The Real-Biased Multiobjective Genetic Algorithm and Its Application to the Design of Wire Antennas

Ricardo M. Ramos, Rodney R. Saldanha, Ricardo H. C. Takahashi, *Member, IEEE*, and Fernando J. S. Moreira, *Member, IEEE*

***Abstract*—This work presents a multiobjective genetic algorithm with a novel feature, the real biased crossover operator. This operator takes into account the function values of the two parents, defining a nonuniform probability for the new individuals' locations that biases them toward the best parents' locations. The procedure leads to better estimates of the Pareto set. The proposed algorithm is applied to the optimization of a Yagi–Uda antenna in a wide frequency range with several simultaneous performance specifications, providing antenna geometries with good performance, compared to those presented in the available literature.**

***Index Terms*—Genetic algorithms, multiobjective optimization, wire antennas.**

I. Introduction

THE OPTIMIZATION of antennas is a long-standing problem. Design specifications considering maximum directivity and impedance matching, together with sidelobe-level requirements, have been gradually introduced since the first attempts to deal with the problem [1]–[4]. In [1], deterministic methods were used to reach the maximum gain, while [2] also deals with the input impedance, in order to maximize the matching with a transmission line. The optimal design of antennas becomes a more complex task when a set of different conflicting specifications is needed. For instance, in wireless applications, the antenna performance over a relatively large spectrum is of primary concern. In such situations, the antenna design is generally guided by the tradeoff between a broadband device and maximum directivity. Furthermore, the sensitivity of the input impedance with the frequency makes the problem more complex. These issues are approached in this paper, which applies multiobjective techniques not considered in previous works [1]–[4].

Due to the problem of intrinsic complexity and the large amount of results available in literature, the optimization of wire antennas can also be used as a test bed for optimization problems. This paper can be also viewed in this way; it presents a new optimization algorithm, the real-biased multiobjective genetic algorithm (RBMGA), in the context of the optimization of antennas. The results that are obtained support the conclusion that the algorithm can efficiently deal with complex practical problems. For the problem at hand (the optimization of an Yagi–Uda antenna), the RBMGA has lead to antenna designs that are superior to ones found in the literature [1]–[4] for the proposed multiobjective design specifications.

Manuscript received June 18, 2002. This work was supported by CNPQ and FINEP-Brazil.

R. M. Ramos is with Faculdade de Ciências Exatas de Pedro Leopoldo, UNIPEL, Department of Computer Science, Pedro Leopoldo, MG, Brazil.

R. R. Saldanha is with the Department of Electrical Engineering, Universidade Federal de Minas Gerais, 31270-010, Belo Horizonte, MG, Brazil (e-mail: rodney@cpdee.ufmg.br).

R. H. C. Takahashi is with the Department of Mathematics, Universidade Federal de Minas Gerais, 31270-010, Belo Horizonte, MG, Brazil (e-mail: rtakahashi@ufmg.br).

F. J. S. Moreira is with the Department of Electronics Engineering, Universidade Federal de Minas Gerais, 31270-010, Belo Horizonte, MG, Brazil (e-mail: fernando@eee.ufmg.br).

Digital Object Identifier 10.1109/TMAG.2003.810505

II. Multiobjective Optimization Problem

The multiobjective optimization problem can be defined as follows. Given a set of objective functions to be minimized, f_i, $i = 1, \ldots, m$, one should find the set $\mathcal{P}$ of solutions defined as follows:

$$\mathcal{P} \triangleq \{x^* \in \mathcal{F} |\ \nexists x \in \mathcal{F} \quad \text{such that} \quad f(x) \leq f(x^*)$$

and

$$f(x) \neq f(x^*)\} \tag{1}$$

in which $\mathcal{F}$ denotes the feasible set. The set $\mathcal{P}$, containing the efficient solutions x^*, is called the Pareto-optimal set.

In the present work, the objective functions are set upon the antenna specifications, aiming the maximization of the directivity, the front-to-back ratio, and the impedance matching, together with the minimization of the half-power beamwidth, over three different frequencies through the antenna bandwidth, resulting in 12 different objectives.

III. Multiobjective GA With Real-Biased Crossover

In the present work, the novel "real-biased crossover operator" is employed for the construction of a multiobjective genetic algorithm. The RBMGA is defined as the successive application of the following operations: 1) population evaluation and fitness function computation for each objective function; 2) multiobjective fitness function evaluation; 3) selection by roulette; 4) real-biased crossover; 5) mutation; and 6) Pareto-set elitism with niche. Operations 1) and 3) are as usual [5]. The other ones are briefly explained here.

Multiobjective Fitness Function Evaluation

After operation 1), each individual x_i has, for the antenna optimization at hand, a set of 12 fitness function values $f_j(x_i)$, for the 12 objective functions ($j = 1, \ldots, 12$). These fitness function values are in the range [0, 1]. The multiobjective fitness function that is assigned to the individual is based on the functional

$$F(x_i) = \min_j f_j(x_i). \tag{2}$$

After this functional evaluation for all individuals, the fitness function is reevaluated with $F(x_i)$ replacing the "objective function."

Real-Biased Crossover Operator

The real-biased crossover is defined as follows.

- The population (with N individuals) is randomly ordered in $N/2$ pairs of individuals. For each pair, the crossover will occur with probability 0.6.
- For each pair subjected to crossover, the fitness function $J(x)$ of the individuals is considered. The individuals (vectors of real parameters) are labeled x_1 and x_2, such that $J(x_2) < J(x_1)$.
- The real biased crossover generates one son individual x_g as

$$x_g = \alpha x_1 + (1 - \alpha) x_2 \tag{3}$$

with α chosen in the interval $[-0.1; 1.1]$, according to the probability distribution defined by

$$\alpha = 1.4\beta_1\beta_2 - 0.2$$

where β_1 and β_2 are random variables with uniform probability distribution inside the domain [0; 1]. These provide a quadratic probability distribution for α which makes the new individual x_g have a greater probability of being closer to x_1 (the best parent individual) than to x_2 (the worst parent individual).
- The other son individual is chosen without bias, i.e., α is chosen in the interval $[-0.1; 1.1]$ with uniform probability.

The specific evaluation of this operator, in the context of mono-objective optimization, can be found in [6].

Reflection Operator

In the case of one individual being out of the admissible range, the reflection method is applied to force the individual back inside the feasible region. For a reflection by the lower limit (x_L) the operation is defined as

$$x_r = x_L + |x - x_L| \tag{4}$$

where x is the individual outside the admissible range and x_r represents the resulting individual, after reflection. The analogous operation is defined for the upper limit.

Mutation Operator

The mutation operator is defined as follows. Each individual in the population can be subjected to mutation, with probability 0.03. If an individual x suffers mutation, the resulting individual x_m is defined as

$$x_m = x + \delta \tag{5}$$

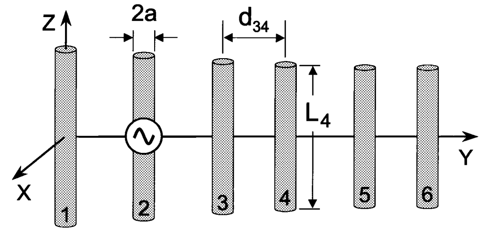

Fig. 1. Six-element Yagi–Uda antenna configuration.

with δ being defined componentwise as

$$\delta_i = 0.05\beta_i(x_r)_i \tag{6}$$

where β_i is a random number with Gaussian distribution, zero mean, and variance equal to one, and x_r is a range vector with lower and upper limits given by x_L and x_U, respectively.

Pareto-Set Elitism With Niche

The subpopulation that is compatible with the "Pareto-set," under definition (1), is extracted from the population. A subset of this subpopulation is deterministically reintroduced in the population, according to the following rule: once an individual is reintroduced in the population, no other individual inside a radius ρ around that individual is reintroduced.

IV. Multiobjective Yagi–Uda Antenna Optimization

In this section, the proposed RBMGA is applied to the optimization of a six-element Yagi–Uda antenna, illustrated in Fig. 1. The element centered at the origin is the reflector, followed by the centered-fed driven element and the four directors. The distances d between consecutive elements (five different distances) and the lengths L of each element are the parameters to be optimized. The cross-section radius a is the same for all elements and is set equal to 0.003 377 wavelengths at 300 MHz.

The design specifications upon the antenna radiation pattern are the highest possible directivity and front-to-back ratio while sustaining a narrow half-power beamwidth on both E and H planes. The impedance matching is attained by requesting an input resistance close to 50 Ω. Such requirements are imposed for three different frequencies (the lower, middle, and upper frequencies) over a 5% bandwidth centered at 300 MHz (292.5–307.5 MHz). All dimensions are given in wavelengths (λ) at 300 MHz.

The electrical characteristics of the antenna, necessary to the establishment of the objective functions, are attained by a numerical analysis based on the method of moments (MoM) [7]. The electric current densities over each dipole element p ($p = 1, \ldots, 6$, according to Fig. 1) are expanded as [7]

$$I_p(z) = \sum_{n=1}^{N} I_{np} \cos\left[\frac{(2n-1)\pi z}{L_p}\right] \tag{7}$$

where N controls the number of sinusoidal basis functions used to represent the current densities and L_p is the length of the pth dipole element. In the present work, $N = 16$ was adopted for all dipoles. The series expansion in (7) is chosen such that $I_p(z)$

TABLE I

GEOMETRIES OF THE THREE ANTENNAS CHOSEN FROM THE PARETO-OPTIMAL SET. DIMENSIONS ARE GIVEN IN WAVELENGTHS (λ) AT 300 MHZ

RBMGA					
Antenna 1		Antenna 2		Antenna 3	
L_p	$d_{p,p+1}$	L_p	$d_{p,p+1}$	L_p	$d_{p,p+1}$
0.508	0.245	0.509	0.221	0.518	0.245
0.489	0.149	0.482	0.143	0.494	0.148
0.438	0.283	0.433	0.254	0.438	0.279
0.405	0.384	0.402	0.371	0.407	0.363
0.433	0.244	0.415	0.270	0.425	0.266
0.420	-	0.390	-	0.412	-

TABLE II

ELECTRICAL CHARACTERISTICS OF THE THREE ANTENNAS OF TABLE I

RBMGA						
Ant.	Freq. MHz	D_o (dB)	FB (dB)	E-plane HPBW	H-plane HPBW	R_{in} (Ω)
1	292.5	11.0	21.4	48.4°	55.3°	52.0
	300	11.6	17.6	44.6°	49.9°	49.6
	307.5	11.5	12.4	39.5°	43.1°	49.9
2	292.5	10.5	18.8	52.3°	61.5°	50.1
	300	11.0	18.6	49.8°	57.5°	50.1
	307.5	11.6	15.8	46.7°	52.9°	50.0
3	292.5	10.9	22.9	49.7°	57.2°	52.8
	300	11.6	17.6	46.2°	52.2°	49.0
	307.5	11.9	12.7	42.0°	46.4°	53.9

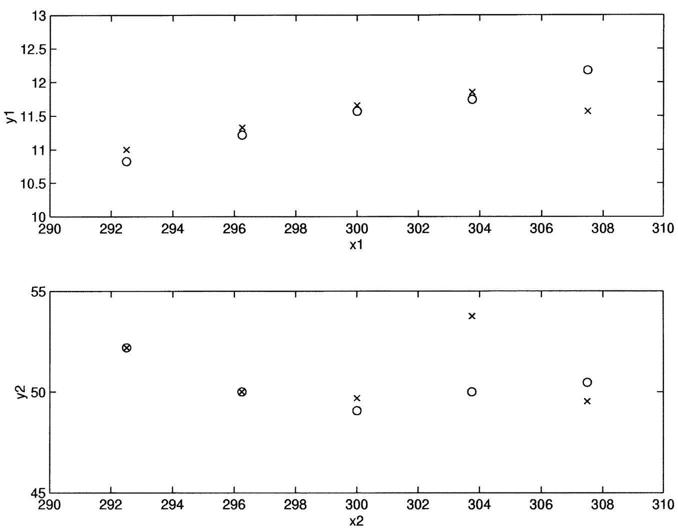

Fig. 2. Directivity and input resistance throughout the bandwidth: NEC2 results (o) and our results ($\times$).

vanishes at the dipole tips, enforcing the continuity condition [7]. Also, note that the azimuthal variation is being neglected, which is reasonable as $a \ll \lambda$. The integral equation to be evaluated is given by [7]

$$E_z^i(\vec{r}) = \frac{j\eta}{4\pi k} \sum_{p=1}^{6} \int_{-L_p/2}^{L_p/2} I_p(z') G_E(\vec{r}\,\vec{r}\,')\, dz' \qquad (8)$$

where $\vec{r}$ and $\vec{r}\,'$ locate the observation and source points, respectively, E_z^i is the z component of the incident electric field, I_p is given by (7), $\eta = \sqrt{\mu/\varepsilon}$, $k = 2\pi/\lambda$, and

$$G_E(\vec{r};\vec{r}\,') = \left[(1+jkR)(2R^2-3a^2)+(kaR)^2\right] \frac{e^{-jkR}}{R^5} \qquad (9)$$

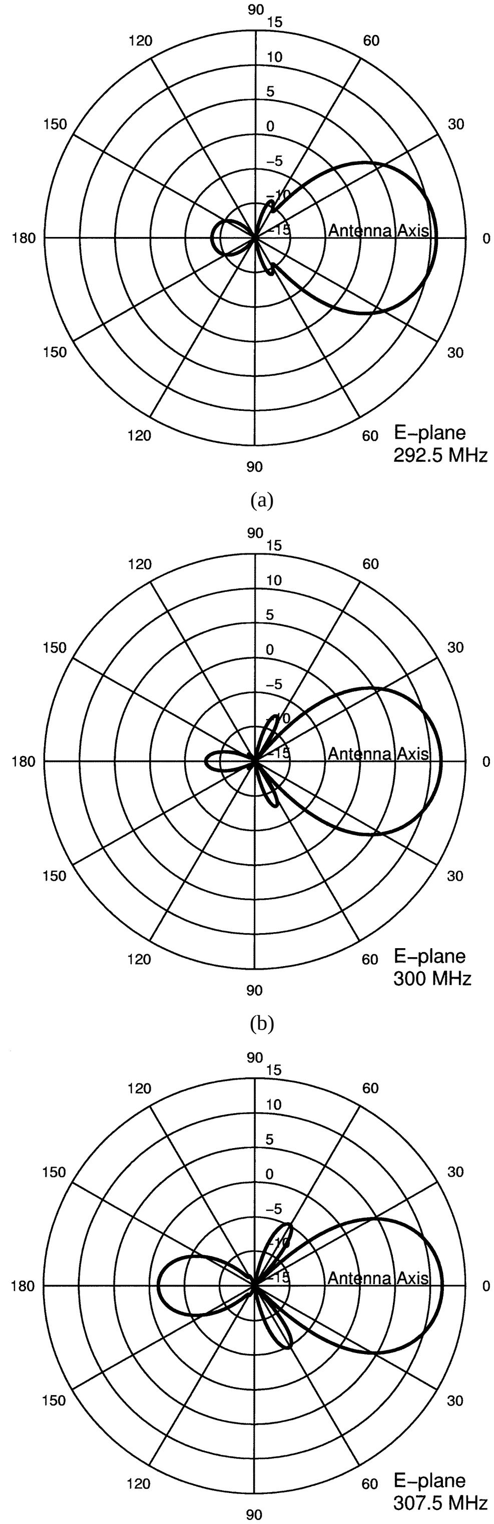

Fig. 3. E-plane radiation patterns of Antenna 1 at (a) 292.5, (b) 300, and (c) 307.5 MHz.

with

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2 + a^2}.$$

Equation (8) is numerically evaluated by the MoM technique using point matching, as discussed in [7]. After determining the

TABLE III
GEOMETRIES OF THE ANTENNAS OBTAINED BY RBMGA (ANTENNA 1) IN [3] AND [4]. DIMENSIONS ARE GIVEN IN WAVELENGTHS (λ) AT 300 MHZ

RBMGA (Ant. 1)		Ref. [3]		Ref. [4]	
L_p	$d_{p,p+1}$	L_p	$d_{p,p+1}$	L_p	$d_{p,p+1}$
0.508	0.245	0.478	0.182	0.50	0.22
0.489	0.149	0.450	0.152	0.46	0.16
0.438	0.283	0.448	0.229	0.40	0.29
0.405	0.384	0.434	0.435	0.40	0.31
0.433	0.244	0.422	0.272	0.39	0.29
0.420	-	0.440	-	0.33	-

TABLE IV
ELECTRICAL CHARACTERISTICS OF THE ANTENNAS OF TABLE III

Ant.	Freq. MHz	D_o (dB)	FB (dB)	E-plan. HPBW	H-plan. HPBW	R_{in} (Ω)
RBMGA (Ant. 1)	292.5	11.0	21.4	48.4°	55.3°	52.0
	300	11.6	17.6	44.6°	49.9°	49.6
	307.5	11.5	12.4	39.5°	43.1°	49.9
[3]	292.5	11.8	16.9	44.7°	50.2°	21.0
	300	12.7	10.5	40.5°	44.5°	49.5
	307.5	11.6	10.3	36.0°	40.1°	1.9
[4]	292.5	9.5	14.3	56.8°	69.2°	54.4
	300	9.7	15.5	55.2°	66.3°	62.1
	307.5	10.0	15.6	53.2°	62.9°	68.3

coefficients I_{np}, the electric current representation given by (7) is applied to determine the radiated field and, consequently, the antenna radiation pattern and input resistance [7].

V. NUMERICAL RESULTS

The RBMGA was applied in the optimization of a six-element Yagi–Uda antenna for the achievement of the following specifications: directivity (D_o) and front-to-back ratio (FB) higher than 10 dB, half-power beamwidths (HPBW) narrower than 60° on both E and H planes, and an input resistance (R_{in}) between 45 and 55 Ω. A Pareto-optimal set with 192 antennas satisfying (1) was obtained, where only 15 fulfilled the above-mentioned specifications. Consequently, the antenna to be implemented should be chosen from this set of 15 elements. Table I presents the geometries of three particular antennas selected from this set, where the element lengths (L_p) and distances between elements ($d_{p,\,p+1}$) are defined as in Fig. 1. The pertinent electrical characteristics of these antennas are summarized in Table II. The strong tradeoff among the front-to-back ratios in the three frequencies should be noted in the data.

Among the three antennas of Tables I and II, the first one (Antenna 1) is the best choice for having a slightly higher directivity compared with Antenna 2 and a better 50-Ω matching than Antenna 3. The electrical characteristics of Antenna 1 are depicted in Fig. 2, which shows the variation of D_o and R_{in} throughout the bandwidth. The numerical results are compared against those provided by the NEC2 code, that is a standard tool, and show a good agreement. The radiation patterns of Antenna 1 at 292.5, 300, and 307.5 MHz are illustrated by Fig. 3.

Tables III and IV compare one of the antennas obtained by the RBMGA (Antenna 1) against two different designs found in [3] and [4]. Comparing the results, it is observed that the RBMGA is capable of yielding antennas with good performance, in the sense that they practically met all the specifications over the desired bandwidth. Note, however, that the results in [3] and [4] were obtained for different design specifications, which means that any comparison should be cautious.

VI. CONCLUSION

The RBMGA has presented a good performance in the design of wire antennas. The usage of a multiobjective approach in this problem was a key issue in getting high performance antennas that feature good parameters in a broad frequency range. The design procedure of generating several Pareto-optimal solutions that are submitted to the human decisor allows the fine adjustment of the selected antenna performance to the requirements of the problem at hand.

The Pareto-optimal set was well mapped, showing that the RBMGA can be an efficient tool for reaching good results in problems with several design objectives.

ACKNOWLEDGMENT

The authors would like to thank Mr. G. J. Burke, from the Lawrence Livermore National Labs, University of California, USA, for running the NEC2 results presented in Fig. 2.

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