WA1-3 10:40

Proceedings of the 1999 IEEE
International Symposium on Computer Aided Control System Design
Kohala Coast-Island of Hawai'i, Hawai'i, USA • August 22-27, 1999

CONTROL SYSTEM DESIGN AUTOMATION WITH ROBUST TRACKING THUMBPRINT PERFORMANCE USING A MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM

K. C. Tan, T. H. Lee and E. F. Khor
Department of Electrical Engineering
National University of Singapore
10 Kent Ridge Crescent Singapore 119260

Abstract: This paper develops a multi-objective evolutionary based methodology for control system design automation of robust tracking thumbprint performances in QFT. Unlike conventional two-stage design approach, the technique is capable of evolving both nominal controller and pre-filter concurrently without the need of QFT bound computation and manual loop-shaping procedure. It is shown that the method can easily accommodate practical soft/hard constraints and allows engineers to examine the different design trade-offs. Validation upon a benchmark QFT design problem illustrates the usefulness of the proposed methodology.

1. INTRODUCTION

Quantitative Feedback Theory (QFT) is well-known as an efficient frequency based robust controller design methodology that maintains system response within pre-specified tolerances despite uncertainties and disturbances [1-3]. It has been successfully applied to various engineering applications such as flight control, missile control, compact disk mechanisms and etc [4, 5]. The basic idea of QFT is to convert design specification on closed-loop response and plant uncertainty into robust stability and performance bounds on open-loop transmission of the nominal system. A fixed structure controller and pre-filter is then synthesized using gain-phase loop-shaping technique so that the two-degree-freedom output feedback system as shown in Fig. 1 is controlled within specification for any member of the plant templates.

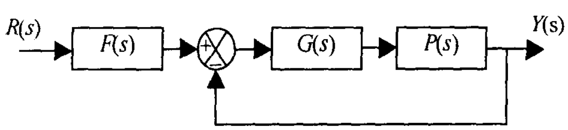


Fig. 1 A typical output feedback control system

In brief, QFT controller design consists of the following steps:

1. For each particular frequency, templates are developed by determining the frequency responses of various plants.

2. A set of QFT bounds in Nichols chart are then computed based upon the chosen nominal plant and the design specifications such as robust margin, robust tracking and disturbance rejection. Taking the worst-case bound at the same frequency point of the intersection of all bounds gives a single QFT bound in the Nichols chart.

3. Loop-shaping design technique is applied to obtain the controller $G(s)$ so that the QFT bounds in Nichols chart at all frequencies are satisfied and the closed-loop nominal system is stable.

4. Lastly, the pre-filter $F(s)$ is designed to position the system within the frequency domain specifications.

To design the controller in Fig. 1, QFT bounds in Nichols chart for each specification of all frequency points must be acquired, which is often an exhaustive trial-and-error procedure [3]. The reason is that, for every frequency point with sufficiently small frequency interval, the template needs to be moved up or down on the Nichols chart until the gain variation of the template is equal to the gain variation allowed for any particular robust specification at that frequency. Besides, only the controller can be synthesized via the QFT bound computation using loop-shaping method. Another independent design task has to be accomplished in order to obtain the pre-filter within a two-stage design framework.

In this paper, a multi-objective evolutionary based methodology for design automation of QFT control system is proposed. Unlike existing methods, the evolutionary technique is capable of obtaining the controller and pre-filter concurrently to meet all performance requirements in QFT without going through the QFT bound computation and manual loop-shaping procedure. In addition, it is capable of evolving a set of non-commensurable solutions that allows the engineers to examine various trade-offs among the different design specifications. Overview of the tracking thumbprint that is useful for formulating the robust tracking specification in QFT is given in Section 2. The QFT design specification of robust tracking thumbprint and other performance requirements are formulated in Section 3. Section 4 briefly describes a multi-objective evolutionary algorithm (MOEA) that is capable of handling both soft and hard design constraints for effective multi-objective optimization. Implementation of the MOEA to a benchmark QFT design problem is illustrated in Section 5. Conclusions are drawn in Section 6.

2. OVERVIEW OF TRACKING THUMBPRINT

Tracking thumbprint specification is often used to determine the robust tracking performance in QFT, which is based upon the satisfaction of an upper and lower bound in time-domain response as shown in Fig. 2. Here, $y(t)_U$ is the upper bound which is usually an under-damped $(m_p, t_p, t_s, t_r, K_m)_U$ step response, while $y(t)_L$ is the lower bound that is often represented by an over-damped step response $(t_s, t_r, K_m)_L$. m_p is denoted as the system overshoot, t_p the peak time, t_s the settling time, t_r the rise time and K_m the gain margin.

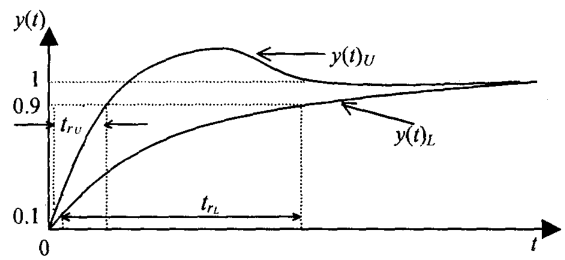


Fig. 2 The tracking thumbprint specification

Based on the time-domain specification of m_p, t_p, t_s, t_r and the required gain margin K_m, the desired control ratio can be modeled in the frequency domain as shown in Fig. 3. Here, T_U is the transformed upper bound and T_L the transformed lower bound, which maybe in the form of a second-order transfer function as given by [2]

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (1)$$

where ω_n is the natural frequency and ζ denotes the damping ratio. Intuitively, ω_n and ζ can be easily determined from the time-domain specifications as given in Fig. 2. The specification of tracking thumbprint in QFT is thus to design the controller and pre-filter in Fig. 1 such that the system frequency responses of all plant templates are inside the region of the desired specified tracking bounds in the Bode plot as shown in Fig. 3.

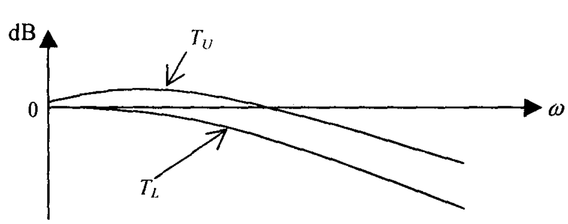


Fig. 3 Bode plot of upper and lower bound tracking model

3. MULTI-OBJECTIVE DESIGN FORMULATION

Apart from the tracking thumbprint specification in QFT, other design objectives such as robust margin performance, high frequency gain and minimal controller order are also important and needed to be satisfied in the design. In contrast to the conventional two-stage loop-shaping approach, these performance requirements can be formulated as a multi-objective design optimization problem. The aim is thus to concurrently design the nominal controller $G(s)$ and pre-filter $F(s)$ in order to satisfy all the specifications such as closed-loop stability, robust tracking performance, robust margin, high frequency gain and minimal controller order as described below:

(i) Stability (RHSP)

The cost of stability, *RHSP*, is included to ensure stability of the closed-loop system, which could be evaluated by solving the roots of the characteristic polynomial. The cost of stability is then defined as the total number of unstable closed-loop poles or the positive poles in the right-hand-side of *S*-plane. Clearly, a stable closed-loop system requires a zero value of *RHSP*.

(ii) Robust Upper Tracking Performance (ERRUT)

The cost of upper tracking performance, given by *ERRUT*, is included for the upper tracking bound specification in Fig. 3. It is computed as the sum of absolute error at each frequency point,

$$ERRUT = \sum_{i=1}^{n} \left| e_u(\omega_i) \right| \quad (2)$$

where n is the total number of interested frequency points; e_u is the difference between the upper bound of the closed-loop system CL_U and the pre-specified upper tracking bound T_U, if the upper bound of the closed-loop system is greater than the pre-specified upper tracking bound or less than the pre-specified lower tracking bound T_L; otherwise, e_u is equal to zero. In Fig. 4, the length of vertical dotted lines at each frequency ω_i represents the magnitude of $e_u(\omega_i)$.

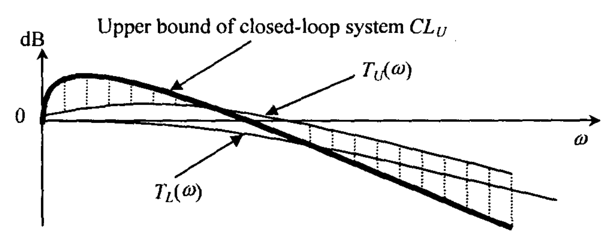


Fig. 4 Computation of upper tracking performance

(iii) Robust Lower Tracking Performance (ERRLT)

The cost of lower tracking performance, given by *ERRLT*, is incorporated for the lower tracking bound specification in Fig. 3. It is defined as the sum of absolute error at each frequency point,

$$ERRLT = \sum_{i=1}^{n} |e_l(\omega_i)| \qquad (3)$$

where n is the number of frequency points; e_l is the difference between the lower bound of the closed-loop system CL_L and the pre-specified lower tracking bound T_L, if the lower bound of the closed-loop system is greater than the pre-specified upper tracking bound T_U or less than the pre-specified lower tracking bound T_L; Otherwise, e_l is equal to zero as illustrated in Fig. 5.

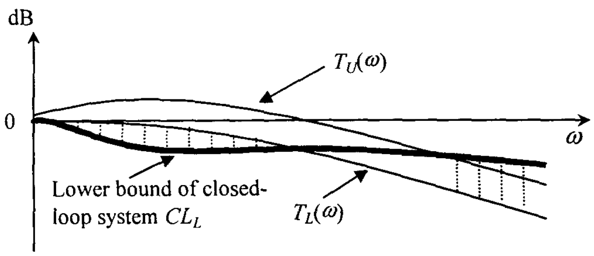


Fig. 5 Computation of lower tracking performance

(iv) Robust Margin (RM)

It is of fundamental importance that control system is designed so that closed-loop stability is preserved in the face of uncertainties. The cost of robust margin, *RM*, is to increase the stability margin of the closed-loop system and is defined as [6]

$$RM = \max\left[\left|\frac{P_i(j\omega)G(j\omega)}{1+P_i(j\omega)G(j\omega)}\right|, \forall P_i \in \boldsymbol{P}, \omega \geq 0\right] \qquad (4)$$

where $\boldsymbol{P}$ is the set of plant templates. Note that a smaller *RM* corresponds to a better robust stability margin of the system.

(v) High Frequency Gain (HFG)

The high frequency gain performance, *HFG*, is included to reduce the gain of loop transmission $L(s) = G(s)P(s)$ at the high frequency in order to avoid the high-frequency sensor noise and the unmodeled high-frequency dynamics or harmonics, which may result in actuator saturation and instability. The high frequency gain of loop transmission $L(s)$ is given as

$$\lim_{s\to\infty} s^r L(s) \qquad (5)$$

where r is the relative order of the $L(s)$. Since only the controller in the loop transmission is to be optimized, this performance requirement is equivalent to the minimization of high frequency gain of the controller or the magnitude of b_n/a_m for a controller structure given as [7]

$$G(s) = \frac{b_n s^n + b_{n-1}s^{n-1} + \cdots b_0}{a_m s^m + a_{m-1}s^{m-1} + \cdots a_0} \qquad (6)$$

where n and m is the order of the numerator and denominator in the controller $G(s)$, respectively.

(vi) Controller Order (CO)

It is often desired that the controller to be designed is as simple as possible, since simple controller requires less computation and implementation effort than a higher-order controller does. Here, the order of controller is included as one of the QFT design specification in order to find the smallest-order controller after all other design specifications have been satisfied.

As addressed in the Introduction, the QFT control system design is basically a multi-objective optimization problem that determines multiple controller and pre-filter coefficients to satisfy a set of non-commensurable and often competing design specifications. Conventional techniques for multi-objective optimization problem include the method of inequalities, goal attainment or weighted sum. To obtain a good solution, however, these approaches require a continuous cost function and a set of precise settings of weights or goals that are usually not well manageable or understood [8, 9]. A powerful multi-objective evolutionary algorithm that is capable of achieving an effective and automated QFT control system design is thus presented in next section.

4. EVOLUTIONARY DESIGN AUTOMATION

Multi-objective optimization seeks to optimize a vector of non-commensurable and often competing objectives or cost functions. Solution to the multi-objective problem is a family of points known as Pareto optimal set, where each objective component of any point along the Pareto front can only be improved by degrading at least one of its other objective components [10]. Ranking scheme based upon the Pareto optimality is regarded as an appropriate approach in coping with multi-objective optimization in evolutionary algorithms. For details of evolutionary algorithms with Pareto ranking and goal/priority information for multi-objective optimization, refer to [8] and [9].

Practical multi-objective optimization problems often involve optimizing a set of objective components, subject to certain constraints to be satisfied. These constraints could be incorporated in the multi-objective cost function as one of the objective components to be optimized. It maybe in the form of a hard constraint where the optimization is directed towards attaining a threshold or goal, and further optimization is meaningless and not desired whenever the goal has been satisfied. In contrast, a soft constraint requires ongoing optimization or to optimize the value of that objective component as much as possible. An easy approach to deal with both soft and hard constraints concurrently in the multi-objective evolutionary optimization is proposed. At each generation, an updated objective function $F_x^{\#}$ concerning the hard and soft constraints for an individual x with its objective function F_x is computed in *a-priori* to the Pareto cost assignment as given by,

$$\forall i = \{1,\ldots,m\},\ F_x^{\#}(i) = \begin{cases} G(i) & if\ [G(i) \text{ is hard}] \& [F_x(i) < G(i)] \\ F_x(i) & otherwise \end{cases} \tag{7}$$

In eqn. 7, any objective component i corresponding to a hard constraint is assigned to the value of the goal $G(i)$ whenever the hard constraint has been satisfied. The underlying reason is that there is no ranking preference for any particular objective component that has the same objective value in an evolutionary process, and thus the evolution will only be directed towards optimizing soft constraints and any unattained hard constraints, as desired.

Convergence trace is an important dynamic behavior in the evolutionary optimization, which provides a useful performance observation of the entire evolution process. For single objective optimization, the convergence behavior is often represented by the performance index versus the iteration or generation. Obviously, this representation is not appropriate for multi-objective optimization that has more than one objective to be optimized at all time. A novel convergence assessment for multi-objective optimization via the concept of population domination and progress ratio is proposed here. In the sense of progress towards the direction that is normal to the trade-off surface formed by the current non-dominated individuals, the progress ratio at any particular generation can be defined as the domination of one population to another. Generally, the progress ratio $Pr^{(n)}$ at generation n can be defined as the ratio between the number of non-dominated individuals at generation n (nondom_indiv$^{(n)}$) *dominating* the non-dominated individuals at generation (n-1) (nondom_indiv$^{(n-1)}$) over the total number of non-dominated individuals at generation n (nondom_indiv$^{(n)}$),

$$Pr^{(n)} = \frac{\text{nondom\_indiv}^{(n)}\ \textit{dominating}\ \text{nondom\_indiv}^{(n-1)}}{\text{nondom\_indiv}^{(n)}} \tag{8}$$

For a normal convergence, the progress ratio should start from a value close to 1 indicating a high possibility for further improvement of the evolution at the initial stage. As the generation proceeds, Pr is expected to be decreased asymptotically towards a small value approaching zero, showing the population is touching the trade-off surface or there is less possibility to produce any new non-dominated individuals dominating the current non-dominated individuals. Intuitively, the evolution is said to be nearly converged at generation n if $Pr^{(n)} \approx 0$, which maybe used as a stopping criterion for the multi-objective optimization. Like conventional convergence trace, the progress ratio Pr cannot reflect how close the population is approaching the usually unknown trade-off surface. However, it provides the important information of the relative progress of the population evolves in the direction that is normal to the trade-off surface formed by the current non-dominated individuals at each generation.

5. A BENCHMARK QFT DESIGN PROBLEM

The benchmark QFT design problem given in [6, 7] is studied here to illustrate the usefulness of the proposed methodology. The QFT control system is shown in Fig. 1, with the uncertain plant set given as

$$P(s) = \frac{ka}{s(s+a)}\ :\ k \in [1, 10],\ a \in [1, 10] \tag{9}$$

The various closed-loop performance requirements for QFT control system design described in Section 3 are formulated as follows:

(i) Robust Stability Margin:

$$\left|\frac{P(j\omega)G(j\omega)}{1+P(j\omega)G(j\omega)}\right| \le 1.2, \forall P \in \boldsymbol{P}, \omega \ge 0$$

(ii) Robust Tracking Performance:

$$T_L(\omega) \le \left|F(j\omega)\frac{P(j\omega)G(j\omega)}{1+P(j\omega)G(j\omega)}\right| \le T_U(\omega)$$

(iii) Upper Tracking Model:

$$T_U(\omega) = \left|\frac{0.6854(j\omega)+30}{(j\omega)^2+4(j\omega)+19.752}\right|$$

(iv) Lower Tracking Model:

$$T_L(\omega) = \left|\frac{120}{(j\omega)^3+17(j\omega)^2+82(j\omega)+120}\right|$$

A wide frequency spectrum covering the ranges of 10^{-2} rad/sec to 10^{4} rad/sec is used to evaluate the robust stability and tracking performances of the system. One major advantage of the proposed evolutionary technique is the ability to directly evolve both the controller $G(s)$ and the pre-filter $F(s)$ simultaneously. For wider choices of solution set, the controller order to be optimized is not fixed and arbitrarily ranges from second-order to fourth-order. For this, an additional variable is added to turn on/off the relevant controller coefficients as appropriate. The filter is fixed to second-order, as it is relevant to the tracking bound in the frequency response which is a second-order transfer function as given in eqn. 1.

The structure of the controller is chosen in the form of a general transfer function as given by

$$G(s) = \frac{\sum_{i=0}^{4} b_i s^i}{\sum_{i=0}^{4} a_i s^i} \quad \forall\, b_i, a_i \in \Re \tag{10}$$

Since the resultant pre-filter must satisfy $\lim_{s \to 0}[F(s)] = 1$ for a step forcing function [6], the structure of $F(s)$ is chosen as

$$F(s) = \frac{1}{1 + \sum_{j=1}^{2} c_j s^j} \quad \forall\, c_j \in \Re \tag{11}$$

To guide the MOEA optimization process, goal and priorities [9] are introduced and their settings are given in Table 1. Here, the stability and robust tracking performance are treated as hard constraints, while robust margin, high frequency gain and controller order are regarded as soft constraints. In Table 1, the cost of stability is given as the first priority because system stability is often the most important requirement in any control system design. The system must also satisfy the tracking thumbprint, robust margin and high frequency gain requirements. The controller order is set as the last priority in order to evolve a set of controllers with the lowest controller order that fulfill all other design specifications. Although determination of the objectives and the priorities may be a subjective matter and depends on the performance requirement, ranking the priorities may be unnecessary and can be ignored for a 'minimum-commitment' design [11]. If, however, an engineer is committed to prioritizing the objectives, it is a much easier task than weighting the objectives. In principle, any number or combination of specifications or constraints can be added to the design if necessary.

Table 1. Goal/priority setting for the benchmark problem

Objective	Cost	Goal	Constraint	Priority
1	*RHSP*	0	hard	1
2	*ERRUT*	0	hard	2
3	*ERRLT*	0	hard	2
4	*RM*	1.2	soft	2
5	*HFG*	6×10^6	soft	2
6	*CO*	4	soft	3

The algorithm was run for 200 generations with a population of 200 to evolve the controller $G(s)$ and pre-filter $F(s)$. At the end of the evolution, all the 200 individuals have met the 6 goals listed in Table 1. Among these, 197 individuals are of 3rd-order controller and 3 individuals are of 4th-order controller. Also, there are 148 non-dominated individuals at the final generation occupying 74% of the population. The progress ratio versus the generation number is plotted in Fig. 7. It can be seen that the progress ratio is relatively large in the beginning and decreases asymptotically as the evolution proceeds or as the population gets closer to the trade-off surface.

For the individuals satisfying the goal, their costs of stability (*RHSP*), robust upper tracking performance (*ERRUT*) and robust lower tracking performance (*ERRLT*) are all equal to zero as according to the hard-setting in Table 1. The resultant cost of robust margin (*RM*) and high frequency gain (*HFG*) are shown in Fig. 8, in which the two costs are plotted together with respect to the index of individuals. As can be seen, the cost of high frequency gain increases with the decreases of robust margin, which indicates a trade-off between the two objectives. This has allowed the engineer to visualize the objectives before final determination of an appropriate controller and pre-filter for the application on-hand.

Fig. 9 shows the robust tracking performance in the frequency domain. Obviously, all the actual closed-loop bounds given by CL_U and CL_L are located successfully within the pre-specified tracking bounds of T_U and T_L. Fig. 10 shows the tracking thumbprint performance in the time-domain for an arbitrary chosen set of 3rd-order controller. Clearly, the evolutionary design has successfully satisfied the required tracking thumbprint performance, with step responses for all plant templates located satisfactory within the required time-domain tacking envelope.

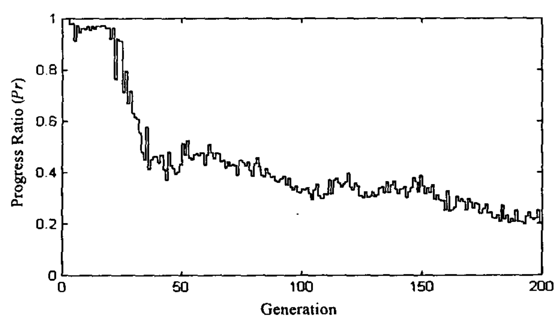


Fig. 7 Progress ratio *Pr* versus generation for MOEA

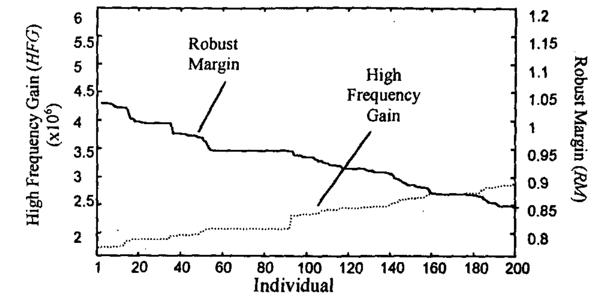


Fig. 8 Trade-off between the *RM* and *HFG*

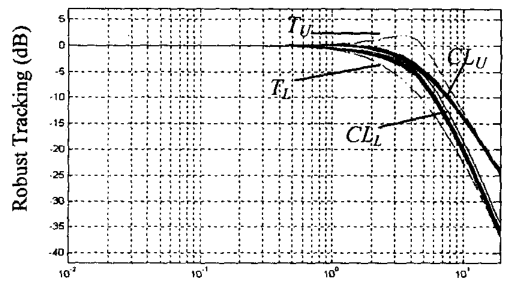
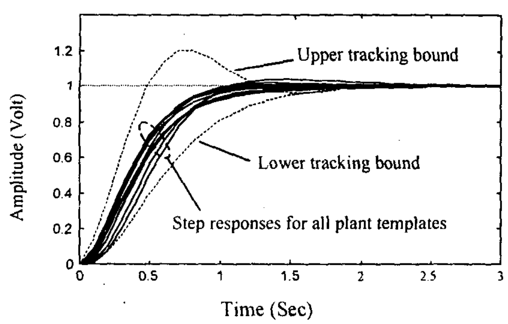


Fig. 9 Robust tracking in frequency response



(a) 3rd order controller

Fig. 10 Thumbprint performance in the time domain

6. CONCLUSION

This paper has developed an evolutionary based controller design methodology to satisfy the robust tracking thumbprint performances in QFT. Unlike conventional two-stage design using loop-shaping methods, the evolutionary technique is capable of evolving both controller and pre-filter concurrently to reduce the design complexity in QFT. In addition, the evolutionary optimization can easily accommodate practical soft/hard constraints and allows engineers to examine different trade-offs among the conflicting design specifications. The advantages of the proposed evolutionary technique are illustrated upon a benchmark QFT design problem. The multi-objective evolutionary design for QFT is currently being applied to multi-input multi-output (MIMO) system and to incorporate other design specifications such as economical cost consideration. Progress and results will be reported in due course.

ACKNOWLEDGEMENT

This work was supported by the National University of Singapore Academic Research Fund under grant NUS Project RP3981620.

REFERENCES

[1] I. M. Horowitz, "Synthesis of feedback system with non-linear time-varying uncertain plants to satisfy quantitative performance specifications," *IEEE Proc.*, vol. 64, pp.123-130, 1976.

[2] I. M. Horowitz "Synthesis of feedback systems with large plant ignorance for prescribed time domain tolerances," *Int. J. Control*, vol. 16, pp. 287-309, 1972.

[3] I. M. Horowitz, "Quantitative feedback theory," *IEE Proc.*, Pt. D, vol. 129, no. 6, pp. 215-226, 1982.

[4] Y. Chait, M. S. Park, and M. Steinbuch, "Design and implementation of a QFT controller for a compact disc player," *Proc. of the 1994 American Control Conf.*, vol. 3, pp. 3204-3208, Baltimore, U.S.A., 1994.

[5] M. Pachter and C. H. Houpis, "A full envelope flight control system design, including aerodynamic control effector failures accommodation, using QFT," *Proc. of the Symp. on QFT and other Freq. Domain Methods and Appl.*, pp. 45-54, Glasgow, Scotland, 1997.

[6] C. Borghesani, Y. Chait and O. Yaniv, *Quantitative Feedback Theory Toolbox User Manual*, The Math Work Inc, 1995.

[7] W. H. Chen, D. J. Balance and Y. Li, "Automatic loop-shaping in QFT using genetic algorithms," *Proc. of 3rd Asia-Pacific Conf. on Cont. & Meas.*, pp. 63-67, 1998.

[8] C. M. Fonseca, *Multi-objective Genetic Algorithms with Application to Control Engineering Problems*, Ph.D. Thesis, Dept. Automatic Control and Systems Eng., *University of Sheffield*, Sheffield, 1995.

[9] K. C. Tan, T. H. Lee and E. F. Khor, "Evolutionary algorithms with goal and priority information for multi-objective optimization," *accepted by Congress on Evolutionary Computation*, 1999.

[10] D. E. Goldberg and J. Richardson, "Genetic algorithms with sharing for multi-modal function optimization," *Proc. Second Int Conf. on Genetic Algorithms*, pp. 41-49, Lawrence Erlbaum, 1987.

[11] K. X. Guan and K. J. MacCallum, "Adopting a minimum commitment principle for computer aided geometric design systems," *in Artificial Intelligence in Design '96 (Gero, J. S. and Sudweeks, F., eds)*, Kluwer Academic Publishers, pp. 623-639, 1996.