A Multi-Objective Approach to Integrated Risk Management

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Abstract. The integrated management of financial risks represents one of the main challenges in contemporary banking business. Deviating from a rather silo-based approach to risk management banks put increasing efforts into aggregating risks across different risk types and also across different business units to obtain an overall risk picture and to manage risk and return on a consolidated level. Up to now no state-of-the-art approach to fulfill this task has emerged yet. Risk managers struggle with a number of important issues including unstable and weakly founded correlation assumptions, inconsistent risk metrics and differing time horizons for the different risk types. In this contribution we present a novel approach that overcomes parts of these unresolved issues. By defining a multi-objective optimization problem we avoid the main drawback of other approaches which try to aggregate different risk metrics that do not fit together. A MOEA is a natural choice in our multi-objective context since some common real-world objective functions in risk management are non-linear and non-convex. To illustrate the use of a MOEA, we apply the NSGA-II to a sample real-world instance of our multi-objective problem. The presented approach is flexible with respect to modifications and extensions concerning real-world risk measurement methodologies, correlation assumptions, different time horizons and additional risk types.

1 Introduction

In the recent study *Trends in risk integration and aggregation* [1] that has been conducted with 31 financial institutions worldwide the Working Group on Risk Assessment and Capital of the Basel Committee on Banking Supervision reports about two major trends in financial risk management. Firstly, the study has identified a strong emphasis on the management of risk on an integrated firm-wide basis. The second emerging trend comprises rising efforts to aggregate risks through mathematical models. At the end of the day banks are highly motivated to approximate their required capital base[3] that serves as a buffer against unexpected losses even more accurate.

[3] In internal banking models this is called economic capital.

While banks undertake high endeavors to gain an integrated sight of their entire business this aim in reality usually still rather resembles a mere vision. In real-world applications different types of risk are still assessed and controlled in a more silo-based manner[4], i.e. market risk is measured separately from credit risk etc. Assuming perfect correlation the resulting risk numbers are often just added up to get an aggregate risk measure. It is clear that this simple method only means a first step to true integrated risk management.

A multi-objective approach is obviously more appropriate under these circumstances. Thus, we propose a MOEA application which supports the silo-based approach currently adopted by many banks. Moreover, our approach allows the use of the Value-at-Risk which is also a commonly used risk measure in many financial institutions (and which we will explain in more detail below).

The remainder of this contribution is organized as follows: In the next section we give a short introduction to the key concepts which constitute integrated risk management. After that, we provide an overview of recent research in the area of integrated risk management and point out important obstacles in real-world applications. In the succeeding section we present our multi-objective approach which fits into current risk management practices and avoids some of the problems mentioned before. The application of a MOEA in our setting is then illustrated for a sample bank by applying the NSGA-II to recent market data. Finally, we give a conclusion and an outlook on possible future developments.

2 Integrated Risk Management

The Basel Committee on Banking Supervision [1] proposes the following definition: "An integrated risk management system seeks to have in place management policies and procedures that are designed to help ensure an awareness of, and accountability for, the risks taken throughout the financial firm, and also develop the tools needed to address these risks."

The core of such an integrated risk management[5] system is represented by an appropriate risk aggregation methodology. In the Basel Committee report [1] this is explained as follows: "Broadly, *risk aggregation* refers to efforts by firms to develop quantitative risk measures that incorporate multiple types or sources of risk. The most common approach is to estimate the amount of *economic capital* that a firm believes is necessary to absorb potential losses associated with each of the included risks."

Risk aggregation makes sense in a variety of different aggregation levels. To obtain a total bank risk measure the risk across different business units and risk types are summarized. Further possibilities include a measure for the total risk in one risk category or an aggregate measure by product or by business unit. Such numbers facilitate internal comparisons across businesses and also between different companies and potential merging partners.

[4] Cf. Pézier [2] and Kuritzkes et al. [3].

[5] Similiar terms are *consolidated (financial) risk management* or *enterprise-wide risk management* (cf. Cumming & Hirtle [4])

Cumming & Hirtle [4] proclaim two main goals that motivate integrated risk management. On the one hand with the safety-and-soundness concern the regulatory authority intends to maintain the stability of the international financial system by avoiding single bank crashes. To ensure that financial institutions hold sufficient amounts of capital to protect their risky positions, the Basel Committee on Banking Supervision has released the first Basel Accord in 1988 (with market risk amendment in 1995) and the new Accord (Basel II) that will probably become effective in 2007. On the other hand the bank's and its shareholders' perspective rather focuses on an efficient allocation of the scarce resource capital by having a more medium-term perspective. Through the integrated view at the entire institution the profitability of certain business lines can be analysed in a better way. Hence it is possible to distribute the available capital according to economically reasonable cost-benefit considerations to the different banking units.

2.1 Current Practice in Risk Management

Financial risks are inherent in financial markets and their management represents one of the main tasks in the business of financial institutions. Firstly, in this entire process the crucial risk types a certain institution faces have to be identified and defined firm-wide. In the second step the possible extents of these risks have to be quantified. As this involves high uncertainties, Alexander & Pézier [5] use the term *risk assessment* rather than *risk measurement*. Along with monitoring and reporting of the results comes the risk controlling function through trading and management action (cf. Alexander [6]). As identified by the Basel Committee on Banking Supervision [7] the main risk sources faced by banks are market, credit and operational risk. In the following section we introduce these key risk factors that we also incorporate into our model which is described later.

In general, **market risk** arises through adverse movements in the market prices of financial instruments. There exist a number of subcategories depending on the considered market factor, for instance *interest rate, equity* or *foreign currency risk*.

A prevalent method to measure market risk is Value-at-Risk (VaR). The formal definition by Frey & McNeil [8] which is derived from Artzner et al. [9] is as follows: Given a loss L with probability distribution $\mathbf{P}$, the Value-at-Risk of a portfolio at the given confidence level $\alpha \in]0,1[$ is represented by the smallest number l such that the probability that the loss L exceeds l is no larger than $(1-\alpha)$. Formally,

$$VaR_\alpha = \inf\left\{l \in \mathbf{R}, \mathbf{P}(L > l) \leq 1-\alpha\right\}. \tag{1}$$

Given a Value-at-Risk of USD 1m for a sample bank portfolio with respect to a risk horizon of 1 day and a confidence level of 99% the following conclusion may be drawn for instance: Within the next 100 days there should occur a maximum of one day with the loss on the current portfolio positions exceeding USD 1m.

The VaR calculation approaches are divided into in parametric (variance-covariance method) and non-parametric methods (historical simulation and Monte Carlo simulation). The main difference is that in the former statistical information is extracted from historical data and then employed into parameters for analytical formulae. The latter approaches perform a full valuation of the portfolio due to a number of risk factor scenarios.[6]

For instance, the historical simulation method which we will use later in our example is conducted as follows. Based on a chosen period of time (e.g. one year with 250 trading days) daily changes in the market risk factors are calculated and then applied to revalue the portfolio in its prevailing composition. By comparing the results with the current portfolio value we get a distribution of likely portfolio value changes within 1 day. Hence we are able to observe the desired quantile (e.g. 99%) directly to obtain the Value-at-Risk figure.

Even though VaR does not represent a coherent risk measure[7] it has still become a state-of-the-art methodology in the risk management of financial institutions. Furthermore in a very recent risk aggregation study performed by Rosenberg & Schuermann [10] the authors have found that the explanatory power of alternative risk measures[8] does not deviate strongly from the conclusions that can be drawn from a Value-at-Risk-based analysis. Due to these results and as we intend to present a real-world application we have decided to build our analyses upon the widespread risk measure VaR[9].

Compared to other risk types market risk measurement and management is rather well developed as extensive research has been carried out in this area (cf. Cumming & Hirtle [4]). Also long historical data sets are available for most of the instruments that are traded on financial markets. Last but not least typical returns of market instruments exhibit the convenient characteristic to resemble the standard normal distribution[10].

In the area of **credit risk** strongly intensifying efforts have been made both in research and practice. Credit risk concerns possible losses through unfavourable changes in a counterparty's credit quality. Within this category falls of course *default risk* in case a contractual partner is not capable to repay his debt anymore. But also possible depreciations in the bond value of an obligor through changes in his individual credit spread may lead to losses stemming from *spread risk* (cf. Crouhy et al. [12]).

Not only the Basel Accords have increased banks' focus on credit risk but also competitive forces to establish adequate credit risk pricing systems. Nowadays

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[6] There are historical factor scenarios in the historical simulation method and simulated scenarios built on stochastic processes in the Monte Carlo approach.

[7] Cf. Artzner et al. [9] for details.

[8] Such as expected shortfall (also known as Tail-VaR), cf. Artzner et al. [9].

[9] For a more detailed illustration to the Value-at-Risk concept cf. [11].

[10] It has to be noted though that empirical data sets usually contain *fat tails*, i.e. the outer quantiles of the observed market distributions possess a higher probability density than assumed by the standard normal distribution. In our latter chosen approach (historical simulation method) fat tails are implicitly modelled through the empirical return distributions.

there exists a number of credit risk models that have emerged as common practices. Also data availability[11] and risk management possibilities have improved substantially in recent years[12].

A risk type that has lately come into focus is **operational risk**, particularly through the new Capital Accord. The Basel Committee defines operational risk as 'the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. This definition includes legal risk, but excludes strategic and reputational risk' [7].

In the literature, there is a number of sophisticated approaches to assess operational risk, e.g. using extreme value theory (cf. Embrechts et al. [14]). In real-world applications however, many sophisticated approaches typically suffer from the absence of sufficiently given input data. Thus, we adopt the straightforward Basel Standardised Approach in our example which is not affected by this data problem. This approach will be introduced within section 4.

Further risks such as *business, reputational* and *strategic risks* are intentionally excluded by the Basel definition from operational risks. Based on the present state-of-the-art these risks are very hard if at all quantifiable. We therefore restrict our focus to market, credit and operational risk.

2.2 Research in the Area of Integrated Risk Management

In a very illustrative way Matten [15] describes the challenges that banks face when controlling their business on an integrated basis while entrapped between supervisory authorities and owners. The author recommends an economic profit concept that subtracts capital cost from profits to efficiently allocate capital within a bank.

Alexander & Pézier [5] propose a straightforward factor model to accomplish an aggregate risk assessment methodology. After the identification of the main bank-wide risk factors[13] aggregate risk measures can be calculated by applying certain correlation assumptions across the risk types. The authors demonstrate that the optimization of risk and return may be improved through the risk integration procedure.

Dimakos & Aas [16] present an approach to model the aggregate economic capital of a financial group taking into account pairwise interrisk correlations. Using a one year time horizon and a 99.97% confidence interval they find a reduction in the overall capital demand by around 20% compared to results obtained through the perfect correlation assumption.

Kuritzkes et al. [3] take the view of the supervisory authority to determine the possible extent of diversification benefits on the minimum capital requirements within financial conglomerates. They suggest a building block approach that aggregates risk at three successive levels in the organisation and find that

[11] Though still lacking behind market risk data due to comparably rare events

[12] A detailed description of prevailing approaches is beyond the scope of this contribution (cf. e.g. Bluhm et al. [13]).

[13] These risk factors include for example interest rate, credit and equity risk

diversification effects are greatest within single risk factors while being smallest within different business lines.

Rosenberg & Schuermann [10] set forth a model that aggregates market, credit and operational risks through the use of copula functions which are a general concept for modelling dependencies between random variables. In their empirical analysis which has been performed with a wide number of publicly available data on financial institutions they find that simply adding up different risks overestimates total risk by more than 40%. The also popular assumption of a joint normality between risk factors underestimates risk by a similar amount. Besides their copula based method they also test a hybrid approximation that surprisingly achieves good results while still being easy to implement. The authors note that operational risk is not only relatively difficult to measure but it also deserves care when being aggregated.

Saita [17] takes a different perspective as he warns of overconfidence in the resulting risk aggregation numbers. Severe consequences may occur when wrong numbers serve as a basis for bonus payments for example. Apart from such model risk issues he addresses business risk and the varying definitions of capital as reference magnitude. Another contribution is a critical comparison of different aggregation approaches that have been proposed recently.

2.3 Obstacles for Integrated Risk Management

A main obstacle to integrated risk management are data problems. There may just be a lack of data as in the area of operational risk management for example. Due to insufficient data statistical methods fail to deliver clear statements in such cases. Sometimes there is enough data but in bad quality, i.e. with missing values or even wrong numbers.

Correlations also emerge in the context of inadequate data supplies. To take into account diversification effects in a reliable way stable correlations are required. If there is only short historical data available or if the parameters prove to be highly volatile empirically found correlations often fail. Additionally in crash situations correlations accross a variety of markets tend to move to one. These extreme cases also have to be taken into consideration when performing a sensible risk assessment. Inadequate correlation assumptions will very likely also lead to wrong incentives within the bank.

It has to be noted in this context that the supervisory authorities are still reluctant to accept internal assessments of correlation and thus diversification benefits. They rather make use of the supposedly conservative assumption of perfect correlation[14]. However the Basel Committee also tries to motivate banks to improve the reliability of their internal correlation estimates [1]. In the future it may well be expected that the use of these proprietary estimates becomes

[14] Alexander & Pézier [5] show that risks can become worse than perfectly correlated if cascading effects are triggered. For example liquidity shortages across markets could worsen a market situation, leveraging correlation effects between two risk categories.

more flexible when computing regulatory capital. Much more experience with correlations is still required though.

A further big challenge in respect of risk interdependencies lies in the correct distribution of diversification benefits between different entitites (e.g. business units or products). To motivate economically reasonable decisions the distribution should be deliberate.

It has already been mentioned above that risk measurement methodologies widely differ across different risk types. To account for the specific risk properties risk horizons usually range from 1 day (market risk) to 1 year (credit and operational risk). Also heterogeneous distributional assumptions hold, e.g. normal distributions for market and skewed distributions for credit risk. This prevents simply summing up the different obtained measures to get an overall risk figure.

Another problem are conceptional requirements. The best approach for integrated risk management seems to be a top-down process. In practical applications mostly bottom-up approaches are used, however. The reasons for this lie within the organisational structure of the institutions and these are hardly changeable.

It has become obvious that the aggregation of risks across these dimensions still turns out to be highly complicated. Unknown correlations between risks and business units, differing risk metrics and time horizons and data problems worsened by heterogenuous IT systems represent the main factors that make an implementation almost impossible. However, the following approach builds right upon these weaknesses of today's widespread structures of financial institutions and allows a bottom-up risk management as it is commonly performed.

3 A Multi-Objective Approach to Integrated Risk Management

In the remainder, we consider a universe of $n \in \mathbf{N}$ investment opportunities (assets or asset classes). Any portfolio consisting of a subset of these assets is specified by an n-dimensional vector

$$X = (x_1, x_2, ..., x_n) \tag{2}$$

which satisfies the conditions

$$\sum_{i=1}^{n} x_i = 1 \wedge \forall i \in \{1, \ldots, n\} : x_i \in [0, 1]. \tag{3}$$

Each decision variable x_i represents the percentage of the bank's current wealth which is to be invested into investment opportunity $i \in \{1, \ldots, n\}$.

The following target functions reflect the usual objectives in a bank's silo-based approach of integrated risk (and return) management. Our first objective is the expected rate of return from a portfolio, given by

$$ret(X) := \sum_{i=1}^{n} x_i r_i \tag{4}$$

where r_i is the expected rate of return of investment opportunity i. This objective is to be maximized.

The second objective function is the Value-at-Risk of the portfolio due to changes of market prices (market risk), denoted by

$$mr(X) := VaR_{marketrisk}(X) \tag{5}$$

where the $VaR_{marketrisk}(X)$ is determined by one of the common calculation methods historical simulation, variance-covariance approach or Monte Carlo simulation. Usually, this objective is short-term oriented, e.g. measured on a time horizon of one or ten trading days, and to be minimized.

Our third objective function is the Value-at-Risk of the portfolio due to credit risk, i.e. defaults of obligors or other losses resulting from changing credit qualities of obligors. It is denoted by

$$cr(X) := VaR_{creditrisk}(X). \tag{6}$$

As mentioned in the first section, the $VaR_{creditrisk}$ is commonly calculated using one of the models CreditMetrics, CreditRisk+, CreditPortfolioView or similar approaches, cf. e.g. Bluhm et al. [13] for an overview of these models. A common time horizon for the calculations is one year, and this risk measure should be minimized.

The fourth objective which is relevant to our context is the required capital for operational risk compensation which we assume to be calculated according to the Basel Committee on Banking Supervision's Standardised Approach (cf. [7], p. 137ff). This yields a target function

$$or(X) := \sum_{i=1}^{n} x_i \beta_i \tag{7}$$

where β_i is specific for the business line in the bank which is affected by the investment $x_i > 0$ into opportunity i.

Summarizing the above definitions and restrictions as well as converting maximization of the ret function into minimization of $-ret$, we obtain the following problem setting:

$$f_1(X) := -ret(X) \tag{8}$$

$$f_2(X) := mr(X) \tag{9}$$

$$f_3(X) := cr(X) \tag{10}$$

$$f_4(X) := or(X) \tag{11}$$

$$X := (x_1, \ldots, x_n) \tag{12}$$

$$\forall i \in \{1, \ldots, n\} : x_i \in [0, 1] \tag{13}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{14}$$

A portfolio X_2 is (weakly) dominated by a portfolio X_1 if the following condition is met:

$$\forall j \in \{1, \dots, 4\} : f_j(X_1) \leq f_j(X_2) \wedge \exists k \in \{1, \dots, 4\} : f_k(X_1) < f_k(X_2) \quad (15)$$

This is compatible to both the usual definition of dominated portfolios in the finance context and the common definition of dominated points in multi-objective optimization.

We assume that the bank is a rational investor, i.e. the bank is not going to invest in a dominated portfolio (cf. e.g. Markowitz [18]). Moreover, we assume that the bank's management prefers to choose from a whole set of individually optimal solutions, particularly by evaluating the trade-off between the desired expected rate of return and the different risks which have to be taken for the respective portfolio. Hence, we search for a set of non-dominated portfolios well-distributed in the four-dimensional objective function space $f_1(X)$ to $f_4(X)$ over the feasible search space which is specified by conditions (12) to (14).

The justification for the use of a heuristic algorithm builds upon the mathematical properties of the objective functions: According to Artzner et al. [9] and Gaivoronski & Pflug [20] the Value-at-Risk risk measure is a nonlinear and nonconvex function and has usually many local optima, hence f_2 and f_3 share this property which is problematic for conventional optimization approaches.[15] From the view of computational complexity, the problem of finding even a single feasible non-dominated point is **NP**-*hard* if the decision variables are restricted to integer values.[16]

Thus, we opt for a heuristic approach to compute approximation solutions. A MOEA is appropriate here since we search for a well-distributed approximation set in a restricted four-dimensional objective function space. In the literature, several different algorithms which actually implement a specific MOEA scheme are discussed, see e.g. Deb [21], Coello et al. [22] and many theoretical and empirical comparisons between the alternative approaches to evolutionary multi-objective optimization. In general, most of these MOEAs should be useful in our problem setting. It has to be pointed out here that it is not the goal of our work to propose a specific MOEA in our context as the best of all these algorithms.

However, for an illustrative example underlining the successful application of a MOEA to our real-world problem of integrated risk management, we have to choose an algorithm. Since the NSGA-II by Deb et al. [23] is an algorithm which has been successfully applied to many problem contexts in general, and more specifically, to other constrained portfolio optimization problems using less than four objective functions (cf. Schlottmann & Seese [24] for a general and [25] for a more specific overview of such studies), we have chosen this algorithm for our illustrative example in the following section.

[15] If we assumed a Value-at-Risk measure for operational risk then this would also apply to f_4.

[16] This can be proven by reducing the standard *KNAPSACK* setting to a discrete version of our problem (cf. e.g. Seese & Schlottmann [19] for a formal analysis in the two-objective function case which can be generalised to more than two objectives). Since we assume real-valued decision variables, this does not apply directly here.

4 An Illustrative Example

We consider $n = 20$ investment opportunities for our sample bank with the characteristics shown in table 1. The historical market data range covers closing prices for the ten traded instruments from 15-MAY-2003 to 30-SEP-2004. The stocks are all traded on the Frankfurt stock exchange. For the 10 loans we assume the bank is hedged against market risk changes, i.e. interest rate risk is not relevant to these instruments. The loans are paying annual net interests. All calculations are based on a decision to be made by the bank's integrated risk manager on 30-SEP-2004.

Table 1. Investment opportunities for sample bank

Quantity	Category	Issuer/Obligor	Coupon	Maturity	Rating
1	Bond	German government (BUND)	6.250%	26-APR-2006	AAA
1	Bond	German government (BUND)	4.500%	04-JUL-2009	AAA
1	Bond	German government (BUND)	5.625%	20-SEP-2016	AAA
1	Bond	Deutsche Telekom (corporate)	8.125%	29-MAY-2012	BBB+
1	Bond	Volkswagen (corporate)	4.125%	22-MAY-2009	A-
1	Equity	BASF AG	-	-	-
1	Equity	Deutsche Bank AG	-	-	-
1	Equity	DaimlerChrysler AG	-	-	-
1	Equity	SAP AG	-	-	-
1	Equity	Siemens AG	-	-	-
2	Loan	Private Obligor	8.000%	30-SEP-2005	BB
2	Loan	Private Obligor	8.000%	30-SEP-2005	BB-
2	Loan	Private Obligor	8.000%	30-SEP-2005	B+
2	Loan	Private Obligor	8.000%	30-SEP-2005	B
2	Loan	Private Obligor	8.000%	30-SEP-2005	B-

For any given portfolio X, the expected rate of return $ret(X)$ is estimated from the historical time series for the ten traded instruments and from the expected annual net interest to be paid by the respective loan obligor.

Moreover, we assume the bank uses historical simulation for the calculation of the function value $mr(X)$ using a confidence level of 99% and a time horizon of 1 trading day. Furthermore, we assume the bank applies the CreditMetrics model by Gupton et al. [26] in the two-state variant described by Gordy [27] to determine $cr(X)$ for a confidence level of 99.9% and a one-year time horizon. At this point it has to be emphasized again that it is an important advantage of our multi-objective approach concerning real-world applications that different risk measures, distinct confidence levels and varying time horizons can be used within the search for non-dominated portfolios without adversely affecting the results.

The function value $or(X)$ is calculated according to the Basel Standardised Approach as specified within the previous section.

We apply the standard NSGA-II implementation provided by Kalyanmoy Deb to this problem instance. Using the genetic variation operators provided in this implementation (simulated binary crossover and a corresponding mutation operator for real-coded genes), we set the crossover probability to 0.8 and the mutation rate to $\frac{1}{n}$.

For the restriction of the decision variables specified in formula (3), we set the bounds for each real-coded gene in the NSGA-II to $[0, 1]$, respectively. In addition, we have to ensure that each portfolio X satisfies $\sum_{i=1}^{n} x_i = 1$. Since we have observed a worse empirical convergence of the algorithm when using an objective function penalty for infeasible individuals (in accordance to other studies in different application contexts), we opt for a simple repair algorithm: Immediately after performing crossover and mutation, every allele value x_j of an offspring individual is re-normed according to

$$\widetilde{x_j} = \frac{x_j}{\sum_{i=1}^{n} x_i} \tag{16}$$

and only the re-normed individuals $\widetilde{X} = (\widetilde{x_1}, \ldots, \widetilde{x_n})$ are considered in the succeeding steps of the NSGA-II.

The following figures 1 and 2 display the objective function values of the final individuals after 50000 population steps in the NSGA-II (100 individuals per population).

In figure 1, the three components concerning the market risk, the credit risk and the expected return of the respective approximated portfolio X are shown. For instance, the bank's risk manager can use this information straightforward to verify the current position of the bank against the drawn portfolios: Assume the bank has a current portfolio status Y. If the risk manager computes $f_i(Y)$ for $i = 1, .., 4$ he can immediately check the bank's current position in the three-dimensional plot. Of course, he also has to check the value $f_4(Y)$ against the objective function values in an additional figure (which we omit here) plotting operational risk and e.g. market risk against the expected return. If the bank's current position Y is dominated by a portfolio X he can directly observe the outcome of possible improvements and derive corresponding managing decisions to move the bank's risk-return profile into an improved position.

It is a striking advantage of the multi-objective view that the risk manager can see the consequences of different decision alternatives concerning different risk sources and the corresponding expected rate of return simultaneously. For instance, in figure 1 he can choose a portfolio that has e.g. high short-term risk (due to the risk horizon of 1 trading day in the market risk calculation) while having low medium-term risk reflected by the 1-year credit risk objective function value and yielding a high expected rate of return. Such portfolios are located in the upper left corner of the figure. If he does not desire a high short-term risk (e.g. if the bank's short term risk limits are low), he can choose a portfolio in the lower right area having higher credit risk and so on.

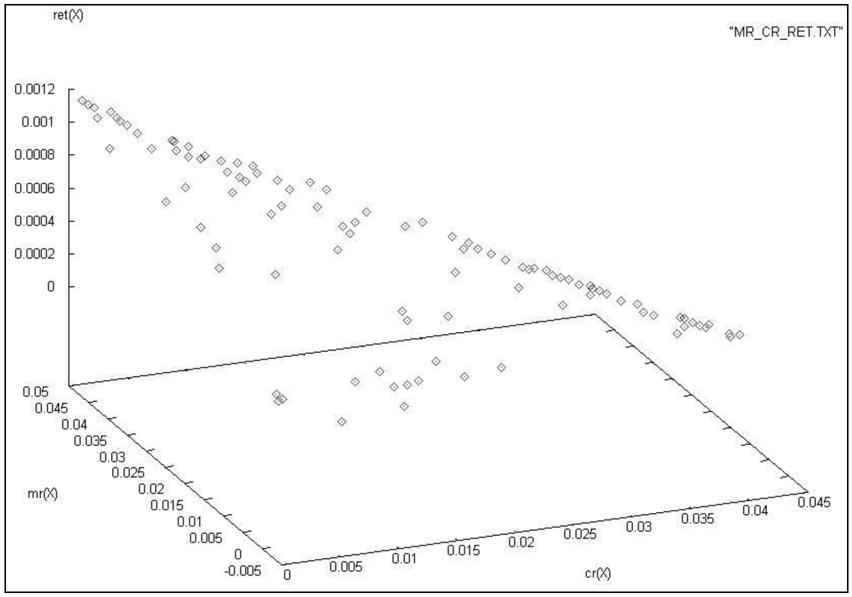

Fig. 1. Projection of mr(X), cr(X) and ret(X)

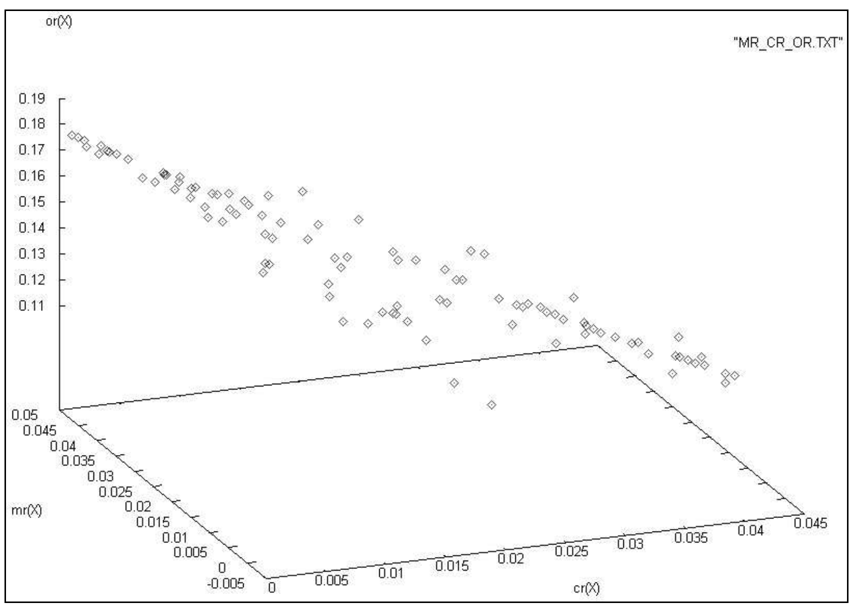

Fig. 2. Projection of mr(X), cr(X) and or(X)

Moreover, figure 2 gives an interesting sample insight into the trade-off between different sources of risk for the given investment opportunities. The mainly negative dependency between the market risk and the credit risk numbers is due to the immunization of the loans against market risk. The degree of operational risk to be taken by the bank is lower for the portfolios having relatively high credit risk and relatively low market risk. Note that in current real-world applications, this trade-off is usually not analyzed in such detail.

The preceding considerations represent a novel approach compared to the current state-of-the-art within the financial industry and the integrated risk management literature. Moreover, it has to be pointed out that the 20 asset example might seem small at first glance, however, we already mentioned that the risk manager can use more global risk factors representing whole asset classes instead of using single assets and the MOEA can of course process larger problems. Thus, the presented approach can be applied even to large portfolios in a top-down approach over different asset classes.

5 Conclusion and Outlook

The integrated management of different sources of risk is one of the largest challenges to the financial industry. Since each risk category has its own specific properties and is particularly measured on a distinct time horizon and an individual confidence level in real-world applications, we have proposed a multi-objective approach to integrated risk management in the previous sections. This approach does not require the aggregation of incompatible risk figures into a single number. Moreover, it does not necessarily require correlations between different risk types which are difficult to estimate in real-world applications due to the lack of data. Instead, the risk manager is provided with a number of solutions which he can use for an analysis of the trade-off between the different risk types and the expected rate of return. The manager can use a silo-based approach for the integrated risk-return management, which is currently standard in many financial institutions. Moreover, due to the use of a MOEA in the search of non-dominated portfolios, the Value-at-Risk which is commonly used in real-world applications, can be kept as a risk measure in the respective category. To illustrate a real-world application of our approach, we have provided an empirical example using the NSGA-II to find approximations of non-dominated portfolios.

For a thorough analysis of the MOEA performance in this area of application, a more detailed empirical study is necessary. Furthermore, our multi-objective approach to integrated risk management might be an adequate real-world application for an empirical comparison between different alternative MOEA schemes. A potential improvement of the approximation algorithm in terms of convergence speed could hybridize MOEAs and problem-specific knowledge, cf. the ideas used within the two-objective function approach presented recently in Schlottmann & Seese [28].

The problem of aggregating the multi-dimensional output to less dimensions still remains if the risk manager desires a single risk figure (although it seems not recommendable, cf. also Cumming & Hirtle [4]). However, a progress in the aggregation of different risk categories from the finance point of view can probably also be integrated into a refined MOEA approach due to its flexibility. In this case, the multi-objective approach would benefit from the development of new financial tools while still being attractive for analyzing the trade-off between different sources of risk and the expected rate of return as pointed out above. In addition, more objective functions of the bank which do not necessarily need to possess convenient mathematical properties might be incorporated quite easily into our MOEA-based approach in the future.

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