Multiobjective Combinatorial Optimization

J. Teghem

Faculté Polytechnique de Mons
9, rue de Houdain 7000 Mons
Belgium
Teghem@mathro.fpms.ac.be

Abstract. The importance of multiobjective combinatorial optimization is emphazised and three approaches – one of them is a heuristic – to generate the set of efficient solutions are presented and compared. Illustrations are made with the multi-objective knapsack problem.

1 Introduction

Multiobjective combinatorial optimization (MOCO) problems have received more attention in the last years. In this paper, we examine how to generate the set E(P) of all efficient solutions of a MOCO problem (P).
MOCO problems are characterized by

- the very large number of efficient solutions ;
- the non convexity of the feasible set requiring to device specific techniques to generate the so-called « non supported » efficient solutions i.e. those which can not be obtained by the optimization of a linear weighted aggregation of the objectives ;
- the specific structure of the constraints (assignment problem, knapsack problems, …) so that to be able to take advantage in a MOCO procedure of the efficiency of the existing methods for the uni-criterion problems (Hungarian method, Martello and Toth method, …) we must preserve this structure avoiding to introduce additional constraints.

We can distinguish three approaches to generate E(P).

2 Direct methods

The idea is to use intensively classical methods for single objective problem P existing in the literature to determine E(P). Each time a feasible solution is obtained, it is compared with the list E*(P) containing all the feasible solutions already obtained and non dominated by another generated feasible solution. Clearly E(P) plays the role of the so-called « incumbent solution » in single objective methods. At each step, E(P) is updated and at the end of the procedure E(P) = E*(P). Such extension of single objective method is specially designed for enumerative procedure based on a Branch and Bound (BB) approach. Unfortunately, in a MO framework, a node of the BB tree is less often fathomed than in the single objective so that logically such MO procedure is less efficient.

3 Two-phase methods

Such approach is particularly well designed for bi-objective MOCO problems.
The first phase consists to determine the set of supported efficient solutions. This phase is based on the parametric single objective problem obtained by linear weighted aggregation of the two objectives. This problem is solved several times for different values of the weights.
The purpose of the second phase is to generate the set of non supported efficient solutions. In the objective space, each non supported efficient solution has its image in a rectangular triangle determined by two adjacent supported efficient solutions. This phase is more difficult to manage and is dependant of the particular MOCO problem analysed ; generally this second phase is achieved using partly a classical single objective CO method.

4 Heuristic methods

It is unrealistic to extend the two previous exact approaches to MOCO problems with more than two criteria or more than a few hundred variables ; the reason is that these methods are too much consuming time.
Because the meta-heuristic - Simulating Annealing (SA), Tabu Search (TS), Genetic Algorithms (GA), …- provide, for the single objective case, excellent solutions in a reasonable time, it appeared logical to try to adapt these metaheuristics to a multi-objective framework.
So we develop a Multiobjective Simulated Annealing methodology called MOSA method.
The principle idea can be resumed in short terms.
One begins with an initial iterate X_0 and initialises the set of potentially efficient points PE to just contain X_0. One then samples a point Y in the neighbourhood of the current iterate. But instead of accepting Y if it is better than the current iterate on an objective , we now accept it if it is not dominated by any of the point currently in the set PE. If it is not dominated , we make Y the current iterate, add it to PE, and throw out any point in PE that are dominated by Y.
On the other hand, if Y is dominated, we still make it the current iterate with some probability. In this way, as we move the iterate through the space, we simultaneously build up a set PE of potentially efficient points. The only complicated aspect of this scheme is the method for computing the acceptance probability for Y when it is dominated by a point in PE.
To define this probability is difficult to avoid to use a scalarizing function; but the value of the weights of this function induces a privileged search direction on the efficient frontier. So to cover all this frontier, it is necessary to define a wide diversified set of weights and to repeat the procedure with each weight. For each one a set PE is generated and these sets are filtered by pairwise comparisons to remove the dominated solutions and to obtain the final approximation of the set of efficient solutions.

5 Comparison

These three approaches will be illustrated on the multi-objective knapsak problem.
Numerical results are presented and the quality of the MOSA method is analysed.