

# A Hybrid Multiobjective Evolutionary Algorithm For Solving Truck And Trailer Vehicle Routing Problems

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**Abstract-** This paper considers a transportation problem for moving empty or laden containers for a logistic company. A model for this truck and trailer vehicle routing problem (TTVRP) is first constructed in the paper. The solution to the TTVRP consists of finding a complete routing schedule for serving the jobs with minimum routing distance and number of trucks, subject to a number of constraints such as time windows and availability of trailers. To solve such a multiobjective and multi-modal combinatorial optimization problem, a hybrid multiobjective evolutionary algorithm (HMOEA) is applied to find the Pareto optimal routing solutions for the TTVRP. Detailed analysis is performed to extract useful decision-making information from the multiobjective optimization results. The computational results have shown that the HMOEA is effective for solving multiobjective combinatorial problems, such as finding useful trade-off solutions for the TTVRP.

## 1 Introduction

Singapore ranks among the top international maritime centers of the world. It is the focal point for some 400 shipping lines with links to more than 740 ports worldwide, (Maritime, 2002). A general model for vehicle capacity planning system (VCPS) consisting of a number of job orders to be served by trucks and trailers daily was constructed for a logistic company that provides transportation services for container movements within the country (Lee et al., 2003). Due to the limited capacity of vehicles owned by the company, engineers in the company have to decide whether to assign the job orders of container movements to its internal fleet of vehicles or to outsource the jobs to other companies daily.

### 1.1 The Trucks and Trailers Vehicle Routing Problem

By analyzing different kinds of job orders received from the company, this paper presents a transportation solution for trucks and trailers vehicle routing problem (TTVRP) containing multiple objectives and constraints, which is extended from the VCPS model with detail maneuver of trailers in a routing plan. In TTVRP, the trailers are resources with certain limitations similar to real world scenarios and the allocation of trailers in different locations could affect the routing plans. The TTVRP is a difficult problem which involves many intricate factors

such as time window constraints and availability of trailers. Instead of handling jobs by the internal fleet of trucks, the jobs can also be considered for outsourcing, if necessary. The routing plans in TTVRP also need to determine the number of trailer exchange points (TEPs) distributed in the region and to cater different types of trailers that are available at the trailer exchange points. In this paper, various test cases for the TTVRP model are generated with random variables simulating the long-term operation of business activities. The management can thus formulate the planning for certain variables, such as the number of trucks (long term capital cost) so that the day-to-day operational cost could be kept at the minimum.

### 1.2 Background on Vehicle Routing Problems

The vehicle routing problem with time windows (VRPTW) diverts from the famous vehicle routing problem (VRP). In this problem, a set of vehicles with limited capacity is to be routed from a central depot to a set of geographically dispersed customers with known demands and predefined time window. Surveys about VRPTW can be found in Solomon (1987), Kilby et al., (2000), Toth and Vigo (2002), Bräysy and Gendreau (2001) etc. In contrast to the TTVRP, the VRPTW neither have any limitation on resources of trailers nor the outsourcing of jobs to external companies.

The vehicle scheduling problem (VSP) (Baita et al., 2000; Dror, 2000) assumed that the routing to different sites can be completed with multiple trips. The objective is to minimize the number of vehicles and the cost function based upon deadheading trips (gas, driver etc) and idling time for the vehicle. Its constraints include the traveling distance and time for normal service and refueling. In contrast to VRP, one customer may be visited more than once or not at all, which is solely depending on the trips data. Although trips in VSP may be analogous to the concept of a job in TTVRP, the VSP does not include the complexity of trailer type constraints.

Chao (2002) presented the problem of TTRP (truck and trailer routing problem), which considers the fleet size of trucks and trailers in the model. In order to provide service to different categories of customers, there are three types of routes in a solution: (1) route that a truck travels alone (2) route that a truck and trailer are required (3) route that trailer is only required at certain sub-tour. Unlike TTRP, the TTVRP requires the trucks to visit trailer exchange points for picking up the correct trailer types depending on the jobs to be serviced. Besides, jobs

that are not routed by self-fleets in TTVRP can be outsourced to external companies.

Generally, vehicle routing problems have been attempted by different approaches ranging from exact algorithms (Applegate et al., 2002; Bard et al., 2002) to heuristics (Breedam, 2002). A number of meta-heuristics such as Tabu search (Cordeau et al., 2001; Lee et al., 2003), simulated annealing (Chiang and Russel, 1996) and genetic algorithms (Gehring and Homberger, 2001) have been applied in large-scale vehicle routing problems. The TTVRP addressed in this paper is NP-hard, which involves the optimization of routes for trucks to minimize routing distance and number of trucks concurrently. Existing routing approaches that strive to minimize a single criterion of routing cost or number of trucks are not suitable for solving such a multi-modal and multiobjective combinatorial problem. The TTVRP should be best tackled by multiobjective optimization methods, which offer a family of Pareto-optimal routing solutions containing both the minimum routing cost and number of trucks.

In this paper, a hybrid multiobjective evolutionary algorithm (HMOEA) that incorporates the local heuristic search and the concept of Pareto's optimality for finding the trade-off is applied to solve the TTVRP. The HMOEA optimizes the objectives concurrently, without the need of aggregating multiple criteria into a compromise function. Unlike conventional multiobjective evolutionary algorithms (MOEAs) that are designed with simple coding or genetic operators for parameterized optimization problems (Knowles and Corne, 2000), the HMOEA is featured with specialized genetic operators and variable-length chromosome representation to accommodate the sequence-oriented optimization problem in TTVRP.

The paper is organized as follows: Section 2 describes the scenario and modeling of the TTVRP. Section 3 gives description to the HMOEA and its various features including variable-length chromosome representation and specialized genetic operators. Pareto fitness ranking and sharing, and local search heuristics are also described in Section 3. Section 4 presents the extensive simulation results and discussions for the TTVRP. Conclusions are drawn in Section 5.

## 2 The Problem Scenario and Modeling

The TTVRP model with detail maneuver of the trailers in a routing plan is extended from a real world VCPS system proposed by Lee et al., (2003). The movement of containers among customers, depots and the port are major transportation job orders considered. A container load is handled like a normal truckload but these loads use containers with a chassis instead of trailers only. From the equipment assignment point of view, a correct trailer type is essential for the routing. For an import job, a loaded container is taken from a port to a customer warehouse and returned empty to the depot. For an export job, however, an empty container is picked up from the depot

and taken to the warehouse before returning loaded to the port. Every job order contains the location of source and destination as well as the customers' information. Load requirement and time windows are specified as hard constraints in the model.

The routing needs to consider both the locations of truck and trailer. Intuitively, there are times when a truck has a correct trailer type and thus can serve a job without going to a trailer exchange point. Otherwise, a truck is required to pick up a trailer (from the nearest TEP where the trailer is available) when it has mismatch trailer type or does not carry a trailer. The number of trailers available at an exchange point depends on how many trailers were picked up and returned to the TEP.

### 2.1 Modeling the Problem Scenarios

Based on the scenarios described, some refinements have been made to the model proposed by Lee et al., (2003). The problem is modeled here on a daily basis where the planning horizon spans only one day. All import and export jobs consist of two sub-trips and a two-day interval at the customer warehouses. The import and export jobs can be broken into two independent tasks, where each of them falls into a different planning horizon. In this way, job orders are broken into sub-job type precisely (referred as a task). Generally a task involves traveling from a source to destination as listed in Table 1.

Table 1 The task type and its description

Task type	Task	Source	Dest.	Trailer type
1	Import job	Port	WH	20
2		Port	WH	40
3		WH	Depot	20
4		WH	Depot	40
5	Export job	Depot	WH	20
6		Depot	WH	40
7		WH	Port	20
8		WH	Port	40
9	Empty container movement	Port	Depot	20
10		Depot	Port /Depot	20
11		Port	Depot	40
12		Depot	Port /Depot	40

\*WH – Warehouse

The number of trailers at TEPs depends on the trailers that are left over from the previous planning horizon. All the pickup, return and exchange activities can also change the number of trailers available. Besides, a number of trailers could also be parked at the customer warehouses instead of the TEPs. All these undetermined factors suggest that the resource of trailers available at each TEP at the initial of planning horizon is random. A truck has to pick up a correct trailer from the nearest TEP if it serves task type 1, 2, 5, 6, 9, 10, 11 or 12 and does not have a trailer or has an incorrect trailer type. For task type 3, 4, 7

or 8, the truck does not need to visit a TEP before servicing the task since the correct trailer has been brought to the place in advanced. In contrast, trucks that serve sub-job type 3, 4, 7 or 8 must not have any trailers. In this case, if a trailer is attached to the truck, it must be returned to a trailer exchange point before servicing the task. For example, a truck that serves sub-job type 7 leaves the destination (port) of a previous task with a trailer. If the same truck is to serve another task type 3, 4, 7 or 8, it must travel to a TEP to drop the trailer obtained previously. Obviously the availability of trailers at TEPs should be updated frequently since the number of trailers changes with the pick-up and return activities.

## 2.2 Test Cases Generation

The TTVRP models various factors affecting the routing performance, particularly on the importance of trailer resources such as the trailers allocation in multiple trailer exchange sites and the location of trailer exchange points. The test cases are generated based on the scenario of one-day activity for a logistic company. The time windows for the source and destination of each job are generated according to the type of jobs. The cost for each task type is based on the way tasks are accomplished, i.e., by self-fleet service or outsourced to external companies. As shown in Table 2, the test cases in this category are divided into 4 groups with different number of tasks in the range of 100 to 132, and all TEPs can contribute to the supply of any demands for trailers.

Table 2 Test cases and the properties

Group	Test case*	Job number	Trailers at each TEP
100	test_100_1_2	100	1 or 2
	test_100_3_4	100	2 or 3
	test_100_2_3	100	3 or 4
112	test_112_1_2	112	1 or 2
	test_112_2_3	112	2 or 3
	test_112_3_4	112	3 or 4
120	test_120_1_2	120	1 or 2
	test_120_2_3	120	2 or 3
	test_120_3_4	120	3 or 4
132	test_132_1_2	132	1 or 2
	test_132_2_3	132	2 or 3
	test_132_3_4	132	3 or 4

\*The last digit denotes the number of trailers allocated for each TEP

## 3 A Hybrid Multiobjective Evolutionary Algorithm

Evolutionary algorithms are global search optimization techniques based upon the mechanics of natural selection, which have been found to be very effective in solving complex multiobjective optimization problems (Burke and Newall, 1999; Jaskiewicz, 2003; Deb, 2001; Knowles

and Corne, 2000). Without the need of linearly combining multiple attributes into a composite scalar objective function, evolutionary algorithms incorporate the concept of Pareto's optimality to evolve a family of solutions at multiple points along the trade-off surface. Several surveys are available for more information of multiobjective evolutionary algorithms, e.g., Coello Coello et al., (2002), Van Veldhuizen and Lamont (2000), and Zitzler and Thiele (1999). Although multiobjective evolutionary algorithms have been applied to solve a number of domain-specific combinatorial problems, such as flowshop scheduling, and timetabling, these algorithms are often designed with specialized genetic representation or operators for specific applications, which are hard to be used directly for solving the TTVRP.

This section presents a hybrid multiobjective evolutionary algorithm designed for solving the TTVRP problem. The program flowchart of the HMOEA is illustrated in Section 3.1. The remaining sections present various features of HMOEA, including the variable-length chromosome representation in Section 3.2, specialized genetic operators in Section 3.3, Pareto fitness ranking in Section 3.4, and fitness sharing in Section 3.5. Following the concept of hybridizing local optimizers with multiobjective evolutionary algorithms for better local exploitations, Section 3.6 describes the local heuristic that is incorporated in HMOEA.

### 3.1 Program Flowchart of HMOEA

The program flowchart of HMOEA is shown in Fig. 1. The simulation begins by reading the information of all tasks. An initial population is then built such that each individual must at least be a feasible candidate solution. The initialization process is started by inserting tasks into an empty route one-by-one in a random order, where any task violating the constraints is deleted from the current route. The route is then accepted as part of the solutions and a new empty route is added to serve the deleted and remaining tasks. This process continues until all tasks are routed and a feasible initial population is built as depicted in Fig. 2.

Once an initial population is formed, all individuals in the population will be evaluated and ranked according to the Pareto ranking scheme. A simple fitness sharing approach (Fonseca and Fleming, 1998) is applied to distribute the population along the Pareto front uniformly. The tournament selection scheme (Tan et al., 2001) with a tournament size of 2 is then performed, where individuals in the population are randomly grouped into pairs and those individuals with a lower rank in partial order will be selected for reproduction. A simple elitism mechanism (Tan et al., 2001) is employed to achieve a faster convergence and better routing solutions. The specialized genetic operators in HMOEA consist of route-exchange crossover and multimode mutation. To improve the local exploitation and internal routing of individuals, simple heuristic is performed at each generation of the HMOEA. It should be noted that the feasibility of all new individuals reproduced after the process of specialized

genetic operations and local heuristic are retained without the need of any repairing mechanism. The evolution process repeats until a predefined number of generations are reached or no significant performance improvement is observed over the last 5 generations.

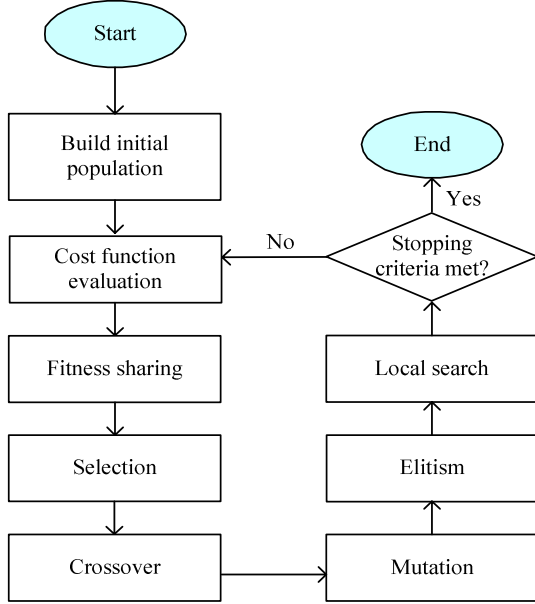


Fig 1 The program flowchart of HMOEA

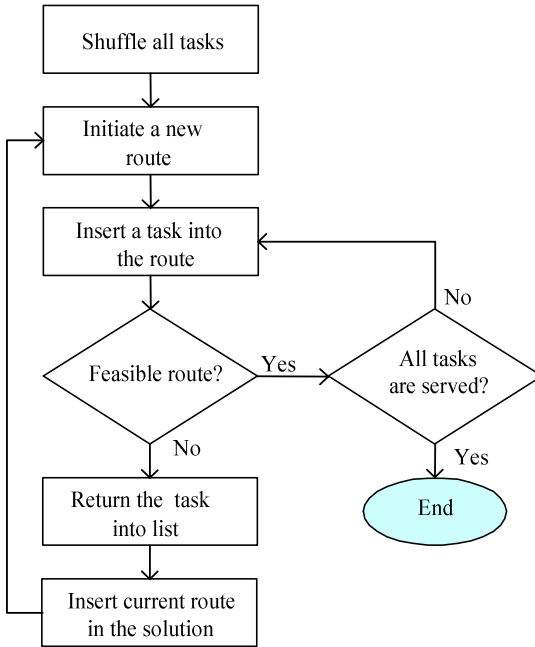


Fig 2. The procedure of building an initial population

### 3.2 Variable-Length Chromosome Representation

The chromosome in an evolutionary algorithm is often represented as a fixed-structure bit string and the bits position in a chromosome are usually assumed to be independent and context insensitive. However, such a representation is not suitable for the order-oriented

combinatorial TTVRP problem, for which the sequence among customers is essential. In HMOEA, a variable-length chromosome representation is adopted, where each chromosome encodes a complete routing plan including the number of routes and tasks served by the trucks, e.g., a route is a sequence of tasks to be served by a truck. A chromosome may consist of several routes and each route is a sequence of tasks to be served. Such a variable-length representation is efficient and allows the number of trucks to be manipulated and minimized directly for the multiobjective optimization in TTVRP. Any task that is not assigned to a route is considered for outsourcing.

### 3.3 Specialized Genetic Operators

Specialized genetic operators of route-exchange crossover and multimode mutation are incorporated in HMOEA as described in the following sub-sections:

#### 3.3.1 Route-exchange Crossover

Classical one-point crossover may produce infeasible routing sequence for combinatorial problems because of the duplication and omission of vertices after reproduction. A simple route-exchange crossover is adopted in HMOEA, which allows good sequence of routes or genes in a chromosome to be shared with other chromosomes in the population. The operation starts by grouping chromosomes into pairs randomly and the crossover is performed according to a predefined crossover rate ( $PC$ ). The operation consists of two independent steps: (1) Two random routes (one from each chromosome) are selected and swapped between the two chromosomes; (2) The route with the highest number of tasks from each chromosome is swapped. To ensure the feasibility of chromosomes after crossover, each task can only appear once in a chromosome. Deleting a task from a route will only incur certain waiting time before the next task is served, and thus will not result in any conflicts for the time windows. Besides, any task that violates the trailer resources constraint will be assigned for outsourcing and hence all the reproduced chromosomes are feasible.

#### 3.3.2 Multimode Mutation

During the crossover by HMOEA, routes' sequence is exchanged in a whole chunk and no direct manipulation is made to the internal ordering of the nodes for the TTVRP. A multimode mutation is adopted in HMOEA to optimize the local route information of a chromosome. A random number is generated to choose between two possible operations. The first operation picks two routes in a chromosome randomly and concatenates the first route to the second route before deleting the first route from the chromosome. In the second operation, the sequence that contains all the outsourced tasks is evaluated as a new route. The approach also checks feasibility on the route in order to delete any task that causes violation, and those deleted tasks will be considered as outsourced tasks.

### 3.4 Pareto Fitness Ranking

The role of HMOEA for multiobjective optimization in TTVRP is to discover such a set of Pareto-optimal

solutions concurrently. The Pareto fitness ranking scheme (Tan et al., 2001) for multiobjective optimization is adopted here to assign the relative strength of individuals in a population. The ranking approach assigns the same smallest rank for all non-dominated individuals, while the dominated individuals are inversely ranked according to how many individuals in the population dominating them based on the following criteria: (1) A smaller number of trucks but an equal cost of routing (2) A smaller routing cost but an equal number of trucks and (3) A smaller routing cost and a smaller number of trucks. Therefore the rank of an individual  $p$  in a population is given by  $(1+q)$ , where  $q$  is the number of individuals that dominating the individual  $p$  based on the above criteria.

### 3.5 Fitness Sharing

A simple fitness sharing (Fonseca and Fleming, 1998) is incorporated in HMOEA to prevent genetic drift, which is a phenomenon where a finite population tends to settle on a single optimum even if many other local optima exist. The sharing approach measures the niching distance in the objective domain to achieve diversity of solutions on the tradeoff curve. The niche radius,  $\sigma$  is a parameter that defines the size of neighborhood. The distance between individuals is normalized to the maximum range of objective space. Let  $dist(x,y)$  be the normalized distance between individual  $x$  and individual  $y$ , the sharing function  $sh$  can be defined as follows,

$$sh(dist(x,y)) = \begin{cases} (1 - dist(x,y)/\sigma)^2 & \text{if } dist(x,y) < \sigma \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

The sharing value of an individual will be increased by other individuals that are found located within the niche radius. The niche count  $nc$  is defined as,

$$nc(x) = \sum_{y \in individuals} sh(dist(x,y)) \quad (2)$$

During the tournament selection, individuals with a lower rank in partial order will be selected for reproduction, where the partial order ranking between two individuals depends on both their Pareto rank and niche counts. Rigorously, the partial order  $\geq_p$  for two individuals  $i$  and  $j$  is defined as,  $i \geq_p j$ , if  $[rank(i) > rank(j)]$  or  $[rank(i) = rank(j) \text{ and } nc(i) > nc(j)]$ .

### 3.6 Local Search Exploitation

As stated by Tan et al., (2001), the role of local search is vital in order to encourage better convergence and to discover any missing trade-off regions in evolutionary multiobjective optimization. In HMOEA, the local search starts by scanning through all routes in a chromosome, where any routes that contain a smaller number of tasks than a threshold are identified. These identified routes will be grouped into pairs randomly. All the tasks in each pair are then combined to form a new route that is sorted in ascending order by the earliest service time. After the merging, feasibility check is performed such that any infeasible tasks are moved to the outsourced list.

## 4 Computational Results

The HMOEA was programmed in C++ based on a Pentium III 933 MHz processor with 256 MB RAM under the Microsoft Windows 2000. Table 3 shows the parameter settings chosen after some preliminary experiments. These settings should not be regarded as an optimal set of parameter values, but rather a generalized one for which the HMOEA performs fairly well over the test problems.

Table 3 Parameter settings

Parameter	Value
Crossover rate	0.8
Mutation rate	0.3
Population size	800
Generation size	1000 or no improvement over the last 5 generations
Niche radius	0.04

This section contains the computational results and analysis of optimization performances for all problem instances. Section 4.1 studies the performance of Pareto-optimality for multiobjective optimization using the test cases. In Section 4.2, the optimization performance of HMOEA is compared with two other multiobjective evolutionary algorithms based upon a few performance measures.

### 4.1 Multiobjective Optimization Performance

#### 4.1.1 Pareto Front

In solving a vehicle routing problem, the logistic manager is often interested in not only getting the minimum routing cost, but also the smallest number of trucks required to service the plan. In order to reduce the routing cost, more number of trucks is often required and vice versa, i.e., the two criteria are noncommensurable and often competing with each other. Fig. 3 shows the evolution progress of Pareto front for 6 random selected test cases. In the simulation, the largest available vehicle number is limited to 35, which is more than sufficient to cater the number of tasks in each test case. The various Pareto fronts obtained at the initial generation (First), two intermediate generations (Int 1 and Int 2) and the final generation (Final) are plotted with different markers as shown in Fig. 3. As can be seen, there is only a small number of non-dominated solutions appeared at the initial generations, which are also congested at a small portion of the solution space. However, as the evolution proceeds, the diversity of the population increases significantly and the non-dominated solutions gradually evolve towards the final trade-off curve. A dashed line connecting all the final non-dominated solutions is drawn for each test case in Fig. 3, which clearly shows the final trade-off or routing plan obtained by the HMOEA. It is noted that the Pareto front includes the plan with zero truck number that subcontracts all tasks to external company, although such a policy is apparently not practical to adopt and is against the will of the logistic management.

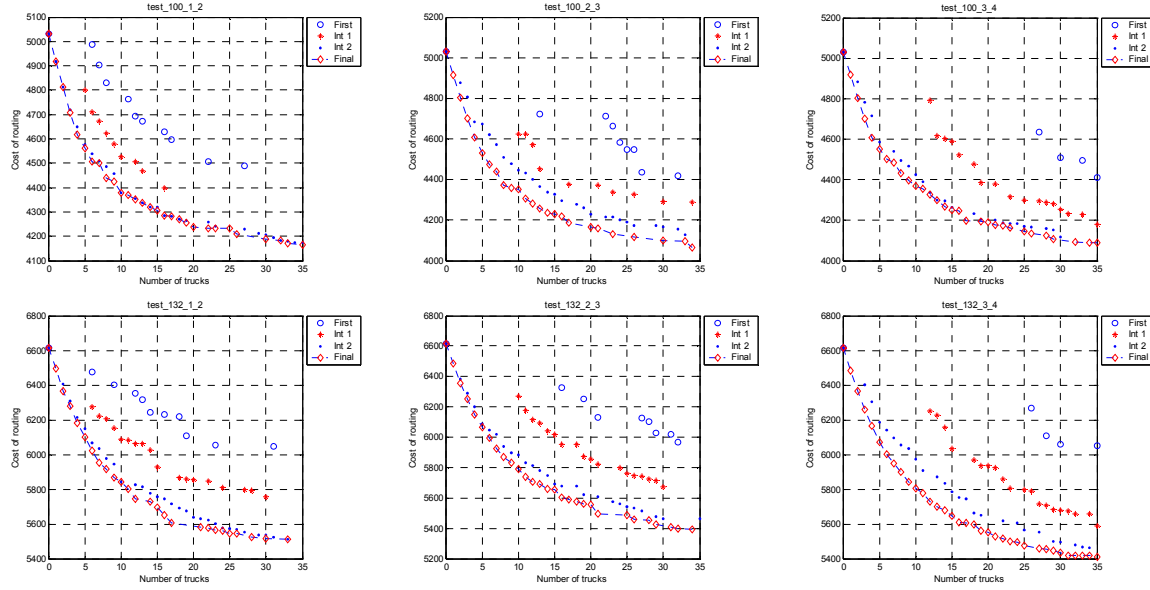


Fig. 3 The evolution progress of Pareto front for test cases

#### 4.1.2 Routing Plan

The average best routing cost for each truck number of the 12 test cases are plotted in Fig. 4, which shows an obvious trade-off between the two objectives of routing cost and truck number in TTVRP. This trade-off curve is useful for the decision-maker to derive an appropriate routing schedule according to the current situation. If the number of trucks available in a company is fixed, the logistic manager can estimate the required routing cost from the trade-off curve in Fig. 4.

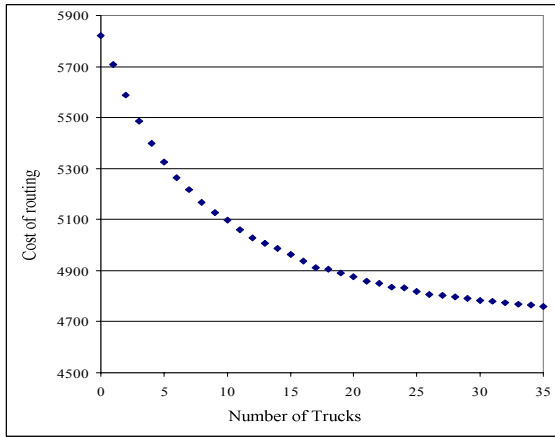


Fig. 4 The trade-off between cost of routing and number of trucks

In contrast, if the manager is given a specified budget or routing cost, he or she can then determine the minimum number of internal trucks to be allocated so that the spending can be kept below the budget allowed. For example, if the routing cost is to be kept below 5100, then the company must allocate at least 10 trucks for serving the task orders. However, if only 15 trucks are allocated

by the company, then the incurred routing cost would be around 4900 to 5000, including the cost payment for outsourced companies.

#### 4.2 Comparison Results

In this section, the performance of HMOEA is compared with two variants of evolutionary algorithms, i.e., MOEA with standard genetic operators as well as MOEA without hybridization of local search. The comparison allows the effectiveness of the various features in HMOEA, such as the specialized genetic operators and local search heuristic, to be examined. The multiobjective evolutionary algorithm with standard generic operators (STD\_MOEA) includes the commonly known cycle crossover and RAR mutation. The cycle crossover is a general crossover operator that preserves the order of sequence in the parent partially and was applied to solve the traveling salesman problems by Oliver et al. (1987). The remove and reinsert (RAR) mutation operator removes a task from the sequence and reinsert it to a random position (Gendreau et al., 1999). The multiobjective evolutionary algorithm without hybridization of local search (NH\_MOEA) employs the specialized genetic operators in HMOEA but excludes the local search heuristic. The experiment setups and parameters for STD\_MOEA and NH\_MOEA are similar to the settings for HMOEA as shown in Table 3.

##### 4.2.1 Average Routing Cost

To compare the quality of solutions produced by the algorithms, the average routing cost (ARC) of the non-dominated solutions in the final population is calculated for various test cases with different number of tasks as shown in Fig. 5. In the figure, the average value of ARC is plotted for each group of the test cases with equal number of tasks. As can be seen, the STD\_MOEA incurs the highest ARC since its operators are not tailored made for the TTVRP problem. According to the no free lunch

theorem (Wolpert and Macready, 1996), any optimization methods should be tailored to the problem domain for best performance. The results in Fig. 5 also illustrate that the HMOEA outperforms NH\_MOEA and STD\_MOEA consistently, which produces the lowest routing cost for all test cases. Since the search space of the multiobjective TTVRP optimization is complex, it is expected that the problem-specific HMOEA should provide an efficient and high-performance routing solution.

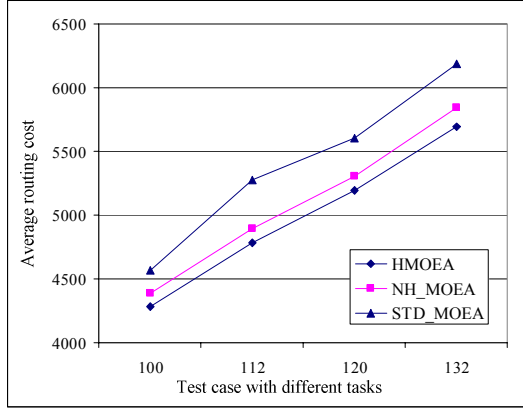


Fig. 5 The average routing cost for various algorithms

#### 4.2.2 Ratio of Non-dominated Individuals

In multiobjective optimization, it is often desired to find many useful candidate solutions that are non-dominated in a population, which could be measured by the ratio of non-dominated individuals (RNI) as proposed by Tan et al., (2001). Given a population  $X$ , the RNI is defined as,

$$RNI(X)\% = \frac{nondom\_indiv}{N} \times 100\% \quad (4)$$

where  $nondom\_indiv$  is the number of non-dominated individuals in population  $X$ , while  $N$  is the size of the population  $X$ . Without loss of generality, Fig. 6 shows the RNI for the three algorithms based on a randomly selected test case 132\_3\_4. As can be seen, the RNI value of STD\_MOEA is the lowest among the three algorithms and in the process of computation, the evolution in STD\_MOEA stopped at around 90 generations as no improvement was observed for 5 generations continuously. The results also show that the search performance of HMOEA for non-dominated solutions is slightly better than NH\_MOEA. Besides, the HMOEA also has the best average RNI of 1.89 as compared to the value of 1.71 and 0.44 for NH\_MOEA and STD\_MOEA, respectively.

#### 4.2.3 Simulation Time

The computational time for different algorithms is studied in this sub-section. The three algorithms adopt the same stopping criteria in the simulation, i.e., the evolution stops after 1000 generations or when no improvement is found for the last 5 generations. Fig. 7 shows the normalized simulation time for the three algorithms based on four

randomly selected test cases, e.g., test\_100\_3\_4, test\_112\_3\_4, test\_120\_3\_4 and test\_132\_3\_4. As can be seen, the STD\_MOEA requires the shortest time to converge or halt the evolution. The optimization results obtained by the STD\_MOEA are much inferior probably because the population in STD\_MOEA has converged prematurely to local Pareto front. The results also show that the computation time required by HMOEA is better than NH\_MOEA for all the instances.

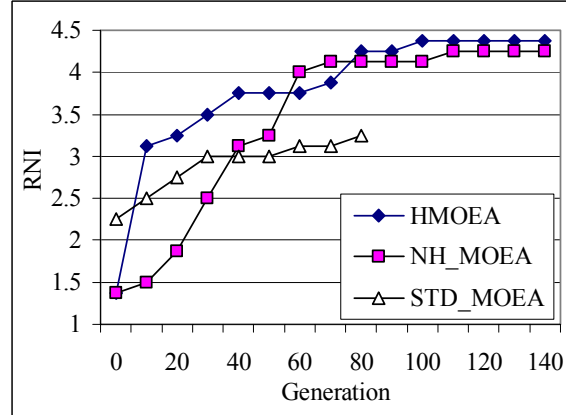


Fig. 6 The RNI of various algorithms for test case 132\_3\_4

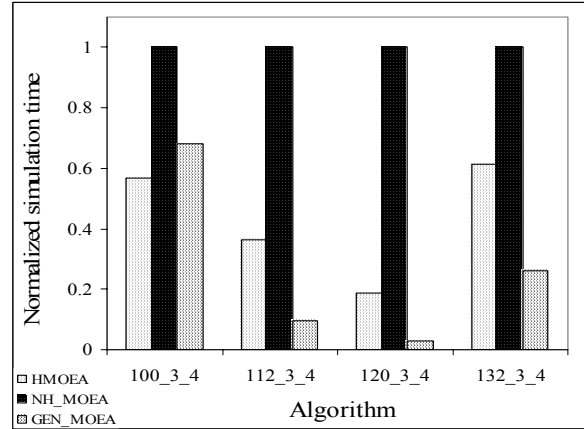


Fig. 7 The normalized simulation time for various algorithms

## 5 Conclusions

A transportation problem for moving empty or laden containers for a logistic company has been considered and a model for the truck and trailer vehicle routing problem (TTVRP) has been constructed in the paper. The objective of the routing problem is to minimize the routing distance and the number of trucks required, subject to a number of constraints such as time windows and availability of trailers. To solve such a multiobjective and multi-modal combinatorial optimization problem, a hybrid multiobjective evolutionary algorithm (HMOEA) featured with specialized genetic operators, variable-length representation and local search heuristic has been applied



to find the Pareto optimal routing solutions for the TTVRP. Detailed analysis has been performed to extract important decision-making information from the multiobjective optimization results. The computational results have shown that the proposed HMOEA and features incorporated are effective for solving multiobjective combinatorial optimization problems such as finding useful trade-off solutions for the TTVRP.

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