outline of limitations and future research activities.

2. Challenges in Integrated Q^T and Q^L Search Space

There are several challenges that can inhibit the wider applications of current optimisation strategies for real world design problems with contradicting Q^T and Q^L information. Some of these are outlined below.

- It is difficult to develop solution strategies that combine both types of information within an optimisation framework since most optimisation techniques deals with Q^T models only.
- Solving real-world problems could present scalability issues. The computational cost required to generate Q^L models when simulating the problem is exponential with increasing number of variables.
- Developing Q^L and Q^T search procedures for objectives greater than two can be complex. Higher number of Q^L objectives has the tendency to increase the fragmentation in the search space. This is largely due to the discreteness in the Q^L search space.
- It is difficult to ensure the appropriate correlation of the granularity of the Q^L models with the measurement scale of the Q^T models. Inappropriate correlation could deceive the genetic search to a local optimum.

3. Related Research

It is observed that although some attempt has been made to handle the Q^L and Q^T knowledge separately within a design optimisation framework, there is not much reported work on handling the two types of knowledge simultaneously within an evolutionary computing based optimisation framework.

Several approaches have been developed such as interval analysis (Moore, 1979), standard sensitivity analysis and probabilistic analysis (Siddall, 1983). Most of these approaches can be used to reason qualitatively about engineering design problems but are incapable of simultaneously dealing with Q^T models within optimisation framework.

There are several approaches developed based on the mathematics of fuzzy sets to incorporate Q^L knowledge into design. Most of the applications of fuzzy sets within the field of decision making consist of fuzzification of classical theories, where the fuzzy theories attempt to deal with the imprecision and vagueness in human reasoning of design variable preferences, constraints and goals. Some of the earlier work dealing with the optimisation of fuzzy systems was by , Tanaka *et al*., (1974), and Zimmerman (1974). Since then several variations of fuzzy based approaches have been reported in literature. Approaches based on fuzzy mathematical programming include fuzzy goal programming, flexible programming, fuzzy multi-objective optimisation, possibilistic programming with fuzzy preference operators and fuzzy linear programming. Antonnsson and Wood (1989) also developed a fuzzy based approach, referred to as the Method of Imprecision for engineering design problems where designers are given preference over a range of design values. Most of these approaches fuzzify the elements (constraints, goals or design variables) of an underlying mathematical formulation and do not combine the Q^L evaluation within the optimisation search.

There are a number of other fuzzy based approaches reported in literature where Q^L knowledge has been used in conjunction with Q^T models. Fuzzy Genetic Algorithms (FGA) manages problems in an imprecise environment. It combines fuzzy concepts with genetic algorithms. Approaches using fuzzy fitness evaluation function for the GA chromosomes has been reported in literature (Dahal *et al.,* 1999). In fuzzy optimisation Hsu *et al.,* (2001) adopted fuzzy optimisation algorithm for determining the optimal gap openings of the programming points in the blow moulding process and in fuzzy controlled simulation optimisation. Roy (1997) developed a design optimisation framework where both types of criteria or knowledge are handled separately. Most of the approaches reported above simply do not provide the means to deal with both Q^T and Q^L information simultaneously within an optimisation framework. Recently, Oduguwa *et al.,* (2003) extended the work of Roy (1997) by developing an integrated Q^T and Q^L evaluation optimisation approach which combines Q^L evaluation from designers with Q^T formulation of the design problem within an optimisation framework. The elaborate approach adopts the principle of multi-objective optimisation to explore the functional relationship between the Q^T and Q^L knowledge.

In previous work, the authors did not justify the functional relationships between the Q^T and Q^L information. The presence of such a functional relationships was treated as an empirical observation from previous work. This however presents a weakness for the proposed solution strategy. Therefore, this paper presents a mathematical justification of the functional relationship between the mixed information. This enhances the rigour of the proposed soluiton strategy. Furthermore, a case study on rod rolling problem is presented to validate the effectiveness of the proposed methodology.

4. Handling Integrated Q^T and Q^L Search Space

Q^T search space in the sense of engineering design problems is such that for every feasible design point identified in the parameter space there is a corresponding objective function value. Therefore it is widely accepted that a functional relationship exists between the design parameters in the parameter space and the objective function values. This functional relationship was nicely defined by Bottazzini (1986). This is stated as follows:

> *Let A and B be two sets, which may or may not be distinct. A relation between a variable element x of A and a variable element y of B is called a functional relation in y if, for all x in A, there exists a unique y in B which is in the given relation with x.*

By the same analogy for solutions lying on the Pareto front, for every Q^T solution to a given design problem, there exists a corresponding Q^L evaluation expressing the designer's opinion about the problem. This Q^L evaluation varies in a unique fashion with the Q^T solution.

The integrated Q^T and Q^L optimisation problem could be viewed as a multi-objective problem and an existing multi-objective optimisation algorithm could be applied to solve such problem. However, this is only applicable in those cases where the two objectives conflicts with each other (i.e only in those cases where Pareto front exist).

For solutions lying on the Pareto front, it is widely acceped that there is a functional relationship between the two objectives. This implies that a multi-objective optimisation algorithm could be applied only to those integrated Q^T and Q^L optimisation problems in which the optimum solutions have a functional relationship between the two objectives. The section that follows presents a mathematical justification for this functional relationship and identifies the conditions under which this is true.

4.1. Mathematical Jusification of Functional Relationship

4.1.1. Theorem

There exist a functional relationship between both Q^T and Q^L Pareto optimal solution of design problems.

4.1.2. Definitions

The following definitions are used in conjunction with the mathematical justification.

Definition 1:
Q^L evaluation is a proposition of the form "if A then B" semantically expressing the designers opinion with respect to inputs of parameter values into the objective function values of a given Q^T model.

This is represented as $\tilde{A} = \{\tilde{f}(x) \mid x \in X\}$ where the tilde represents the fuzziness in the Q^L evaluations, and modelled as stated in definition 2.

Definition 2:
If X is a collection of objects denoted generically by x then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs: $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$ where $\mu_{\tilde{A}}(x)$ is the membership function of x in $\tilde{A}$ which maps X to the membership space M.

Definition 3:
The Q^T model and the Q^L model (obtained from definition 2) represent two independent objective spaces explaining different behavioural aspects of an overall design problem.

Definition 4:
Two propositions $\tilde{A}$ and $\tilde{B}$ are equivalent if and only if the membership function values induced by $\tilde{A}$ and $\tilde{B}$ are equal such that: $\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x), \forall x \in X$

Definition 5:
QT objective function value is a function of the form $y = f(x)$, $x \in X$, where y is the objective function value and x is a location in the search space.

The theorem is therefore stated mathematically as follows:

Let $\{\tilde{A} \in \Re^n \times I \mid I\ [1,0]\}$, $B \subseteq \Re$ such that:

$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$

$B = \{f(y) \mid y \in Y\}$

Then $K = \{[\mu_{\tilde{A}}(x), f(y)] \mid (x, y) \in X \times Y\}$ *is a functional relationship on the Q^L evaluation A and the associated objective function value of the Q^T model.*

4.1.3. Identification of validity conditions

This section identifies the conditions under which the proposed theorem is valid. For a functional relationship to exist between the variables x and y, such that $y = f(x)$, where $x \in X$, this condition; $x_1 = x_2 \Rightarrow y_1 = y_2$ must be satisfied.

The condition is a standard for functional relationships in Q^T based models. However, this condition is specified as proposition P1, for functional relationship between both Q^T and Q^L solutions to exist.

P1: $f(x_k) = f(x_{k+1}) \Rightarrow \tilde{f}(x_k) = \tilde{f}(x_{k+1})$

P2: $\tilde{f}(x_k) \neq \tilde{f}(x_{k+1}) \Rightarrow f(x_k) \neq f(x_{k+1})$

Proposition 1 (P1): $f(x_k) = f(x_{k+1}) \Rightarrow \tilde{f}(x_k) = \tilde{f}(x_{k+1})$

This proposition states that for a given set of identical objective function values ($f(x_k)$, $f(x_{k+1})$), the associated Q^L evaluations are equal. The equality expression on the right hand side is treated in accordance with definition 4. There are clearly two cases to be considered in this proposition.

Case I: $x_k = x_{k+1}$
This is the case when the two designs are the same. This implies that both their Q^T and Q^L evaluations are also equal. Therefore, P1 is unconditionally true for cases where the two designs under consideration are the same.

Case II: $x_k \neq x_{k+1}$
This is the case when the two designs are different. If the Q^T evaluations of two different designs are equal, then one of the following conditions is true:

- The corresponding Q^L evauation of the two designs are equal
- The corresponding Q^L evaluations of the two designs are different

However as stated in this proposition, the proposed theorem is valid only if the equality of the Q^T evaluations of the two different designs implies the equality of the corresponding Q^L evaluations. This condition is mathematically stated in Lemma 1. Therefore Lemma 1 provides a necessary condition for the proposed theorem to be valid.

Lemma 1: $\{f(x_k) = f(x_{k+1}) \Rightarrow \tilde{f}(x_k) = \tilde{f}(x_{k+1})\} \mid (x_k \neq x_{k+1})$
This lemma provides a condition that must be satisfied for the solutions to lie on the Pareto front. Here two different designs having identical Q^L evaluations implies that the imprecision in human reasoning perceives both solutions as being identical. The difference in the designs is not perceived to be sufficiently enough to result to different Q^L evaluations.

The above discussion reveals that a multi-objective optimisation algorithm could only be applied to those integrated Q^T and Q^L problems whose optimum solutions satisfy Lemma 1. The converse of LI is P2, however it should be noted that L1 is equivalent to the conditions above since $A \rightarrow B \Rightarrow !B \rightarrow !A$.

4.2. Solution Strategy for integrated Q^T and Q^L Search Space Problems

The fundamental principle for combining the Q^T and Q^L information is based on transforming the Q^L information into cardinal information with the subsequent use of multi-criteria method. Evolutionary multi-objective optimisation solution approach is proposed as a solution strategy for the integrated Q^T and Q^L search space problem as discussed above. The rational for adopting this strategy is based on the following considerations:

- A multi-objective GA is applied for solving the Q^T and Q^L where the cardinality of the objectives is greater than one.
- This problem is such that the nature of this relationship exhibits a conflict.
- This can be applied to problems that is assumed to have Pareto front. Here, Pareto front implies a functional relattionship between the two objectives for solutions lying on the Pareto front.

- A structured method is required to explore the conflicting behaviour of the two objectives.

The following conditions are presented for which the solution strategy applies.

- The Q^T and Q^L objectives derived in relation to definitions 3 represent objective cardinality greater than one.
- The proposed theorem is valid only if the equality of the Q^T evaluations of two different designs implies the equality of the corresponding Q^L evaluations(Lemma 1).

4.3. Solution Approach

The optimisation algorithm as shown in Figure 1 is based on the genetic algorithm (GA) integrated with a fuzzy reasoning module.

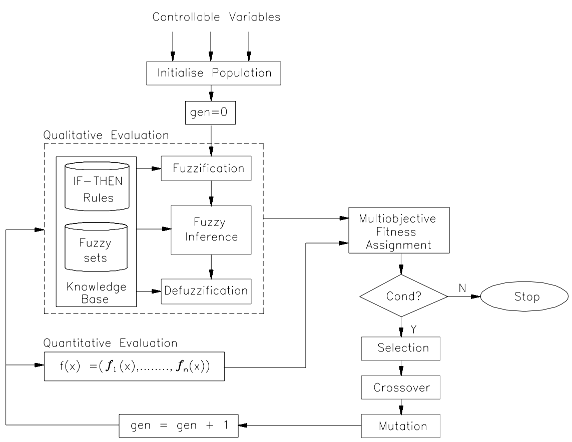


Figure 1: Optimisation for Integrated Q^T and Q^L Search Space

The fuzzy reasoning module consists of fuzzification, fuzzy inference and defuzzification routines. Values of the decision variables from individual members of the population are fuzzified, and fuzzy IF-THEN rules are applied within the fuzzy inference mechanism. The evaluation of a proposition produces a single fuzzy set associated with each model solution variable. An appropriate method of defuzzification is used to find a scalar value and the corresponding membership grade that best represents the information contained in the consequent fuzzy set.

The Q^L evaluation module outputs both the scalar value and the associated membership value from the defuzzified fuzzy set. The scalar value represents the approximate Q^L evaluation of the design problem. This value represents the goodness of the individual based on Q^L model and is used as the Q^L objective function value to rank part of the fitness of the individual. The Q^L fitness evaluation also takes into account the membership grade to ensure that membership grade below a selected threshold is penalised using the penalty function method. The Q^L objective value is represented formally as: $\{\tilde{f}(\vec{x}) \mid \mu(\tilde{f}(\vec{x})) > \alpha\}$, where $\tilde{f}(\vec{x})$ is the defuzzified domain value, $\mu(\tilde{f}(\vec{x}))$ is the membership function value and α is a threshold set for the membership function value. Similarly, the Q^T evaluation represents the Q^T evaluation of the design problem. This value represents the goodness of the individual based on the Q^T model and is used as the Q^T objective function value ($f(\vec{x})$) to rank part of the fitness of the individual.

Final fitness solution of each member of the population is based on a Pareto dominance ranking mechanism that considers the objective function values from both the Q^T and Q^L models expressed as: $\vec{f}_i^t(\vec{x}) = \{(f_i(\vec{x}), \tilde{f}_i^t(\vec{x})) \mid \mu_i(\tilde{f}(\vec{x})) > \alpha\}$, where i is a member of the population . In order to select the fittest member of the population, each individual is ranked based on the Pareto dominance criteria stated in section 2 and shown in the algorithm in **Error! Reference source not found.**. The multi-objective ranking mechanism then performs a non-domination ranking procedure on each member where it is assigned a ranking value based on its location in the objective space.

5. Rod Rolling Design Problem

The proposed approach is illustrated using a rod rolling design problem. The rod rolling process is a continuous manufacturing process whereby a square billet (dimensions ranging from 100mm to 150mm) referred to as the stock is deformed into a rod size ranging from 5mm to 12mm. The rod design problem is a two-objective optimisation problem (maximising the shape of the rod profile and minimising the deformation load). It is used to illustrate an optimisation problem based not only on Q^T information but also on the engineer's Q^L knowledge for solving complex engineering design problems.

The shape condition is a roundness measure of the rod profile often measured using classical numerical models. Since the rod profiles tend to emerge as non-smooth, most of the shape conditions evaluated using classical models do not tend to correlate with the designer's representation. Here, a Q^L model is proposed to capture the designer's representaion of the shape condition.

In this study, the shape and the load required for rod deformation are modelled using fuzzy reasoning and meta-modelling technique respectively. The simultaneous optimisation of both responses is treated as a multi-objective problem. The problem is considered multi-objective in nature due to the conflicting relationship between the two objectives. In practice, for a given stock size a perfect shape condition requires large roll pockets. This implies a high contact of the stock with the roll, which results in high loads.

5.1. Experimental Procedure and Model Development

A single roll pass was modelled using the ABAQUS Explicit FE simulation software. The case study described in this paper deals with the shape, and load optimisation of a single oval to round wire rod pass. The geometrical parameters relevant to the present study that affects these objectives were solicited from the domain expert and categorised as: (a) initial thickness (h_1), (b) initial width

(w_1), (c) work roll radius (R), (d) pass radius (*Pr*), (e) roll gap (*Rg*) and (f) temperature (*T*).

The genetic search for optimal solution requires a model definition that quantifies the 'goodness' of each solution according to the formulation of the optimisation problem. Here specific model details of the objectives are shown in the sections that follow. Details of the Q^T and Q^L model development process for shape and load are detailed elsewhere (Oduguwa and Roy, 2003), and therefore are omitted in this paper.

5.1.1. Quantitative Modelling

Advanced computational simulation is becoming a key component of engineering research and product development. However despite improvement in both hardware and software the function evaluation tends to be computationally expensive.

In order to address these problems, approximate metamodels are developed using Response Surface Methods (RSM). The metamodel is a typical example of functional approximation defined as a model of an underlying simulation model. The RSM is used in this study since it is one of the most popular method of constructing approximate models in the design optimisation literature (Montgomery and Peck, 1992). RSM can be used to create smooth approximations of the response data. In its simplistic sense, RSM involves (a) choosing an experimental design for generating data, (b) choosing a model to represent the data, and then (c) fitting the model to the observed data.

Q^T models of the responses were generated by fitting a second order model (main effects, interaction effects and quadratic effects). The fit with the lowest sum of squares error (highest R^2) was selected, this resulted in the following experimental models (initial stock area/roll area (SAR), form factor (FF) and the roll radius/material height ratio (RRMR)) as predicted using ANOVA.

$$\text{Load} = -2023520.422 + 50112.96\, h_1 - 853.369\, h_1^2 + 35728.755 w_1 + 434.706\, w_1^2 - 39003.709\, Pr + 604.149\, Pr^2 + 19369.967\, R_g - 1271.041 R_g^2 + 578.474\, R_r - 2.206\, R_r^2 + 2799.198\, T - 1.039\, T^2 + 1135\, h_1 w_1 - 177.396\, h_1 Pr - 305.971\, w_1 Pr - 2075.8333\, w_1 Rg + 57.274\, w_1 Rr - 77.103\, w_1 T + 417.083\, PrRg + 14.183\, PrT - 0.413\, RrT \quad (1)$$

$$\text{SAR} = -1.976 + 0.1106\, h_1 - 0.00157\, h_1^2 + 0.184\, w_1 - 0.0012\, w_1^2 - 0.104\, Pr + 0.0025\, Pr^2 + 0.0046\, Rr - 2.708\text{E-}6\, Rr^2 + 11.24\text{E-}6\, T + 0.0026\, h_1 w_1 - 5.728\text{E-}4\, h_1 Pr - 1.0455\text{E-}4\, h_1 Rr - 0.002\, w_1 Pr - 0.0036\, w_1 Rg - 1.207\text{E-}4\, w_1 T \quad (2)$$

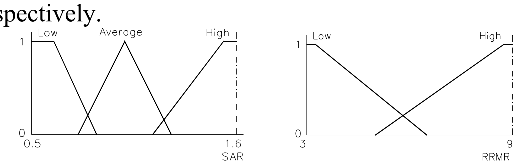
$$\text{RRMR} = 6.155 - 0.375 h_1 + 0.0056 h_1^2 + 0.061 Rr + 5.877\text{E-}5\, h_1 Rg - 9.319\text{E-}4\, h_1 Rr - 1.267\text{E-}5\, RgRr \quad (3)$$

$$\text{FF} = 11.109 - 0.190\, h_1 + 0.0022\, h_1^2 - 0.525\, w_1 + 0.0077\, w_1^2 + 0.0022\, Pr^2 + 0.176\, Rg + 0.00561\, Rg^2 - 0.0035\, Rr + 5.191\text{E-}6\, Rr^2 - 0.0061\, T + 9.765\text{E-}7\, T^2 - 8.722\text{E-}4\, h_1 w_1 + 0.0011\, h_1 Pr + 7.015\text{E-}5\, h_1 Rr - 0.0061\, w_1 Rg - 4.1\text{E-}5\, w_1 Rg + 2.532\text{E-}4\, w_1 T - 0.0011\, PrRg - 1.695\text{E-}4\, RgRr + 4.319\text{E-}5\, RgT \quad (4)$$

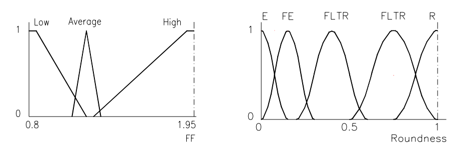
5.1.2. Qualitative Modelling

Q^L evaluation of the design solutions is performed by the experts to determine the suitability of each design. In this study, fuzzy modelling technique is used to represent the Fuzzy models that were developed by formulating the response variables of equations 2, 3 and 4 as the antecedent part of the rules, and modelling the expert's reasoning of the FE outputs as the consequent part of the rules. Nine rules were developed. Details of the rule development process are as follows.

For each of the response variables, the fuzzy sets shown in Figure 2a-Figure 2c were created with triangular membership functions and their corresponding linguistic labels *low*, *average* and *high*. These fuzzy sets correspond to the expert's interpretation of the variable's behaviour with respect to the rod roundness phenomenon. The membership function for each fuzzy variable shows the degree of membership of each value in the variable's fuzzy sets for the range of interest. For example, Figure 2a shows the membership function for stock area (SAR). The membership functions for the h/w ratio and form factor were also developed as shown in Figure 2b and Figure 2c respectively.



(a) Fuzzy Set SAR **(b) Fuzzy Set RRMR**



(c) Fuzzy Set Form Factor **(d) Fuzzy set Roundness**

Figure 2: Membership Functions for Rolling Variables

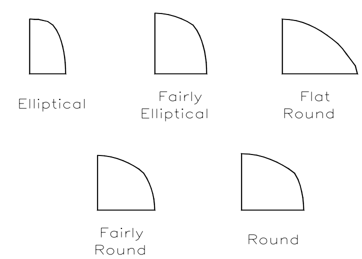


Figure 3: Classification of FE Rod Shape Profiles

The consequent part of the fuzzy rule was developed to represent the expert's Q^L evaluation of the roundness of the rod profile. This was achieved by initially classifying the FE output of the rod profiles into five main categories as shown in Figure 2d. These five categories were then formulated into the following five fuzzy sets (as shown in Figure 3) with bell shaped membership functions. The corresponding linguistic labels are: *Elliptical (E), Fairly Elliptical (FE), Flat Round (FLTR), Fairly Round (FR),* *and Round (R)*. These represent the way experts' reason about the roundness of the rod profile.

A rule base that specifies the Q^L relationship between the output parameter (shape condition) and the input parameters: initial stock area/roll area (SAR), form factor (FF) and the roll radius/material height ratio (RRMR) formulated as shown in Table 1. These rules were developed by interactive interview with the domain experts. For example rule 1 shows that, if the area ratio is *average*, the RRMR is *low*, and the form factor is *low* then rod profile is predicted as '*round'*.

Table 1: Fuzzy Rule Base

Rule no	SAR	RRMR	FF	Roundness
1	Ave	L	L	R
2	H	H	L	FLTR
3	H	L	L	FLTR
4	Ave	H	H	FE
5	Ave	H	Ave	FR
6	L	L	H	E
7	Ave	L	H	E
8	L	H	H	E
9	Ave	H	L	R

The compensatory weighted mean operator was used to aggregate the fuzzy sets in the antecedent part of the rule. This ensures that the cumulative effect of the other rules influences the determination of the strain distribution. These fuzzy sets were then converted into a scalar value using the centroid method of defuzzification in the final step of the fuzzy inference cycle.

The fuzzy sets, input, output fuzzy variables and fuzzy rule base all constitute the Q^L model that is used within the optimisation module to evaluate the Q^L aspect (shape condition) of the design problem. These fuzzy sets are then converted into a scalar value by a chosen method of defuzzification in the final step of the fuzzy inference cycle. A centroid method of defuzzification is used in this study. The defuzzified scalar value best represents the fuzzy solution sets.

6. Definition of the Optimisation Problem

The rod design problem is a two objective optimisation problem. The aim of this module is to solve a two objective rod design optimisation problem using simplified method of dealing with the membership function. The design problem consists of two cardinal objectives: to maximise the shape of the rod profile using Q^L models of the rod profile and minimise the deformation load using the Q^T model. The multi-objective optimisation problem is formulated as shown below:

Minimise	Load	$f_1(\mathbf{x}) = P$	(5)
Maximise	Shape	$f_2(\mathbf{x}) = \tilde{A}(x)$	(6)
Subject to		$P > 0$	(7)
		$\mu_{\tilde{A}(x)} > 0.5$	(8)

where: fuzzy terms are denoted by the tilde, $\mu_{\tilde{A}(x)}$ is the membership grade of the shape condition. P is the Q^T model given by equation 1 in section 5.1.1. Equation 7 is a constraint that ensures the deformation load is not negative while equation 8 controls the influence of the membership function values on the search space. These constraints were dealt with using the penalty function method.

NSGAII (Deb *et al.,* 2000) was adopted in the study since it is one of the most popular multi-objective GA. NSGAII was used to rank each member of the population in terms of the fitness from the Q^T model and the Q^L model. Fitness from the Q^L model consists of defuzzified scalar values and the associated membership grade from the fuzzy inference mechanism. This describes the shape condition of the rod profile and the deformation load for the rod design. Solutions having membership grades below a chosen threshold (0.5 in this study) are considered infeasible for the rod problem and are replaced by feasible solutions obtained by conducting a local search. Similarly, fitness from the Q^T models expresses the positive load deformation.

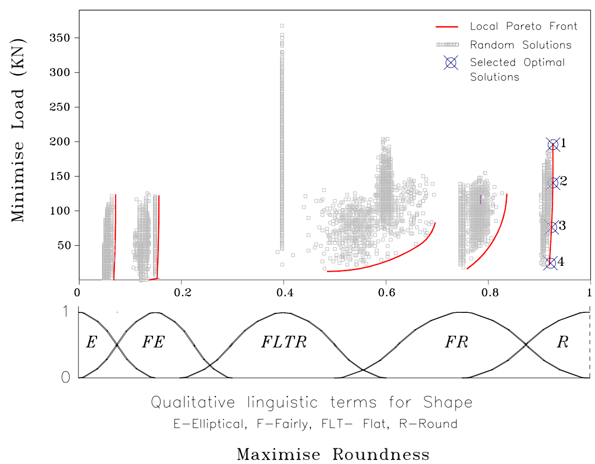


Figure 4: Q^T and Q^L Search Space of Design Problem

7. Test Results and discussion

The proposed approach was used to optimise two objectives: the maximisation of the fuzzy output values (defuzzified domain value and the associated membership grade) and the minimisation of the deformation load. The Q^T and Q^L models outlined in section 5.1 were used for this purpose. The proposed algorithm was implemented for the test problem using C++ code on a Pentium 4 PC. Ten independent GA runs were performed in each case using a different random initial population. A population size of 100 was used with a generation of 1000 iterations. In most of the cases examined, seven out of ten runs obtained similar results.

A random search was conducted on the problem to explore the nature of the search space. Figure 4 shows the search space of the multi-objective Q^T and Q^L search space in this study. Here the functional relationship of the Q^T and Q^L objectives is analysed with respect to the theorem given in section 4 for Pareto optimal solutions given in Table 2. P1 supports the functional relation for any combination of solutions 1,2,3 and 4. Where P1 states that for a given set of identical objective function values ($f(x_k)$, $f(x_{k+1})$), the associated Q^L evaluations are also equal. Since ($a \rightarrow b$) is the same as (not a $\rightarrow$ not b), therefore for functional relationship to exist between Q^T and Q^L information for any two different design ($x_k \neq x_{k+1}$) solutions to lie on the Pareto front, the different Q^T evaluations should also imply different Q^L evaluations. For example 1 and 4, the QT objective values are 179.55 and 28.6 respectively while the equal Q^L evaluations are also R{0.823(0.93)} and Fr{0.51(0.91)}. This illustration confirms the functional relationship between the Q^T and Q^L objectives, and it also indicates the conflicting nature of this relationship. This conflicting behaviour and the two-objective cardinality therefore confirm the appropriateness of the multi-objective solution approach.

In the sections that follows, the NSGAII results of the multi-objective problem and some of the challenges poised by optimising within integrated Q^T and Q^L search space are discussed.

7.1. Experimental Results

The trade-off solutions between roundness and load located in the optimal region by the NSGA II optimisation algorithm is shown in Figure 5. Despite the complexity of the problem, NSGAII was able to find solutions in the optimal region of the design space. Non-dominated solutions were obtained from the experimental runs. The Pareto optimal solution plot shows the spread of the optimal solutions in the two dimensions. Figure 5 also shows a selection of optimal solutions and their variable values from the experimental runs. It demonstrates the diversity of the vectors of the decision variables in the parameter space. Since solutions on these fronts are all equally good, further higher level criteria could be applied to select a suitable solution for the problem.

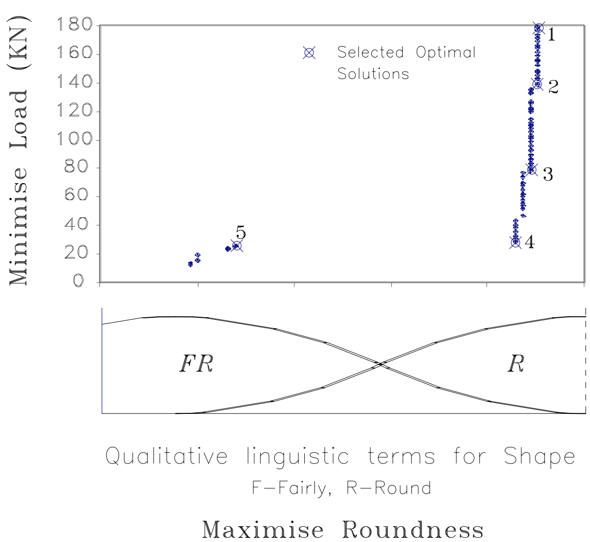


Figure 5: NSGA II Pareto Solution Plot

Table 2: Selected Solutions

No	(h)	(w)	Pass Rad	Roll Gap	Roll Rad	Temp	Shape	$\mu(f(x))$	Load (KN)
1	29.1	22.2	30.1	4.3	254.2	769.3	R	0.823(0.93)	179.55
2	28.1	21.2	30.4	4.3	254.2	731.5	R	0.824(0.92)	138.5
3	28.1	22.2	30.4	3.6	114.4	728.1	R	0.72(0.92)	78.2
4	29.5	19.2	30.9	0.9	119.8	1070	R	0.51 (0.91)	28.6
5	27.8	19.1	26.8	1.1	162.6	1062	FR	0.72(0.77)	26.1

Selected solutions shown in Table 2 provide insight into both the parameter and the objective space, which is useful for designers to select suitable solutions for the given problem. For example, solutions number 3 and 5 has a roundness and load value of (round , 78.2 KN) and (flat round, 26.1 KN) respectively. Since both solutions are equally good, designers might prefer to tradeoff solution in the parameter space by selecting solution 3 based on lower temperature (728.1°C). Similarly, designers might prefer to tradeoff solution in the objective space by selecting solution 5 based on lower deformation load (26.1KN).

7.2. Limitations

Three main issues are discussed that reflect the limitations of the proposed approach to design optimisation problems with Q^L evaluation.

- Visibility of the Q^L parameter space to the search algorithm is lost due to the transformation of the Q^L information into cardinal information. As a result, it becomes difficult to control the equivalent correlation between granularity of the Q^L models with the measurement scale of the Q^T models.
- The approach is mostly suitable for real world problems with lower number of objectives, as higher number of Q^L objectives has the tendency to increase the fragmentation in the search space. This is largely due to the discreteness in the Q^L search space.
- The approach mainly deals with Q^T and Q^L information that are conflicting in nature. It is not suitable for mixed form of information that are complementary in nature. This limitation is due to the fundamental solution strategy adopted in the approach.
- The Lemma 1 condition limits the application to problems that satisfies this condition.

7.3. Future Research

Future research activities are required to address the main limitations described in the previous section and the challenges outlined in section 2. This section briefly describes main research directions.

- Studies are required to develop optimisation algorithms that can deal with various combinations of Q^T and Q^L information in a single framework. This can improve the robustness of such techniques for real world problems.
- Scalability of integrated Q^T and Q^L design optimisation strategies to higher dimensional problems is an important success criteria for wider applications. This is influenced by the feature of the problem (large number of parameters) and the nature of the resulting

search space (fragmentation). This is due to the discontinuity present in real world problems and Q^L design space.

- Techniques are required for representing the native parameter space of the Q^L information within the optimisation framework. This could provide capabilities for tuning the correlation between the granularity of the Q^L models with the measurement scale of the Q^T models. Search algorithm that considers such features of the problem should give better performance.

8. Conclusions

Most real world engineering design problems can be Q^T and Q^L in nature. A review of the literature reveals that most optimisation algorithms are not capable of dealing with such mixed information simulataneusly within a design optimisation framework. This paper proposes a methodology to deal with the challenges posed by integrated Q^T and Q^L search space in real world optimisation problems. The mathematical proof of the solution strategy is also presented. A case study based on multi-objective rod rolling problem is presented to validate the effectiveness of the proposed metholodology.

The results obtained show Q^T solutions and their functional relationships with the Q^L evaluations in the optimal region of the search space. This demonstrates that the proposed solution approach can be used to solve real world problems having integrated Q^T and Q^L information. The paper concludes with a brief outline of limitations and future research activities.

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