APPENDIX 3

PROMETHEE

The PROMETHEE methods (Preference Ranking Organisation METHod for Enrichment Evaluations) belong to the family of outranking methods. The method permits to maximise a set of objectives and minimise another set simultaneously. Figure A3.1 illustrates the preference function used to maximise an objective, while Figure A3.2 shows the preference function used in the case of *minimisation*.

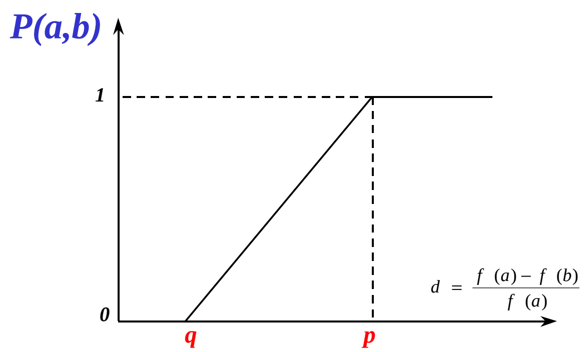

Figure A3.1. Preference function used for objectives to be maximised.

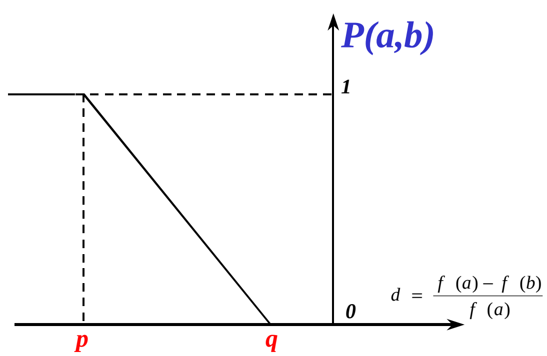

Figure A3.2. Preference function for objectives to be minimised.

For each objective a triplet (p, q, w) is introduced, where w is the weight, the values p and q are the preference and the indifference thresholds respectively. They are both positive for objectives to be maximised with the condition p>q (Figure A3.1). In the case of minimisation the thresholds p and q are negative, with p<q (Figure A3.2).

Preference threshold (p) : In the of maximisation, if the absolute value of difference between two solutions is higher than *p*, that means that this difference is significant, and the solution representing the highest performance is better (preferred) than the other. The same reasoning can be done in case of minimisation.

Indifference threshold (q) : If the absolute value of difference between two solutions is lower than *q*, that means that this difference is not significant, and the two solutions are practically equivalent.

Weight (w) : The weight *w* or in other words *coefficient of importance* means that if a criterion is attributed a weight of 3, and another a weight of 2, that means that 2 (respectively 3) points gained with the first criterion can be compensated by 4 (respectively 5.5) points gained in the second. Is supposed that the values *p* and *q* are the same for the two criterion.

In order to take into account the amplitudes of deviations between evaluations, (Brans, 1994) associated to each objective one preference function $P_j(a, b)$. This preference function gives the degree of preference of a solution $a \in S$ over another solution $b \in S$ for an objective *j*. Let *S* be the set of *N* possible solutions or alternatives which are evaluated trough the *k* objective $f_1, f_2, \ldots, f_k$. The preference functions are used to establish *in-two* comparisons between alternatives for each objective. These preference functions will be based on the relative variations between two objective evaluations. If we consider an objective *j* to be optimised, the preference of alternative *a* over alternative *b* will be computed as a function of:

$$d_j = \frac{f_j(a) - f_j(b)}{f_j(a)}$$

where $f_j(a)$ and $f_j(b)$ are the evaluations of alternative *a* and *b* respectively for objective *j*; d_j is the relative variation between *a* and *b* evaluations for objective *j*.

We consider a normalised degree, so that $0 \le P(a, b) \le 1$ and:

- $P_j(a, b) = 0$ if $d \le 0$, no preference or indifference
- $P_j(a, b) \approx 0$ if $d > 0$, weak preference
- $P_j(a, b) \approx 1$ if $d >> 0$, strong preference
- $P_j(a, b) = 1$ if $d >>> 0$, strict preference.

When the relative variation d_j (of objective *j*) between the evaluation of solution *a* and solution *b* is lower or equal to the difference threshold, it means that it is too small to be significant and the preference of *a* over *b* is null in that case. Alternatives *a* and *b* are thus indifferent in such case for objective *j*. On the contrary, when d_j is larger or equal to the preference threshold *p*, it means that d_j is large enough to attribute a maximum preference $(P_i(a, b)=1)$ of *a* over *b*. Between these two thresholds, we have a 'fuzzy 'region alongside the indifference and the strict preference thresholds.

A multiple objective preference index *p(a, b)* of *a* over *b* can then be defined taking into account all the objectives:

$$p(a,b) = \sum_{j=1}^{k} w_j P_j(a,b)$$

where $w_j>0$ $(j=1,...,k)$ are weights associated to each objective. These weights are positive real numbers and do not depend on the scales of objectives. Note, that if all weights are equal *p(a, b)* is simply the arithmetic average of all the $P_j(a, b)$ degrees $(j=1,...,k)$.

p(a, b) expresses how and with which degree *a* is preferred to *b*, and *p(b, a)* how b is preferred to a, over all the objectives. For each pair of solutions *a* and *b* the values *p(a, b)* and *p(b, a)* are computed. Thus, a complete valued outranking relation between solutions is obtained.

Let us consider how each alternative *a* is facing the *N-1* other ones. We define the two following outranking flows :

- the positive outranking flow :

$$f^{+}(a) = \frac{1}{n-1} \sum_{b \in A} p(a,b)$$

- the negative outranking flow :

$$f^{-}(a) = \frac{1}{n-1} \sum_{b \in A} p(b,a)$$

The positive outranking flow expresses how much each alternative is outranking all the others. The higher $f^+(a)$, the better the alternative. $f^+(a)$ represents the power of a, its outranking character. The negative outranking flow expresses how much each alternative is outranked by all the others. The smaller $f^-(a)$, the better the alternative. The $f^-(a)$ represents the weakness of *a*, its outranked character. Since the aim is the complete ranking of alternatives (solutions), we consider the net outranking flow which can be formulated as:

$$f(a) = f^{+}(a) - f^{-}(a)$$

This flow gives us a ranking, called the PROMETHEE II complete ranking, between the different solutions. Here are the rules defining this ranking:

$$\begin{cases} aP^{II}b & if \quad f(a) > f(b) \\ aI^{II}b & if \quad f(a) = f(b) \end{cases}$$

It means that solution *a* is preferred to solution *b* if and only if $f(a) > f(b)$, and that solution *a* and *b* are indifferent if and only if $f(a) = f(b)$.

References

(Brans, 1994) Brans J.-P. and Mareschal B., 'The PROMCALC & GAIA decision support system for multicriteria decision aid', Decision Support Systems, North-Holland, Vol. 12, pp. 297-310, 1994.