

# Rotordynamic Optimization of Large Turbo Systems using Genetic Algorithms

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Luleå, March 2006

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## Abstract

In engineering design, nature has often been the source of inspiration. It is easy to point out solutions in nature that are optimal in some sense. One example is the roughness of the surface of a shark's skin. This is designed by nature to minimize the resistance when the shark swims in the water. Another example is the shape of an egg shell. This is an optimal load carrying structure which often is found in engineering design applications. An even more fascinating question is how nature has found these optimal solutions? The answer to this question is *evolution*.

Instead of just analyzing and copying optimal structures invented by nature it seems reasonable to mimic the process how nature has came up with these solutions. Research on how these ideas can be interpreted and used in engineering design started in the early seventies and has now become a large field known as Evolutionary Algorithms (EAs). During the past decade these methods have emerged as potent tools for engineering design optimization. Some of these methods are especially suited for problems which involve multiple objectives such as almost all real engineering design problems.

Just until recently, these methods have seldom been used in the area of rotordynamical design. This thesis deals with the question how these methods can be adapted and applied in order to improve the design and design process of large rotor-bearing system. A hypothesis for this work is that EAs are suitable to use in the late design process of these systems. The aim of this work is to evaluate this hypothesis by studying real applications found in industry.

This thesis comprises an introductory part and five appended papers. The introductory part is divided into four different chapters. In the second chapter the concept of engineering design optimization is introduced. In the third chapter Genetic Algorithms (GAs) is presented. Finally, the analysis and design of rotor-bearing systems are introduced and discussed. The purpose with the introductory part is to introduce and prepare the reader to the concepts presented in the papers. The introductory part may serve as a start point for newcomers interested in these areas.

The appended papers deal with different rotor-bearing system optimization problems and how these can be formulated and solved with GAs. Paper A introduces a constraint handling technique based on concepts found in multiobjective GAs. In Paper B the multiobjective optimization of a generator is presented and discussed. In Paper C and Paper D the constraint handling technique introduced in Paper A is used for two different rotor-bearing system where the actual bearing geometry parameters are used as design variables in the optimizations. In Paper E the feasibility of site balancing rewinded turbo generators is investigated by the use of a multiobjective GA.

## Keywords

Rotordynamics, Optimization, Genetic Algorithms, Turbo Generators, Balancing, Journal Bearings





## **Thesis**

This thesis comprises an introductory part and five appended papers.

## **Paper A**

Angantyr, A., Andersson, J. and Aidanpää, J-O., 2003, “Constrained Optimization based on a Multiobjective Evolutionary Algorithm”, In Sarker R. et al. (Eds.), *Proceedings of the Congress on Evolutionary Computation*, Canberra, Australia, IEEE-Press, **3**, pp. 1560-1567.

## **Paper B**

Angantyr, A. and Aidanpää, J-O., 2004, “A Pareto Based Genetic Algorithm Search Approach to Handle Damped Natural Frequency Constraints in Turbo Generator Rotor System Design”, *ASME Journal of Engineering for Gas Turbines and Power*, **126**, pp. 619-625.

## **Paper C**

Angantyr, A. and Aidanpää, J-O., 2004, “Optimization of a Rotor-Bearing System with an Evolutionary Algorithm”, In Bohn, D. (Ed.), *Proceedings of The 10<sup>th</sup> International Symposium on Rotating Machinery*, March 7-11, Honolulu, Hawaii.

## **Paper D**

Angantyr, A. and Aidanpää, J-O., 2006, “Constrained Optimization of Gas Turbine Tilting Pad Bearing Designs”, Under publication in *ASME Journal of Engineering for Gas Turbines and Power*.

## **Paper E**

Angantyr, A., 2006, “Feasibility of Site Balancing Rewinded 2-pole Turbo Generators”, *Accepted for presentation on ASME Power*, May 2-4, 2006, Atlanta, GA.



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## Appended papers

<b>Paper A</b>	Constrained Optimization based on a Multiobjective Evolutionary Algorithm
<b>Paper B</b>	A Pareto Based Genetic Algorithm Search Approach to Handle Damped Natural Frequency Constraints in Turbo Generator Rotor System Design
<b>Paper C</b>	Optimization of a Rotor-Bearing System with an Evolutionary Algorithm
<b>Paper D</b>	Constrained Optimization of Gas Turbine Tilting Pad Bearing Designs
<b>Paper E</b>	Feasibility of Site Balancing Rewinded 2-pole Turbo Generators



# 1 Introduction

The body of this thesis is an introduction to the subject of the research. This first chapter gives a background and defines the research. In the following chapter, the matter of engineering design optimization is introduced and discussed from a practical point of view. The third chapter gives a more detailed description of Genetic Algorithms (GAs) which is on class of Evolutionary Algorithms (EAs). Different types of real coded GAs have been used extensively during the work. Then, the dynamical design and analysis of large rotor-bearing systems are dealt with. In this thesis, large turbo systems or large rotor-bearing systems means gas turbines, turbo generators, etc, which usually are supported by hydrodynamic journal bearings. Finally, the appended papers are summarized, discussed and concluded.

The author's aim with the outline of this thesis is that each chapter can be read independently depending of the reader's interest and prior knowledge. A reader with no prior knowledge in the area of rotordynamics and GAs is encouraged to read all the chapters. The purpose of the introductory part of the thesis is to give the reader enough understanding in order to follow the concepts in the appended papers.

## 1.1 Background

The design of high speed rotating machines started in the late 19<sup>th</sup> century by for example the cream separator by De Laval. In the early 20<sup>th</sup> century the development of the steam turbine started. During the 2<sup>nd</sup> world war the development of the turbo jet engine accelerated. Today there is no striking difference between a modern gas turbine for power production and a turbo jet engine from the 1950s. Most of today's high speed rotating machines are based on concepts invented decades ago. This means that most of the development in design of these machines will be small improvements of existing machines. Still this is important since even small improvements in performance of several MW machines may yield substantially increased profit. Perhaps even more important factors in the design of these machines are to reduce costs and increase reliability. The design process of large rotor-bearing systems of today is often an iterative refinement of known concepts. The detailed design process is definitely an iterative procedure or as Rajan et al. in [51] state *"The design of a rotor-bearing system is an iterative process in which the parameters that influence the design are modified until the desired design objectives are achieved"*.

Apparently there is a potential to make use of optimization methods in the design process of rotor-bearing systems. Work with optimization has been done for rotor systems with magnetic bearings. See for example [59], [58] and [57] for further references. A rotor system with ball bearings was studied and optimized by Lee and Choi in [38]. In [45] Montusiewicz and Osyczka optimized a spindle supported by hydrostatic bearings. The eigenvalues of the system are often used as constraints or objectives in the optimization of rotor-bearing systems, [21] and [20]. An optimization with complex eigenvalue constraints and the mathematical model based on the state vector approach is discussed by Chen and Wang in [7]. In [56] Shiau and Chang studied a rotor-bearing problem with multiple objectives.

Various optimization methods have been known for decades. Still the use of optimization techniques in the practical design of rotor-bearing system is limited at industrial companies. An interesting question to pose is: *Why is not optimization techniques*

*used more frequently in industry?* An obvious answer to this question is that it is seldom straight-forward to formulate optimization problems for real-world design problems where several objectives and constraints exist. It may even be difficult to distinguish the objectives and the constraints. Another possible answer to the question is that most traditional optimization methods are not suitable for real-world problems with several non-linear objective functions and constraints that are difficult to satisfy.

In the late 1960s ideas of search and optimization methods based on mechanisms found in natural evolution began to pop up. The natural evolution is slow, and so are most of these methods compared to many other optimization techniques. In the beginning, these methods did not receive much attention. However, due to the rapid development of computers it is now practically possible to apply these methods on many different problem areas. The research about these methods has now become a large field itself. The main cause of this recovery is that many difficult problems can be solved by these methods. In the next two chapters, the background and use of EAs and GAs is described in more detail.

The ideas to use EAs or GAs in the field of rotordynamics are not new. An early application was [29] by Genta and Bassani. Later work with a variant of a GA was done by Choi and Yang, [9] and [10]. In [8] Choi and Yang studied the optimum placement of two eigenvalues for a rotor supported by hydrodynamic journal bearings with a GA. The bearing design parameters were not used as design variables. In [11] Choi et al. optimized several objectives for a low pressure steam turbine with a weighted sum and a variant of a GA. The turbine was supported by hydrodynamic journal bearings and the bearing width and clearance was used as design variables but nothing was said about the actual bearing model.

In most of the previously cited references, the mathematical models of the rotor-bearing systems are too simple to reflect the dynamical properties of large flexible rotor systems. This implies that the formulations of the optimization problems are easy and straight-forward. For a designer working with more advanced models, several problems arise which have not yet been properly addressed. There will for example exist more objectives and constraints. In this work, analysis models are used with an industrial degree of complexity. The purpose is to evaluate and explore the possibilities to use GAs as search methods on this kind of problems. It should be clear that validation and verification of the analysis models are important in order to get reliable results. Validation and verification is however not the purpose for this work. In fact one may instead argue that the accuracy of the models affects how some of the constraints are set. Still the engineer is left with a problem to push the model to the limiting conditions.

Rotor-bearing system optimization problems are, as almost all real-world optimization problems, constrained. GAs have been criticized for the lack of robust and general constraint handling methods. This is also an area that this thesis should contribute to.

## **1.2 Research question**

This research relates to the field of rotordynamics and optimization by EAs and particularly GAs. The focus of the research as well as the research question have changed and evolved during the work. However, the research question that best represents the work is formulated as:



*- How can evolutionary optimization techniques be adapted and applied to improve the design and design process of large rotor-bearing systems?*

### **1.3 Scope and research approach**

This work spans two very different research areas (rotordynamics and GAs). A hypothesis for this work is that GAs are suitable to apply in the late design process of large rotor-bearing systems and that the use of GAs will lead to better performance and more cost effective machines. The aim is therefore to evaluate this hypothesis by studying problems gathered from industry. The work should indicate the potential and limitations of using these modern optimization methods in the late rotordynamic design of large rotor-bearing systems.

### **1.4 Motivation and relevance**

Much of the traditional research within the rotordynamic community is applied and focused on how to model and describe different physical phenomena found in industrial problems. As the knowledge on how to develop models increases, an efficient use of these models becomes more important. At industrial companies working with large rotor-bearing systems the knowledge level is high on how to create models for analysis. On the other hand there is usually a lack of awareness on how to efficiently make use of the models in the product development process. In the best case, some parametric studies are performed. As the number of parameters and objectives increases, it becomes a difficult task for an engineer to optimally design the system. It may even be difficult to define what an optimal system means.

Hopefully, this work leads to an increased knowledge in industry on how to formulate optimization problems and efficiently make use of some modern optimization methods. An increased knowledge in this domain may not only have effect on the detailed design stage but also on other stages in the design process.

## 2 Engineering design optimization

In this chapter the concept of engineering design optimization is introduced. The stand point is more from practical use of search and optimization methods than discussion of theorems and rigorous mathematical proofs. First some concepts are defined and the mathematical definitions of some important engineering design optimization problems are given. Finally practical aspects of engineering design optimization are highlighted and some methods suitable to use in many real-world applications are discussed.

### 2.1 The optimization problem

In this section the mathematical definitions of two important optimization problems are stated. The constrained single objective problem is defined. Then the multiobjective problem and Pareto optimality are defined. But first, some other required concepts are described.

#### 2.1.1 Definitions

Except for the definition of the *feasible region* or *feasible space* in section 2.1.2 the definitions follow [50].

An *objective function* is a function that describes some value of the design. The objective function/functions should be minimized or maximized. *Constraints* are restrictions imposed on the design.

*Design parameters* are data that define the design. *Design variables* are the design parameters that are subject to change during the optimization process. Design variables may be *continuous* or *discrete*. A discrete design variable can only take predefined values and the problem is said to be of combinatorial type. Two types of discrete design variables are *integer type* or *categorical type*. Between categorical design variables there is no defined regular order. A typical categorical design variable is bearing type for example. Bearing width can for example be a continuous design variable if it can take any real value. If the bearing width is chosen from a standard catalogue it may however be an integer design variable.

The *search space* or *design space*, denoted  $S$ , is the set of designs that is spanned by the design variables. For continuous design variables this is usually defined by upper and lower bounds for the design variables called *side constraints*.

#### 2.1.2 The constrained single objective problem

An optimization problem can be formulated as a constrained single objective problem. This means that one single objective is chosen. If there exist other objectives these may be formulated as constraints. This is also called the non-linear programming problem (NLP). With  $k$  inequality constraints and  $m$  equality constraints it is formulated as

$$\begin{aligned}
 &\text{Minimize } f(\mathbf{x}) \\
 &\text{subject to} \\
 &g_i(\mathbf{x}) \leq 0 \quad i = 1, \dots, k, \\
 &h_i(\mathbf{x}) = 0 \quad i = 1, \dots, m,
 \end{aligned} \tag{1}$$

where  $f(\mathbf{x})$  is the objective function,  $g_i(\mathbf{x}) \leq 0$  the  $i^{\text{th}}$  inequality constraint and  $h_i(\mathbf{x}) = 0$  the  $i^{\text{th}}$  equality constraint. Hereinafter minimization is assumed which is no lack of generality since maximization is achieved if the objective is multiplied by  $-1$ .  $\mathbf{x} = [x_1, x_2, \dots, x_n]$  is the vector of the  $n$  design variables. For continuous design variables  $\mathbf{x} \in S \subseteq \mathbb{R}^n$ . The search space  $S$  is usually defined as an  $n$ -dimensional rectangle by the side constraints,  $x_i^l \leq x_i \leq x_i^u \quad i = 1 \dots n$ . The *feasible region* (or *feasible space*) is here defined as the region of  $S$  for which the inequality and equality constraints are satisfied. Hence,  $F \subseteq S$ . The optimal solution is denoted  $\mathbf{x}^*$ . A constraint is said to be active at the point  $\mathbf{x}^*$  if  $g_i(\mathbf{x}^*) = 0$ . By default all equality constraints are active at all points of the feasible space.

In Paper A a new ranking scheme for GAs to handle the constraints in (1) is introduced. The formulation (1) is also used in Paper C and Paper D.

### 2.1.3 The multiobjective problem

In many cases when several design objectives exists it is not possible to formulate the problem as a constrained single objective problem. Another way to pose the problem can be to formulate it as a multiobjective optimization problem. An unconstrained multiobjective problem with  $k$  objectives is formulated as

$$\begin{aligned} &\text{Minimize } \mathbf{F}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x})] \\ &\text{subject to} \\ &\mathbf{x} \in S \end{aligned} \tag{2}$$

where  $S$  is the search space defined as an  $n$ -dimensional rectangle if the design variables are continuous. Of course the multiobjective optimization problem also can be constrained and defined by discrete design variables.

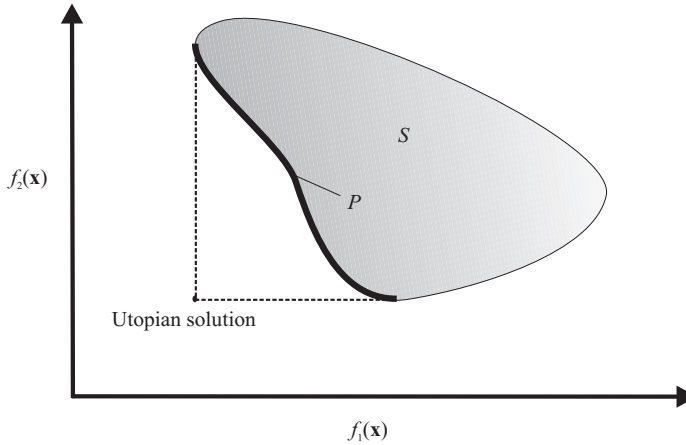
The problem now is to search for solutions which minimize all the objectives  $f_i(\mathbf{x})$ . The ideal solution which minimizes all the objectives simultaneously is called the utopian solution. In the general case when some objectives are in conflict with each other it is however not possible to find the utopian solution. The solution that is searched for should in this case be a member of the non-dominated set of solutions. According to [1] a solution  $\mathbf{x}$  is said to dominate a solution  $\mathbf{y}$  if the following holds:

$$\forall i \in \{1, 2, \dots, k\}: f_i(\mathbf{x}) \leq f_i(\mathbf{y}) \quad \text{and} \quad \exists j \in \{1, 2, \dots, k\}: f_j(\mathbf{x}) < f_j(\mathbf{y}). \tag{3}$$

What Equation (3) says is that a solution dominates another solution if it is better in at least one objective and not worse in the other objectives. With the terminology used here and the definition according to Deb [17] the non-dominated set of solutions is defined as: *Among a set of solutions  $S$ , the non-dominated set of solutions  $P$  are those that are not dominated by any member of the set  $S$ .* This set is also called the Pareto optimal set of solutions and illustrated in Figure 2.1.

A short discussion of possible techniques to solve (3) will be done in section 3.4. Let's just mention that there exist methods that try to find the whole Pareto optimal set in one single optimization run. A multiobjective GA, which is one such method, will be

discussed later. Paper B and Paper E presents rotor-bearing optimization problems of type (2).



**Figure 2.1** Search space  $S$ , Pareto optimal set  $P$  and the utopian solution (assuming minimization) illustrated in the objective space for a problem with two objectives.

## 2.2 Practical aspects of engineering design optimization

In order to achieve a successful final design, a good conceptual design is of course required. Since most products actually are based on re-design and modifications of existing concepts, optimization in the late design stage may be motivated. Optimized products are especially important for high performance, large and expensive or high volume products. An important aspect is that there are always some overhead costs related to performing optimizations in the design process. Before one starts with any optimization one should therefore always pose the question: *Does the potential gain in product performance, cost, etc motivate the use of optimization methods in this particular case?* If the answer is yes or probably one might proceed and start formulating and solving the problem. Worth to note is that the use of optimization techniques is not only restricted to the late design stage. A requirement is however that a mathematical model that describes the value exists.

Most books on engineering design optimization go directly into the details of the algorithms. An engineer working with design optimization is usually more interested to solve the problem than in details of the algorithm. Therefore stable and robust algorithms that work for a large number of problems are motivated. Before even thinking about how to solve it, the engineer is faced with the important challenge to formulate the problem. A few questions and aspects that affect the problem formulation are:

- Which design variables should be chosen?
- What is the objective and what are the constraints?
- Often a mix of design variables exists (continuous, discrete, categorical).
- Almost always several objectives exist. How should these be handled?

In practice, the choice of design variables is often given by the fact that all design parameters are not possible to change. The chosen design variables should be as independent as possible and have effect on the response of the system. Design parameters that only have minor effect on the system response could probably be excluded as design variables.

In reality it is often difficult to decide what are the objectives and what are the constraints? If several objectives exist, the formulation might be a multiobjective optimization problem (2). If some of the objectives can be formulated as constraints, this might be preferable since the problem will in general become easier to solve.

An optimization problem with only discrete design variables is a combinatorial problem with a finite set of solutions. If some design variables are of continuous type, the search space is a set of infinitely many solutions. Many real-world design problems involve mixed types of design variables, for example, such as design of rotor-bearing systems when different bearing types are considered.

Before the formulation of the problem is done it is always important to get as much information as possible about the system. The strategy used by the author in the papers of this thesis is summarized as follows.

1. Chose design variables
2. Perform a numerical experiment
3. Formulate the optimization problem
4. Optimize
5. Post-optimal analysis

The purpose of the initial numerical experiment is to get an understanding of the design variables effect and to get an overview of what possibly can be achieved in the optimization. It may for example also show that some design variables are not relevant and therefore can be excluded. Design of experiments (DOE) is a large field, see Montgomery [44]. In Paper B, Paper C and Paper D a simple but informative method is used. Solutions are simply generated randomly in the search space. Even though more advanced methods exist this method should not be neglected since it may serve its purpose for large and complex problems with many constraints and possibly several objectives.

When an optimal solution is found it is of interest to know how robust the solution is. This is investigated in the post-optimal analysis phase. A robust solution is a solution that gives only small effects in response for perturbations in the design parameters. In the case of a multiobjective problem, robustness may well be the base for the final decision of which solution to choose from the Pareto optimal set.

## **2.3 General purpose optimization methods**

The purpose for this section is to present a short review of some of the most important methods that can be an alternative to EAs in practical engineering design optimization. It should be clear that a comprehensive survey of all existing optimization methods is far beyond the scope for this section. The interested reader is therefore encouraged to read the introductory book by Onwubiko [50] or the more comprehensive book by Rao [52].

Traditionally there has been a large focus on methods that use gradient information in the search process, i.e. gradient based methods. Gradient based methods are efficient for convex objective functions and if the number of design variables is not too high. A drawback that frequently appears for objective functions with multiple local optima is that the result is dependent on an initial start guess. Engineering design problems often involves non-convex objective functions or even disjointed search spaces if discrete design variables are involved. The focus in this section is therefore towards methods that are better suited for this kind of problems. Still one should not reject the use of gradient based methods on problem with continuous design variables and convex objective functions.

A subject closely connected to engineering design optimization is DOE, Montgomery [44]. Before an optimization is conducted it is necessary to have some knowledge of the behavior of the system. This is where DOE come in. DOE may also be used in the post optimal analysis phase. Methods related to DOE and robustness of the system is response surface methods (RSM) [62]. In these methods an initial experiment is set up and conducted. The objective function is then usually approximated by a second order polynomial response surface. The experiment may be repeated to obtain a more accurate response surface. These methods are suitable if the number of design variables is low. Another more advanced approximation technique for problems with costly and noisy objective functions is to train neural networks [34] to simulate the behavior of the system.

There are different ways to classify search and optimization methods. In [33] Hajela reviews some non-gradient based search and optimization methods. He also distinguishes between zero-order methods for local search and methods for global search. A selected set of these methods is listed in Table 1.

**Table 1.** Non-gradient based search and optimization methods classified according to Hajela [33].

	Zero-order methods for local search	Methods for global search
Deterministic	Hookes-Jeeves [50]	Sequential quadratic programming [50]
	Nelder-Mead simplex [50]	-
	Complex [5]	-
Stochastic	Random walk [33]	Simulated Annealing (SA) [37]
	-	Evolutionary Algorithms (EAs)

Zero-order methods use only the objective function value and no gradient information. The progress in the optimization process for the deterministic methods is based on predefined rules. In the stochastic methods, a certain amount of randomization is used in the search process. These methods are computationally expensive in terms of the many required objective function evaluations but they are also robust. During the past decade stochastic global search methods has become more and more important as tools in engineering design optimization problems. In [33] Hajela says that *“These methods have emerged as potent tools for locating optimal designs in problems that are generally regarded as difficult.”* Simulated Annealing (SA) is based on ideas from statistical mechanics and thermodynamics. If a metal piece is slowly cooled from an initially high temperature it takes the state which minimizes the potential energy. This is completely analogous to the

working principles of SA. EAs are based on principles found in natural evolution. The background of these methods is described in some more detail in the next section.

It is worth to note that there exists no “best method” for all cases. Even in a single case, a best method may not exist. In the general case, probably the best choice is a hybrid method that starts with a global search method to locate the interesting region of the search space followed by a fast local search method.

## **2.4 Evolutionary Algorithms**

EAs is a class of global search algorithms inspired by natural evolution. Several different types of EAs exist. Genetic Algorithms (GAs), Evolution Strategies (ES), Evolutionary Programming (EP) and Genetic Programming (GP) are some of the most known. In this thesis, real coded GAs (explained in section 3.1) have been used extensively. Real coded GAs are in many aspects similar to ES. Therefore a short review of the history for GAs and ES is given in the next section.

### **2.4.1 Historical perspective**

During the early 1970s Holland [36] and his students presented the first work in the field of GAs. At almost the same time, Rechenberg [53] and colleagues worked with ES. ES and GAs are quite similar but have important differences. ES work with real design variables while GAs work with a binary coding of the design variables, see section 3.1. Furthermore, the search operator in ES is a mutation operator. In a GA this is a secondary operator since the primary search operator is a crossover operation. During the 1980s real coded GAs began to pop up. At this point it becomes difficult to distinguish between ES and GAs. In chapter 3 the background and principles of GAs are given in more detail.

### **2.4.2 Why and when to use EAs?**

The natural evolution is a slow process. Hence, EAs are also quite slow compared to many other optimization algorithms, i.e. on continuous and convex functions. But it is also important that they are robust algorithms. EAs are a good choice for multimodal and noisy objective functions. It is also shown that EAs are suitable on many combinatorial problems with large search spaces. If the number of design variables is high, EAs may be one possible choice. EAs may also be the choice if there is a mix of different types of design variables. EAs should not be used if local search efficiency is of importance, nor should they be used when the objective function evaluations are computationally expensive.

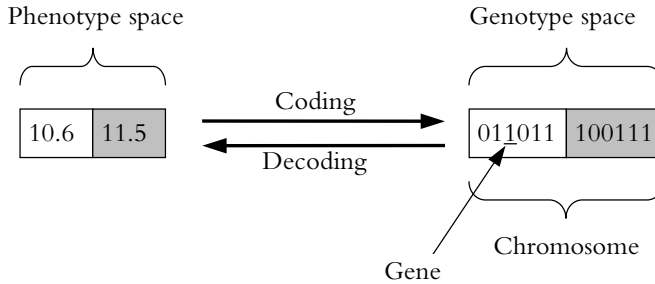
### 3 Genetic Algorithms

The purpose of this chapter is to introduce the reader into the basics of GAs. Since GAs is a huge field itself, it should be clear that what will be presented here are only selected parts which the author finds most necessary.

GAs is one type of EAs which originates from the work by Holland [36] in the early 1970s. Two other good text books on the subject are [31] by Goldberg and [16] by Davis. Traditionally GAs works with binary coding of the problem. This will be explained in the next section. Nowadays there exist GAs that work on data structures which are more similar to the specific problem, for example real numbers. The book by Gen and Cheng [27] gives some introduction to this subject. Two more recent comprehensive books that may serve as references of the fundamentals of GAs and especially multiobjective GAs are [14] by Coello Coello et al. and [17] by Deb.

#### 3.1 Coding of design variables

Coding of the problem is the first central step to understand how GAs work. Let's now consider a possible vector of design variables to an optimization problem with two continuous design variables,  $\mathbf{x} = [10.6, 11.5]$ . These two real numbers can be described (coded) as a string of binary digits. The chosen length of this string and range for the design variables (side constraints) determines the precision of the decoded values. If a string length of a single design variable is 6 and the side constraints are  $9.0 \leq x_i \leq 13.0$  the binary coding of the considered vector of design variables becomes like in Figure 3.1. See for example [17] for how to binary code real valued design variables.



**Figure 3.1** An example of binary coding of two real design variables.

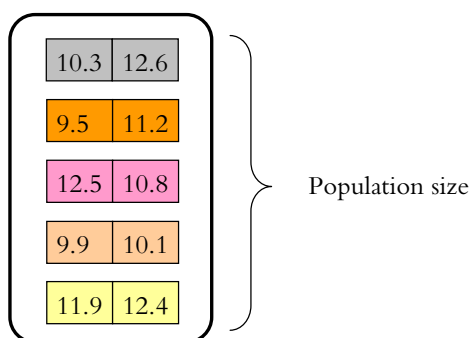
The string of binary digits is called a chromosome. Sometimes it is also called genome. Each single binary digit in the chromosome is a gene. All the operations in a GA are performed on the genotype representation of the design variables. There exist GAs which operate on the design variables in the phenotype space directly, hence genotype = phenotype. For real valued design variables these are said to be real-number encoded, [27]. However, in this case actually no coding and decoding is performed at all. Evaluation of the objective functions and constraints is done for the phenotype representation of the design variables.



If the underlying problem easily is described by real valued design variables, the real-number encoding is preferable. This since a binary encoding in this case gives severe drawbacks with Hamming cliffs, [27]. In most cases it is preferable to choose a coding that gives a genotype representation which is as similar as possible to the phenotype representation.

### 3.2 The population

One of the most important things about GAs, in contrast to many other optimization methods, is that they work with a set of solutions to the problem (a set of chromosomes). Later it hopefully will become apparent why it may be an advantage to work with several solutions to the problem simultaneously. The set of solutions is called the population. Each chromosome in the population is an individual. Before the evolution of the population can start, a population must be initiated. Most often this is done by random generation of individuals. Hence, no guess of a single start point is necessary. If a real-number encoding is assumed, a possible population for the hypothetical problem described by two real design variables is illustrated in Figure 3.2.



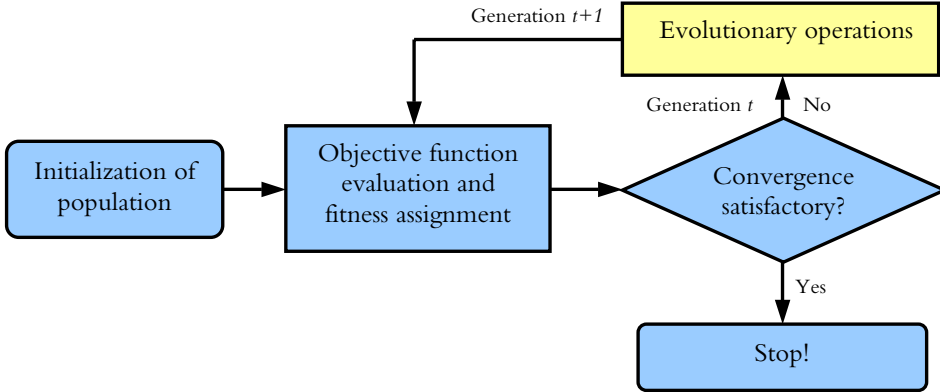
**Figure 3.2** A population with size 5 for a problem described by two real design variables.

The uniqueness of each individual in the population in Figure 3.2 is here illustrated by the fact that each individual has a unique color. The size of the population, i.e. the number of individuals in the population, is an important parameter in a GA. A large population size gives a more robust search in terms of finding the global optimum. However, a large population size also slow down the convergence rate. A good choice of population size is dependent of the problem in order to achieve a well behaving GA. It should also be mentioned that the performance of a GA depends on the choice of other operators. So the choice of population size is also coupled to the choice of these operators.

### 3.3 Evolution of the population

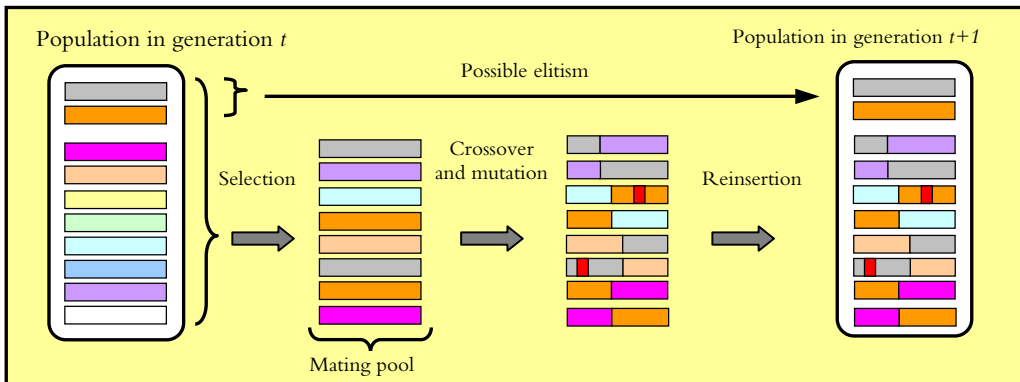
Now when the concept population is defined it is time to explain the evolutionary process. The individuals in a population will undergo different operations inspired by the natural evolution. Let's call the population generation  $t$  at one instant in time. After the evolutionary operations have been performed a new population has been created. This new population is generation  $t+1$ . In the new generation there exist new individuals that

have been produced based on the information from its parents in the previous generation. If an elitist strategy is used, the new generation can also contain individuals that have survived from the previous generation. The overall working principles of a GA are shown in Figure 3.3 and Figure 3.4.



**Figure 3.3** Flowchart for the working principles in a GA.

First the population is initiated, usually by random generation of individuals. The next step is to evaluate the objective function/functions and possibly constraint violations. The objective function is of course evaluated for the phenotype representation of the individuals. Then each individual is assigned a fitness value. This is a measure of how good the individuals are, the higher fitness the better individual. Observe that the fitness value is not necessarily the same as the objective function value. Then a convergence criterion is evaluated. This can be a predefined number of generations, a predefined minimum difference to a goal value or something similar. For the work in this thesis, the GAs are always run for a predefined number of generations. If convergence is not satisfactory, the evolutionary operations are repeated. By the term “evolutionary operations” is here meant selection, crossover, mutation and reinsertion. A more detailed illustration of the evolutionary operations is shown in Figure 3.4.



**Figure 3.4** Operations performed in the evolutionary process in a GA.

Figure 3.4 shows the performed operations on a population in order to create the next generation. Each colored rectangle represents a unique individual in the population. At this point it does not matter if a real coding or a binary coding is used.

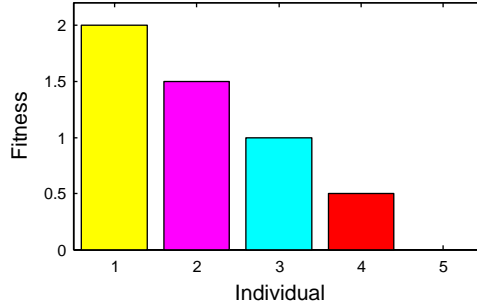
After fitness has been assigned to each individual in generation  $t$ , a portion of these individuals are selected by some method for mating. This new intermediate set of individuals is called the mating pool. There exist many different selection operators of which some are explained in section 3.3.2. The selection operator should prefer “good” individuals rather than “bad” individuals. In the multiobjective case or constrained single objective case, it is not straight forward to define what a “good” or “bad” individual actually is. This question is dealt with in Paper A for the constrained single objective optimization problem (1).

The genes from pair-wise chosen individuals in the mating pool (the parents) are then mixed in what is called the crossover operation. Two new individuals (the offsprings or children) are now created. The probability for crossover to occur is usually high (around 0.9). The crossover operation is illustrated by the mixing of colors in Figure 3.4.

At low probability the genes are then mutated. This is illustrated by the red color in Figure 3.4. Now a portion of the population in generation  $t$  is replaced by the new offsprings and generation  $t+1$  is created. The elitism showed in Figure 3.4 is simply to copy some of the best individuals in generation  $t$  into the next generation. In the appended papers the term “generation gap” occurs. This is the ratio of the population that is replaced in each generation, i.e. the size of the mating pool. Elitism can also be introduced via the reinsertion scheme. It should be mentioned that there are large differences how the crossover operator and mutation operator works depending on how the design variables are coded (binary or real). In the next sub sections these different operations will be described in some more detail.

### 3.3.1 Fitness assignment and ranking

In this sub section it is assumed that the unconstrained single objective problem is considered. This implies that the term “good individual” is obvious. The fitness of an individual is a measure of how good an individual is, the higher fitness, the better individual. Fitness can be assigned proportionally to the objective value but usually the fitness is determined by the ranking of the current population, i.e. rank-based fitness assignment. In rank-based fitness assignment the population is first sorted according to the objective value. Then fitness is assigned according to some scheme (linear or non-linear). Rank-based fitness assignment is in general more robust since no scaling problem with the objective value occurs, [6]. A frequent problem with proportional fitness assignment is that good individuals receive too high fitness which may lead to premature convergence in the population. This can be avoided with rank-based fitness assignment. Figure 3.5 shows the fitness for each individual in a ranked population with five individuals.



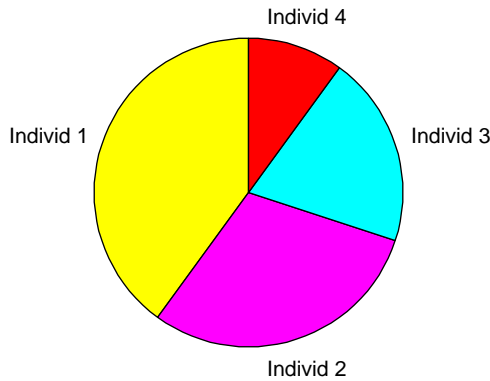
**Figure 3.5** Linear fitness (selective pressure 2.0) for a ranked population with 5 individuals.

In Figure 3.5 a linear fitness assignment with selective pressure 2.0 is used. This means that the fitness is assigned according to a linear distribution. The best individual (No. 1 in Figure 3.5) gets a fitness value of 2.0 and the worst individual gets 0. Selective pressure 2.0 means that the best individual should have twice as high probability to be selected for mating compared to the average individual.

### 3.3.2 Selection

When fitness is assigned to each individual, some of these should be selected to be a member of the mating pool. The selection criteria should preferably be based on the fitness values in some way. There exist many different selection procedures, tournament selection [32], stochastic universal sampling [3], roulette wheel selection, etc. In the work of this thesis the roulette wheel selection method is most frequently used and is therefore briefly explained here.

Let's first imagine a roulette wheel that has as many slots as there are individuals in the population. The size of each slot should be proportional the corresponding fitness. Such a wheel is shown in Figure 3.6.



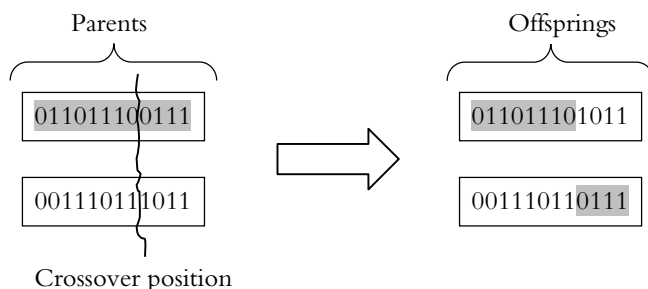
**Figure 3.6** Roulette wheel with slot sizes proportionate to the assigned fitness.

Now the ball is thrown. The individual on which the ball hits is chosen for mating. This procedure is repeated until the whole mating pool is full. Observe that there is more likely to find multiple copies of individuals with high fitness rather than individuals with low fitness in the mating pool. The fifth individual gets no chance to reproduce since its fitness value was zero.

### 3.3.3 Crossover

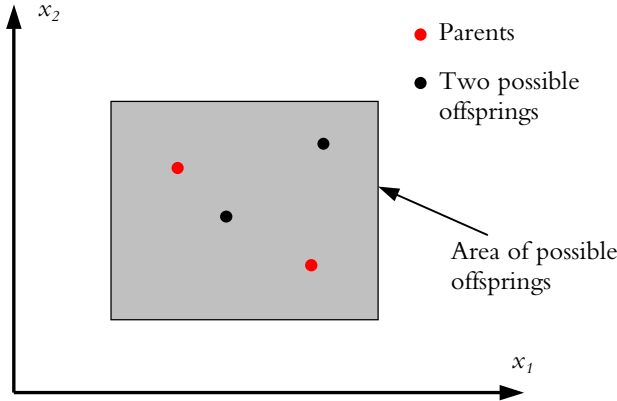
Crossover is the operation that mixes the genetic information from parents in the mating pool in order to create offsprings. The term “crossover” originates from the early binary coded GAs and is therefore straight-forward to understand for binary coded design variables. In the case of real coded design variables, crossover is a more awkward concept. In this case it is better to refer to blending operations of the design variables rather than crossover. Still the term crossover is used for real coded design variables in many text books, so also in this thesis.

First, a crossover operation (single point) for binary coded design variables is explained. A pair of individuals (the parents) is picked from the mating pool. Then the chromosomes of the parents are divided at a randomly generated crossover position. Now the first part of the chromosome from parent 1 goes to offspring 1, the second part goes to offspring 2. For parent 2 the reverse holds. This is illustrated in Figure 3.7.



**Figure 3.7** Single point crossover for binary coded design variables.

Crossover operators used for real coded design variables tries to create one or more offsprings in the neighborhood of its two parents. In contrast to the single point crossover operator, most of these operators work with a single gene (a design variable in this case) at a time rather than the whole chromosome. What mainly differ different crossover operators for real coded design variables is the probability distribution for creation of the offsprings. For example the simulated binary crossover (SBX) by Deb and Agrawal [18] will at high probability create an offspring with design variables close to similar to one of its parents. The SBX crossover operator is used in Paper E. In Paper A, Paper C and Paper D the BLX- $\alpha$  crossover operator is used, [22]. For this operator the offsprings are created with uniform probability within a hyper cube slightly larger than the hyper cube spanned by the two parents. This is illustrated for two design variables ( $x_1$  and  $x_2$ ) in Figure 3.8.  $\alpha$  is a constant that defines the size of the hypercube for possible offsprings.

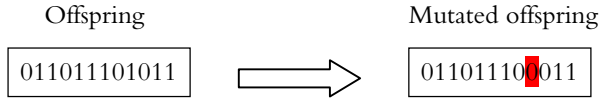


**Figure 3.8** The BLX- $\alpha$  crossover for two a real valued design variables.

Again it should be mentioned that what is presented here is only a few of all existing crossover operators. In a GA selection and crossover are the main search operators. In order to increase the robustness of a GA (in terms of chance to find the global optimum), mutation is introduced. This is the subject for the next sub section.

### 3.3.4 Mutation

When the offsprings have been created, these may at low probability be mutated. Mutation in this case means a random change of the genes in the offsprings chromosomes. In the case of binary coded design variables this simply is to flip 0 to 1 or vice versa as shown in Figure 3.9.



**Figure 3.9** Binary mutation of a gene in an offspring's chromosome.

Also when it comes to mutation of real coded design variables there exist a large number of different mutation operators. Most of them work in such way that the variable that will be mutated is changed to a new value in the neighborhood of its original value. One such mutation operator could be to add a zero-mean Gaussian probability value to the original value as in Equation (4).

$$x_i^{mutated} = x_i^{old} + N(0, \sigma_i) \quad (4)$$

In Paper A to Paper D the mutation operator by Mühlenbein and Schlierkamp-Voosen [46] is used. This works in a similar way as Equation (4) but with a different probability distribution. It also generates the mutated variable in a closed predefined range. Different papers report results for optimal mutation probability. In [46] Mühlenbein and

Schlierkamp-Voosen used the mutation probability  $1/n$ , where  $n$  is the number of design variables, so this seems to be a good value for a variety of problems. For an overview of many crossover and mutation operators for real coded GAs, see Herrera et al. [35].

### 3.3.5 Reinsertion

The last step before a new generation is created is to insert the offsprings into the population and replace some of the parents. This may be done in many different ways. The most obvious one is to replace parents that have low fitness (fitness based reinsertion). This is the reinsertion scheme used for the GAs in Paper A to Paper D. Another scheme may be to replace a parent in the neighborhood of the offspring that is to be inserted (local reinsertion). Depending on how the reinsertion is done, the GA may behave very differently. But as always, the behavior of the GA is dependent on all the performed operations.

## 3.4 Multiobjective GAs

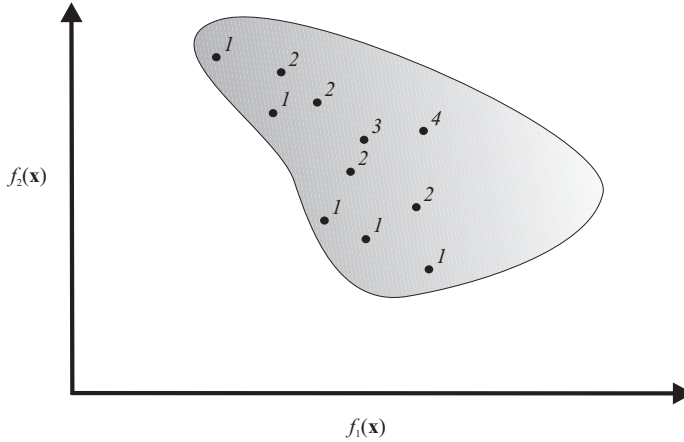
It has already been mentioned that a constrained problem and a multiobjective problem may be two different formulations of the same underlying problem. Hence, multiple objectives and several constraints are somehow related. In Paper A an alternative constraint handling method based on concepts found in multiobjective GAs is presented. In Paper B and Paper E multiobjective generator rotor-bearing problems are studied and solved with different multiobjective GAs. Therefore a brief introduction to non-dominated multiobjective GAs is given in this section.

There exist several different methods [1] to handle the multiobjective problem (2). Probably the most widespread approach is the weighted sum where each objective is assigned a weight and added together into a single objective function. In this way the preferences of the different objectives are set before the optimization is started. The searched solution is most likely a single solution of the Pareto optimal set. Another method is the e-constraint approach where objectives are directly treated as constraints. If there is no preference between the objectives, each solution in the Pareto optimal set is equally good as all other members of the Pareto optimal set. Hence, all solutions in the Pareto optimal set are searched for. It is possible to find this set with a weighted sum or an e-constraint method but the optimization must be run several times with different weights or constraint levels. This is where GAs has an advantage.

Since the search is done with a set of solutions (the population) it is possible to evolve the whole Pareto optimal set in a single optimization run. During past 10–15 years the research about multiobjective GAs has considerably increased. For a complete background into the subject of multiobjective GAs the books by Coello Coello et al. [14] and Deb [17] are recommended.

The most central operation in a multiobjective GA is the ranking of the individuals. Often this is done by some sort of non-dominated ranking. Figure 3.10 shows the non-dominated ranking by Goldberg [31] for a hypothetical population of solutions to a problem with two objectives. In Goldberg's ranking scheme, the non-dominated individuals in the current population are identified. These individuals receive rank 1 and are removed from the population. The non-dominated individuals in the remaining population are then assigned rank 2. This scheme is repeated until all individuals have been assigned a ranking. Goldberg's ranking is one ranking scheme based on non-

domination. Another similar ranking scheme for multiple objectives is the one by Fonseca and Fleming used in the MOGA, [25]. Preferably the information from non-dominated individuals should not get lost. Therefore a considerable degree of elitism is used in many multiobjective GAs.



**Figure 3.10** Goldberg's non-dominated ranking for a set of hypothetical solutions (a population) to a problem with two objectives (minimization is assumed).

Non-dominated ranking is not the only key for a successful multiobjective GA. If the final generation should be more or less evenly spread near the Pareto set, a diversity preserving operator must be used. In [25] Fonseca and Fleming used a sharing method (found in [27]) in order to obtain spread solutions. This simply means that the fitness for clustered solutions is degraded so that diverse solutions are more likely to be preferred. The same sharing method is applied in the multiobjective GA used in Paper B. The NSGA-II by Deb et al. [19] uses non-dominated ranking and a special crowded tournament selection operation for diversity preservation. The NSGA-II is used in Paper E.

### 3.5 Constraint handling

In an unconstrained single objective problem the ranking of individuals is trivial. When constraints are introduced this task becomes trickier. For example: An individual that not satisfies the constraints may still have a good objective value and contain important genetic information. Should this individual then just be low ranked and probably rejected for mating? Paper A presents an alternative ranking scheme where this question is addressed. The method is shortly explained later in this section but first another widespread method is presented.

A comprehensive survey is given by Coello Coello in [13] of the wide variety of existing constraint handling methods for EAs. Probably the most popular technique is to use penalty functions. In a penalty method a term is added to the objective function if a constraint is not satisfied. One such method for problem (1) is described by



$$\phi(\mathbf{x}) = f(\mathbf{x}) + \sum_{i=1}^k r_i \cdot G_i + \sum_{i=1}^m c_i \cdot H_i \quad (5)$$

where the penalty functions can be

$$G_i = [\max(0, g_i(\mathbf{x}))]^\beta, \quad (6)$$

$$H_i = |h_i(\mathbf{x})|^\gamma. \quad (7)$$

The parameters  $r_i$ ,  $c_i$ ,  $\beta$  and  $\gamma$  are all parameters that should be set. Penalty methods are generic but a major drawback is that many parameters need to be given good values in order to achieve a well performing GA. Some guidelines for penalty methods are given by Richardson et al. in [54]. The penalty method described by Equation (5) to Equation (7) is one of many different variants. There are for example other methods with dynamic penalty parameters and different penalty functions. In [40] several different penalty methods for GAs are compared.

In [12], [41] and [26] it is indicated that the constrained optimization problem may be handled as a multiobjective optimization problem. Since many multiobjective GAs have been quite successful during the last decade this seems to be a promising idea. To treat constraints as objectives via (6) and (7) and directly apply a multiobjective GA based on non-domination would however not be efficient. This since compromise solutions of constraint satisfaction and minimum objective values are then searched for. Instead, the solution that minimizes the objective and satisfies the constraint is the solution that should be searched for.

In Paper A an alternative generic constraint handling method is introduced. First the constraints are formulated as objectives, similar to Equation (6) and (7). Then these new objectives are ranked according to Goldberg's non-domination ranking (Figure 3.10) and the function defined  $rank_2(\mathbf{x}_i)$  returns this ranking for the  $i^{th}$  individual in the current generation. A function defined  $rank_1(\mathbf{x}_i)$  gives the ranking according to the original objective. Fitness is then assigned in a regular manner according to the new objective function

$$\phi(\mathbf{x}_i) = \frac{N}{P} rank_1(\mathbf{x}_i) + \frac{P-N}{P} rank_2(\mathbf{x}_i) \quad (8)$$

where  $P$  is the population size and  $N$  is the number of feasible individuals in the current population. Observe that if no feasible individual is present ( $N = 0$ ), only the ranking according to the constraints is active. If all individuals are feasible ( $N = P$ ), the population is ranked only according to the objective. This gives a dynamic behavior of the GA. If the feasible region is found, the population can oscillate and explore the boundary of the feasible region if the global optimum is located outside the feasible region.

So, why propose another constraint handling method? One strong argument is that the method does not require any tunable parameters. Furthermore, in Paper A it is shown that this method is more robust than many other penalty based methods. It is also used in Paper C and Paper D with good experiences.

### **3.6 Some remarks on GAs**

So far an introduction of how GAs work has been given. Nothing is however yet said about why they work. Goldberg has in [31] explained why binary coded GAs works with the discussion about schemata. Yet there exist no solid proofs for convergence for real coded GAs. Still it is empirically shown by all successful applications that GAs are interesting methods for difficult real-world problems. Much of the current research about GAs (for example Paper A) is experimental.

As mentioned earlier, one of the most important features of a GA is that the search is done with a set of solutions. This also gives some interesting possibilities where one is that it is possible to evolve an approximation to the Pareto optimal set of solutions in a multiobjective problem. Another is that it is possible to locate several local optima if niching is introduced in a single objective multi modal problem. The knowledge of several local optima is important if the robustness of the solutions is considered. It may be better to choose a locally optimal solution than the global optimal solution if the former is more robust. Another interesting thing about GAs and robustness of solutions is that it is possible to evolve only robust solutions and reject solutions from narrow optimum. This can be done by disturbing the design variables or other parameters during the evolutionary process [1]. The population based model also implies that GAs are inherently parallel. This since the objective functions to be evaluated for one single generation may be evaluated in parallel. The GA used in Paper D is parallelized and run on a Linux cluster of standard PCs.

## 4 Rotor-bearing system analysis and design

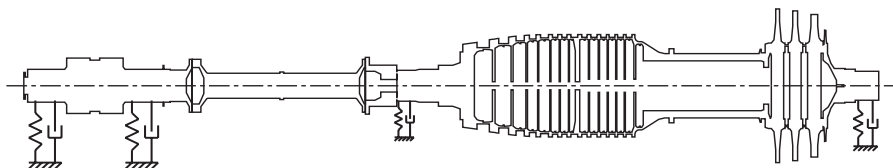
The purpose of this chapter is to present some methods to analyze flexible rotor bearing systems. In Paper E the balancing of turbo generator rotors is considered, therefore an introduction to balancing of flexible rotors will also be given in this chapter. Finally the design process and analysis of large rotor-bearing systems is introduced and discussed.

### 4.1 Rotordynamic analysis

This thesis deals with the optimization of turbo generator and gas turbine rotor-bearing systems. The rotors in these systems are long compared to their diameters. This implies that they must be handled as slender and flexible rotors. Since the geometry of these rotors is too complex in order to be handled by continuous models, approximate discretized models need to be used. This section deals with the analysis of multi DOF (degree of freedom) models. For a deeper insight into the fundamentals of rotordynamics the reader is referred to the books by Genta [28], Vance [61] and Yamamoto and Ishida [60].

#### 4.1.1 Discretized rotor-bearing systems models (multi DOF models)

An example of a rotor-bearing system that must be analyzed using a discretized model is shown in Figure 4.1.



**Figure 4.1** Gas turbine rotor with intermediate shaft, pinion and bearings.

In a discretized model, the rotor is divided into a discrete number of elements where each node usually is described by two translational and two rotational degrees of freedoms. The equations of motion of a single element may be obtained by for example the law for conservation of momentum or Lagrange's equation. The transfer matrix method (found in [28]) has traditionally been frequently used in rotordynamics, for example to calculate the critical speeds. The advantage of this method is that it is fast since large matrix operations are not required. The involved matrix has the same size as the matrices at the element level.

Another popular method to discretize and handle differential equations for continuous systems is the finite element method (FEM). FEM formulations and application to the dynamics of mechanical systems is found in [39] and [15]. A formulation for rotordynamics that also handles the gyroscopic effect is given by Nelson and McVaugh in [47]. In FEM the displacement field is approximated by certain shape functions within each element. Then the element matrices are formulated using an energy approach. Finally, the system matrices are assembled from the element matrices. The size of the system matrices is the same as the number of degrees of freedoms.

A third approach is the so called lumped parameter method. In this approach the continuous system is approximated by rigid bodies (the nodes) coupled by springs and dampers. In fact the transfer matrix method is a sort of lumped parameter method. In a lumped parameter method it is straight-forward to obtain the inertia matrix (mass matrix) but the stiffness matrix may be more difficult to find. A possibility is also to mix the lumped parameter approach and FEM. This is usually done in FEM and modal analysis in order to obtain the stiffness matrix and a diagonal inertia matrix.

It should be mentioned that the rotordynamical analysis in the appended papers is done with an in-house code based on the transfer matrix method. However, hereinafter in the thesis it is assumed that either a FEM approach or a lumped parameter method is used to obtain the system matrices.

Assume now that  $\mathbf{q}$  is a displacement column vector in real coordinates that defines the position of each node (two translational DOFs and two rotational DOFs per node) with respect to a fixed reference frame. Then the assembled matrix equation of motion for a discretized rotor system is formulated as

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{\Omega G} + \mathbf{C})\dot{\mathbf{q}} + \mathbf{Kq} = \mathbf{f}(t). \quad (9)$$

The rotational frequency is  $\mathbf{\Omega}$  and the inertia matrix  $\mathbf{M}$ . If a lumped approach is used, this matrix is diagonal and if a consistent approach is used it is a banded matrix.  $\mathbf{C}$  is the damping matrix and  $\mathbf{G}$  is the skew-symmetric gyroscopic matrix. The stiffness matrix for the shaft and the supports is  $\mathbf{K}$ . The right hand side of Equation (9) describe the forces acting on the system.

If the rotor is supported by journal bearings the stiffness matrix can be non-symmetric which implies that instability can occur. This will be discussed later. The exciting force considered in the appended Paper D and Paper E is mass unbalance. Furthermore, the only contribution to the damping matrix  $\mathbf{C}$  in the appended papers comes from damping in the bearings and supports. In Figure 4.1 the supports are illustrated by springs and dampers but it should be mentioned that Equation (9) is not restricted to these support models. More advanced modeling of the stator structure is possible. If a FEM approach is used, the question is how to discretize the stator or support structure, choose element formulation and how to assemble the matrices. If the number of degrees of freedoms becomes too high, one possibility is to use a reduction technique to reduce the computational effort in the analyses of Equation (9), Genta [28].

#### 4.1.2 Analysis by the state vector approach

There are different ways to analyze Equation (9). Here the state vector approach is used. The state vector approach is often used in control of dynamical systems or numerical simulations. It is also an interesting method for analysis of problems of type (9) since the presence of damping and the gyroscopic matrix are easily handled. The state vector approach is general since it gives the most general form of eigenvalue problem for mechanical systems. Furthermore, it does not require any particular shape of the matrices as long as the mass matrix is invertible. The mass matrix can for example be lumped or consistent.

Let's first assume that the number of degrees of freedom is  $N$ . Hence, the size of the matrices in (9) is  $N \times N$ . Now a new coordinate vector  $\mathbf{x}^T = \{\mathbf{q}^T, \dot{\mathbf{q}}^T\}$  is created. Then

Equation (9) can be expanded to a set of  $2N$  differential equations of first order illustrated by the matrix equation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b} \quad (10)$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}(\Omega\mathbf{G} + \mathbf{C}) \end{bmatrix} \text{ and } \mathbf{b} = \left\{ \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{f}(t) \end{bmatrix} \right\}. \quad (11)$$

The total solution to the differential Equation (10) is the sum of the homogeneous solution and the particular solution. First the homogeneous solution is considered, i.e. the solution to Equation (10) if  $\mathbf{b} = \mathbf{0}$ .

If a homogeneous solution on the form  $\mathbf{x}_h(t) = \mathbf{z}e^{\lambda t}$  is assumed and inserted into Equation (10) the eigenvalue problem  $\mathbf{A}\mathbf{z} = \lambda\mathbf{z}$ ,  $\mathbf{z} \neq \mathbf{0}$  arise. The eigenvalues are  $\lambda_j$  and the corresponding eigenvectors are  $\mathbf{z}_j$ . Since  $\mathbf{A}$  is non-symmetric the eigenvalues and eigenvectors appears in complex conjugate pairs as

$$\begin{aligned} \lambda_j &= \alpha_j + \beta_j i, \quad \lambda_j^* = \alpha_j - \beta_j i \\ \text{and} \\ \mathbf{z}_j, \quad \mathbf{z}_j^* \end{aligned} \quad (12)$$

according to Inman [24]. It should be mentioned that  $i = \sqrt{-1}$ . The complex eigenvectors are also referred to as complex mode shapes. Sometimes these are referred to only as modes in this thesis. The homogeneous solution to Equation (10) is then

$$\mathbf{x}_h(t) = \sum_{j=1}^N \left( c_j \mathbf{z}_j e^{\lambda_j t} + c_j^* \mathbf{z}_j^* e^{\lambda_j^* t} \right). \quad (13)$$

The solution to Equation (13) is real. The complex conjugate constants  $c_j$  and  $c_j^*$  are determined by initial conditions.

The homogeneous solution is seldom of interest in the analysis and design of large rotor bearing systems. More interesting is however the complex eigenvalues and the physical interpretation of these. In the underdamped case,  $\beta_j$  in (12) is the damped natural frequency for the  $j^{\text{th}}$  mode according to Inman, [24]. The corresponding undamped natural frequency and modal damping ratio associated with the  $j^{\text{th}}$  mode are

$$\omega_j = \sqrt{\alpha_j^2 + \beta_j^2} \quad \text{and} \quad \zeta_j = \frac{-\alpha_j}{\sqrt{\alpha_j^2 + \beta_j^2}}. \quad (14)$$

A negative damping ratio ( $\alpha_j$  is positive) implies that the mode is unstable and the amplitude of vibration grows exponentially in time. In the next section the interpretation

of the concepts complex eigenvalues and complex modeshapes are discussed in some more detail.

In Paper B to Paper E the complex eigenvalues of the rotor-bearing systems are considered. In these papers the term *complex eigenvalues* is sometimes mixed with the natural frequencies and damping ratios. However, from Equation (12) and (14) it should be clear how these concepts are related for the underdamped case (which is almost always the case for large rotor bearing systems).

In Paper E the term *root locus* plot is used. The root locus plot is a plot of the complex eigenvalues in the complex plane or a plot of the damping ratios vs. natural frequencies. The root locus plot is usually used to study how the stability and natural frequencies changes as a parameter is varied.

Now when the homogeneous solution is given the *particular solution* (or *steady-state solution*) is discussed. The force acting on the system is here assumed to be the mass unbalance force. This is the only force considered in the appended papers. The particular solution to Equation (9) or (10) then describes the motion of the rotor if it is run at constant speed for a long time so that the transient has died out (it is assumed that all modes are stable). The mass unbalance force can be described as

$$\mathbf{f}(t) = \mathbf{f}_c \cos(\Omega t) + \mathbf{f}_s \sin(\Omega t) \quad (15)$$

which implies that a particular solution of the form

$$\mathbf{x}_p(t) = \mathbf{c} \cos(\Omega t) + \mathbf{s} \sin(\Omega t) \quad (16)$$

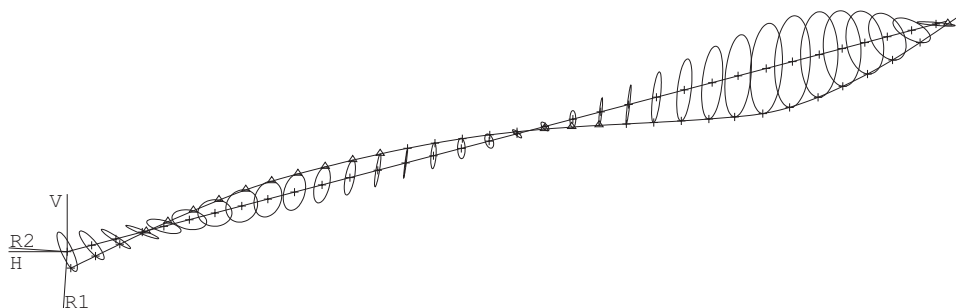
can be assumed. Combining Eq. 15 and Eq. 16 with Eq. 10 and Eq. 11 gives the solution

$$\mathbf{c} = -\left[\Omega \mathbf{I} + \frac{1}{\Omega} \mathbf{A}^2\right]^{-1} \left\{ \frac{1}{\Omega} \mathbf{A} \mathbf{b}_c + \mathbf{b}_s \right\} \text{ and } \mathbf{s} = \frac{1}{\Omega} \{ \mathbf{A} \mathbf{c} + \mathbf{b}_c \} \quad (17)$$

where

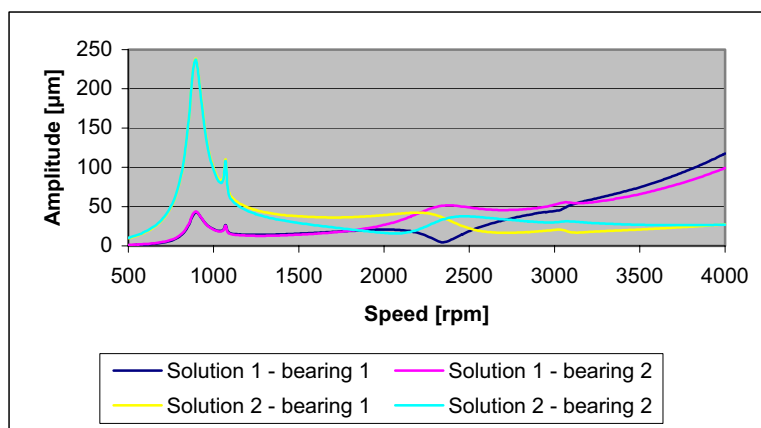
$$\mathbf{b}_c = \begin{Bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \mathbf{f}_c \end{Bmatrix} \text{ and } \mathbf{b}_s = \begin{Bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \mathbf{f}_s \end{Bmatrix}. \quad (18)$$

The motion of a point on the rotor is an elliptical whirling orbit described by Eq. 16. Figure 4.2 shows an example of the whirling orbit due to a mass unbalance for a rotor studied in Paper E.



**Figure 4.2** Whirling orbit at 3000 rpm due to a mass unbalance for a rotor studied in Paper E. Forward whirl is denoted by + and backward whirl by  $\Delta$ .

Often the most interesting result is the major axis of the elliptical whirling motions, i.e. the maximum amplitude of vibration. If this analysis is repeated for a range of rotational speeds the analysis is usually referred to as an *unbalance response analysis*. Figure 4.3 shows an example from Paper E of the unbalance response (i.e. the major axis of relative vibrations in two bearings) for the same rotor with two different mass unbalance distributions.



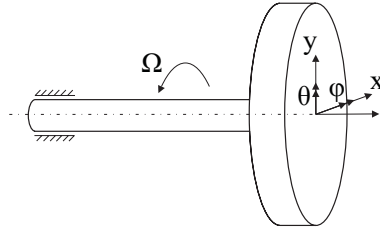
**Figure 4.3** Unbalance response (relative amplitude for major axis) in two bearings for two different unbalance mass distribution.

To conclude, the state vector approach gives a straight-forward way to explain and define the concepts of complex eigenvalues and stability. However, it must not necessarily be used to solve the unbalance response problem. The drawback with the state vector approach is the required computational effort since it involves the matrix  $\mathbf{A}$  which is of twice the size as the matrices in (9). It can also be difficult to use approximate methods to solve the eigenvalue problem since the structure of the matrix  $\mathbf{A}$  is not banded. Still it should be clear that with the computational tools of today it is an interesting method for medium sized problems. As information, the time required to

solve the eigenvalue problem for a rotor with 800 degrees of freedoms using standard routines in MATLAB is in the order of seconds on a standard laptop from year 2001.

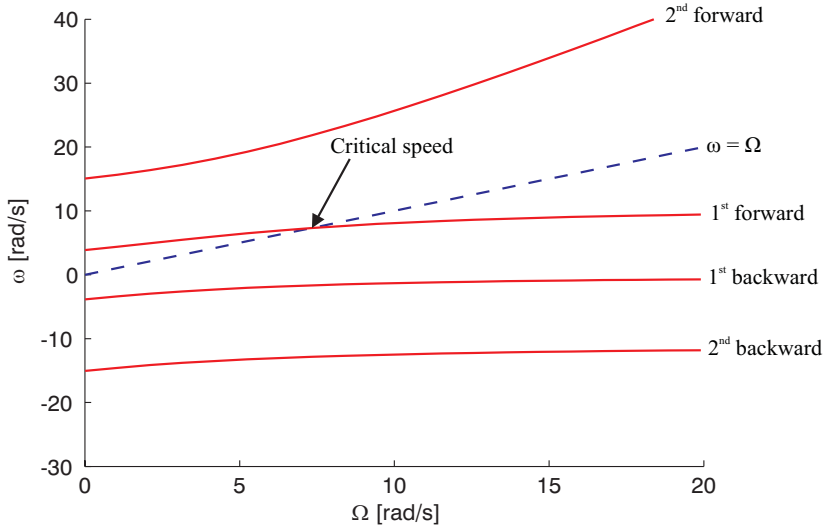
#### 4.1.3 Gyroscopic effect and critical speeds

In this section a discussion about how the rotational speed affects the eigenvalues and eigenvectors is given. From Equation (9) and (11) it is easy to realize that the rotational speed  $\Omega$  affects the eigenvalues and the eigenvectors. The gyroscopic effect introduced by the matrix  $\mathbf{G}$  gives a stiffening effect that leads to an increase of the natural frequencies that correspond to forward whirl modes as the rotational speed increases. This is usually illustrated in a so-called Campbell diagram where the natural frequencies are plotted vs. the rotational speed. To illustrate this for a simple rotor, Figure 4.4 is first considered.



**Figure 4.4** A 4-DOF overhang disk rotor.

This rotor can be modeled as a 4-DOF model indicated by the coordinates in the figure. Since this rotor has four degrees of freedoms it should have four natural frequencies. An example of a Campbell diagram for this type of rotor is shown in Figure 4.5.



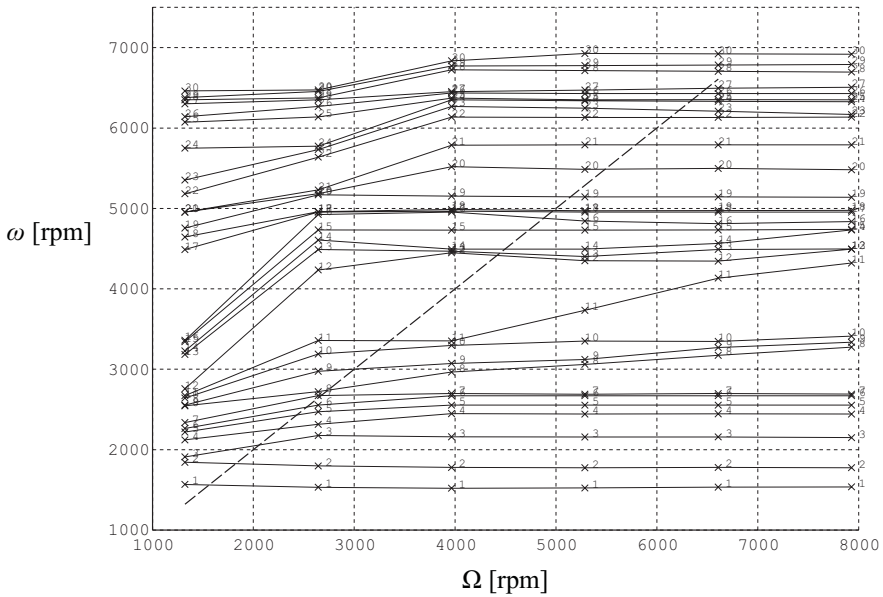
**Figure 4.5** A typical Campbell diagram for a 4-DOF disk rotor.



The line for  $\omega = \Omega$  is marked in Figure 4.5. The speeds at which the curves for the natural frequencies intersect this line are called the *critical speeds* (with respect to the unbalance response). Observe that the critical speeds do not only depend on the natural frequencies but also on the type of excitation. There also exist other types of excitation forces which gives other critical speeds, see Genta [28].

From Figure 4.5 it can be seen that two of the natural frequencies are negative. Here this means that these frequencies correspond to backward whirl modes. In other words, they correspond to modes that whirl in opposite direction to the rotational direction indicated by  $\Omega$  in Figure 4.4. It is a common practice to plot the Campbell diagram with the negative values for the backward whirl modes for simpler rotor systems, see Genta [28]. However, from Equation (14) it is not clear that the natural frequencies can become negative. In fact, the information whether a complex mode whirls forward or backward is in the eigenvector.

In the case of a continuous rotor approximated by a discrete model, the meaning of a forward or a backward mode is sometimes lost. In some cases forward and backward whirl can coexist for a single mode of such a rotors. Therefore the Campbell diagram is often plotted only with positive values of the natural frequencies as in Figure 4.6.



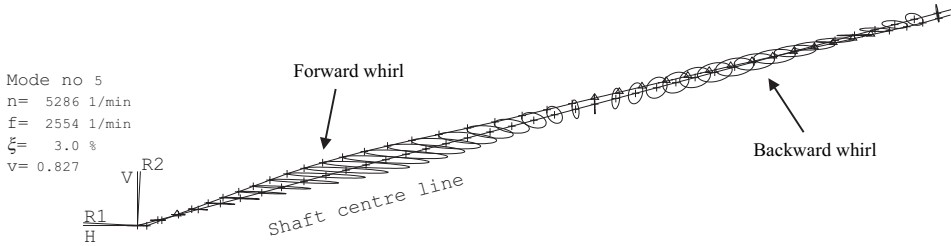
**Figure 4.6** Campbell diagram for a gas turbine rotor with stator structure.

Figure 4.6 shows the Campbell diagram for a long and slender rotor. This rotor is supercritical since the operational speed is larger than 6000 rpm and there are several critical speeds below.

The complex eigenvalue analysis is done at discrete values of the operational speed. Therefore it is a problem to evaluate how the eigenvalues should be connected in the diagram. This may be of interest if the critical speeds have to be calculated by interpolation of several natural frequencies. It is not trivial to do since one cannot easily

define the shape of each mode and identify the same mode at another rotational speed since each mode shape changes as the rotational speed changes. Even if one could identify the same mode for two different rotational speeds, the result could become completely wrong. An example is the two forward modes for the 4-DOF rotor shown in Figure 4.4. Although the curves for the natural frequencies in the Campbell diagram are continuous (Figure 4.5), the modes shapes are almost the same for first forward mode at  $\Omega = 0^+$  and second forward mode as  $\Omega \rightarrow \infty$ .

An example of a complex mode for a gas turbine rotor-bearing system that shows both forward whirl and backward whirl motion is shown in Figure 4.7. Usually these modes are referred to as mixed modes.

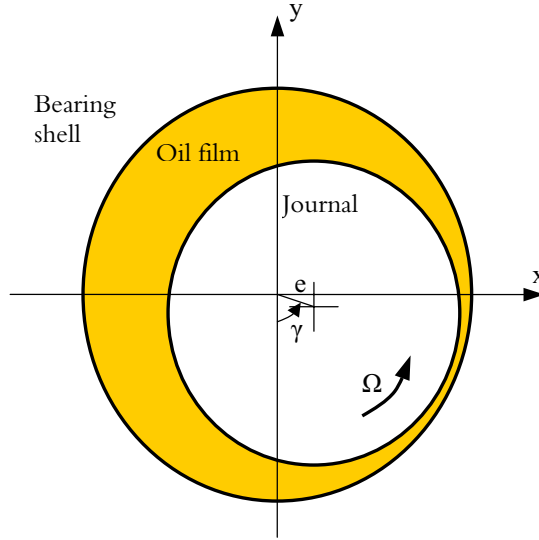


**Figure 4.7** A complex mixed mode for a gas turbine. Forward whirl is denoted by + and backward whirl by  $\Delta$ .

It should also be mentioned that it is not only the gyroscopic effect that affects the Campbell diagram shown in Figure 4.6. Also the bearings are involved since the bearing's stiffness and damping depend on the rotational speed. The effect of journal bearings is the topic for the next section.

#### 4.1.4 Effects of journal bearings

The rotor systems considered in this thesis are supported by fluid film hydrodynamic journal bearings or hereinafter referred to as journal bearings. Hydrodynamic means that the pressure in the oil film is built up by the relative motion between the journal and the bearing shell caused by the rotation of the shaft. The main reason for using this type of bearings in these applications is that they have high load carrying capacity and damping compared to for example rolling element bearings. An important aspect in the design of these bearings is that they may become unstable above a threshold speed. This instability is usually referred to as *oil whip* characterized by subsynchronous whirling according to Vance [61]. The concept of oil whip should not be mixed up with *oil whirl*. Oil whirl is a whirling motion of the rotor that occurs with about half the rotational frequency. This whirling motion is stable and does not generally cause any problems. The cause of oil whirl cannot be predicted by linear theory. Before the origin of instability in journal bearings is discussed, a brief introduction to the governing equation for fluid film bearings is given. First Figure 4.8 is considered.



**Figure 4.8** Exaggerated schematic sketch of a cylindrical journal bearing.

Figure 4.8 shows an exaggerated sketch of the shaft, oil film and journal housing for a cylindrical bearing. The position of the shaft center is defined by the distance  $e$  and angle  $\gamma$ . Often the non-dimensional eccentricity  $\varepsilon = e/\Delta R$  where  $\Delta R$  is the radial clearance is used instead of  $e$ . Figure 4.8 shows a probable position of the shaft if the conditions are stationary, i.e.  $\partial e/\partial t = \partial \gamma/\partial t = 0$ , and the shaft is loaded by gravity in the negative  $y$ -direction. From system dynamic point of view, the pressure that acts on the shaft is the most interesting quantity. The governing equation for the pressure in the thin oil film is Reynolds equation. Reynolds equation in cylindrical coordinates can be written as

$$\frac{1}{6} \left[ \frac{1}{R^2} \frac{\partial}{\partial \theta} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) \right] = \Omega \frac{\partial h}{\partial \theta} + 2 \frac{\partial h}{\partial t} \quad (19)$$

according to Genta [28] where  $R$  is the radial of the journal,  $h$  the film thickness,  $\mu$  the viscosity and  $p$  the pressure. There exist analytical solutions to Equation (19) for long or short bearing assumptions. However, for many practical bearing geometries Equation (19) (or equations similar to Equation (19)) has to be solved by approximate numerical methods, for example a finite difference method. Clearly the dynamics of a rotor supported by journal bearings is a coupled elastohydrodynamical problem.

From the rotordynamic point of view, the stiffness and damping in the bearing is of interest. This can be interpreted as the partial derivatives of the forces from the oil film that acts on the shaft (integrated pressure distribution) with respect to the coordinates  $x$  and  $y$  and the velocities  $\dot{x}$  and  $\dot{y}$ . An assumption that often is used in rotordynamics (so also for the analyses in the appended papers of this thesis) is to assume that the bearing works under stationary conditions. Hence, due to some preload the rotor whirls around a fixed point defined by  $\varepsilon$  and  $\gamma$ . Since the stiffness and damping is highly non-linear with

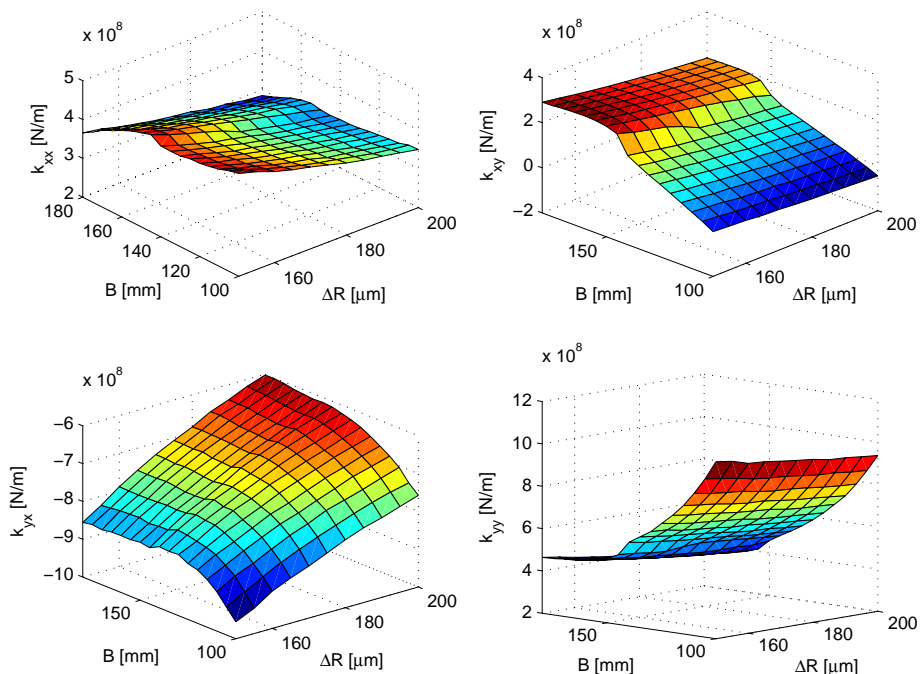
respect to  $\varepsilon$ , this assumption is only valid for small whirl amplitudes around the stationary point. Under this assumption the forces from the oil film that acts on the shaft in the x and y-direction is described by the linear model

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = - \begin{bmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} - \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} \quad (20)$$

which is similar to the model in [49]. The stiffness and damping matrices in Equation (20) are assembled into the systems stiffness and damping matrices ( $\mathbf{K}$  and  $\mathbf{C}$ ) in Equation (9). According to [49]  $c_{xy} = c_{yx}$  so the damping matrix is symmetric. However, in most cases the stiffness matrix in Equation (20) is non-symmetric or even worse, the coefficients  $k_{xy}$  and  $k_{yx}$  can have different signs. This means that the system can become unstable since energy from the rotation of the shaft can be transferred into the lateral vibrations of the shaft. Observe that if the damping in the bearing is large enough, positive damping for all eigenvalues may still be achieved even if the bearing stiffness matrix is non-symmetric.

It should also be mentioned that there exists other types of bearings with different dynamical properties than the cylindrical bearing. Offset halves, elliptical (or sometimes called lemon bore) or tilting pad bearings are a few types just to mention. The difference compared to the cylindrical bearing is the bearing shell geometry. Tilting pad bearings are interesting since these do not have any (or at least very small) destabilizing cross coupling stiffness coefficients. The damping is however low for these bearings and the load carrying capacity not as high as for cylindrical bearings. When it comes to bearing design, other aspects than dynamical properties are also important objectives. These can be cost, power loss and load capacity, etc. A frequently used trick to increase the damping in the systems is to use a squeeze film damper. This is in principle a cylindrical bearing where the rotational relative motion is constrained so that the “shaft” only can translate in the viscous fluid. This damper is placed outside the actual bearing. Since no squeeze film damper is used in the studied systems in the appended papers no further discussion about this topic is therefore done.

For the bearing analyses in the appended papers C, D and E, a solver based on ALP3T [43] is used. This can handle different types of bearing geometries. It should also be mentioned that Reynolds equation (19) is not the whole truth since a temperature dependent viscosity model and cavitation conditions are implemented in the code. Typical output is the stationary point ( $\varepsilon$  and  $\gamma$ ), bearing stiffness and damping coefficients, power losses and bearing temperatures. An example of how the bearing stiffness coefficients vary with bearing width ( $B$ ) and radial clearance ( $\Delta R$ ) for a preloaded cylindrical bearing is shown in Figure 4.9.

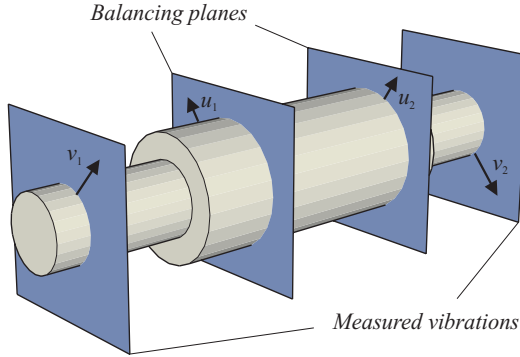


**Figure 4.9** Stiffness coefficients as functions of bearing width and radial clearance for a cylindrical bearing.

Three things are worth to note from Figure 4.9. The first is that the bearing clearly is much stiffer in the vertical direction (y-dir) than in the horizontal direction (x-dir) due to the preload in the negative y-direction. Secondly, the cross coupling coefficients ( $k_{xy}$  and  $k_{yx}$ ) are in the same order of magnitude as the horizontal stiffness ( $k_{xx}$ ). Thirdly, the cross coupling coefficients have different signs so that the bearing stiffness matrix is non-symmetric. Hence, this bearing may introduce instability if the damping is not high enough.

## 4.2 Balancing of flexible rotors

Since all real rotors have imperfections in form of mass unbalance vibrations will occur. *Balancing* means here to minimize the vibrations caused by the mass unbalance. *Balancing plane* means an axial position on the rotor where a compensation weight (balancing weight) can be applied as illustrated in Figure 4.10. Paper E deals with the feasibility of on-site balancing rewinded turbo generator rotors. Therefore an introduction to the balancing of flexible rotors is here given.



**Figure 4.10** Example of a rotor with two balancing planes and two positions for vibration measurements. The vectors  $u_1$  and  $u_2$  represents the balancing correction weights. The vectors  $v_1$  and  $v_2$  represents the amplitude and phase of the vibrations in some fix direction, usually in horizontal or vertical direction.

A complete review of balancing methods is given in [23]. Most of the proposed balancing methods are variants of two different methods the *modal balancing method* [4] and the *influence coefficient method* [30] (also well described in [48]).

The principles of modal balancing method will shortly be mentioned. In this method each mode in the interesting speed range is balanced separately and it is assumed that the modes are orthogonal. Hence, the distribution of the first applied compensation mass (balancing weights) is such that it only affects the first mode. The distribution of the second applied balancing weights only affects the second mode and so on. Although the modal balancing method has limited practical interest the knowledge of the mode shapes is essential when rotor design and balancing plane design is considered. In  $N$ -plane balancing the modal eccentricity up to the  $N^{\text{th}}$  mode is diminished and if  $(N+2)$ -plane balancing is used also the forces transmitted to the bearings (in the case of two bearings) can be diminished according to [60].

The influence coefficient method is based on the assumption that there is a linear relationship between the applied balancing weight and vibrations such that

$$\mathbf{v} = \mathbf{r}\mathbf{u} + \mathbf{v}^0 \quad \text{or} \quad \begin{Bmatrix} v_1 \\ v_2 \\ \dots \\ v_i \\ \dots \\ v_m \end{Bmatrix} = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1j} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{2j} & \dots & r_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ r_{i1} & r_{i2} & \dots & r_{ij} & \dots & r_{in} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ r_{m1} & r_{m2} & \dots & r_{mj} & \dots & r_{mn} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ \dots \\ u_j \\ \dots \\ u_n \end{Bmatrix} + \begin{Bmatrix} v_1^0 \\ v_2^0 \\ \dots \\ v_i^0 \\ \dots \\ v_m^0 \end{Bmatrix}. \quad (21)$$

The complex components  $(v_i)$  of the vector  $\mathbf{v}$  represent the vibration in some direction (amplitude and phase relative to a fix point on the rotor) at a certain position and rotational speed. The balancing weight at position  $j$  is described by the complex quantity  $u_j$  (mass times radius and phase angle). The generally unknown matrix  $\mathbf{r}$  with complex components is called the influence coefficient matrix. The vibrations are described by the

vector  $\mathbf{v}^0$  if no balancing weights are applied. Observe that  $m$  describes at how many positions and rotational speeds the vibrations are measured. Generally the bearings are measured at speeds near the critical speeds and the operational speed. The number of balancing planes is  $n$ .

The influence coefficient matrix is determined such that first the vibrations are measured without any balancing weights, i.e.  $\mathbf{v}^0$  for  $\mathbf{u} = \mathbf{0}$ . Then a test weight is applied in the first balancing plane such that  $\mathbf{u}^1 = \{u_1^1 \ 0 \ \dots \ 0 \ \dots \ 0\}^T$  and the vibrations are measured again (now called  $\mathbf{v}^1$ ). It should then be clear that  $\mathbf{v}^1 = \mathbf{r}\mathbf{u}^1 + \mathbf{v}^0$ . Now the influence coefficients for the first column of  $\mathbf{r}$  are determined by

$$r_{i1} = \frac{v_i^1 - v_i^0}{u_1^1}. \quad (22)$$

The procedure to apply a test weight and measure the vibrations is then repeated for each balancing plane and the influence coefficient matrix is thus determined.

If  $m = n$ , that is the vibrations are measured at as many positions and speeds as there are balancing planes, the vibrations can theoretically be zero (at the defined speeds and positions) by applying the balancing compensation weights  $\mathbf{u} = -\mathbf{r}^{-1}\mathbf{v}^0$ . However, in most practical applications  $m > n$  which means that Equation 21 is an overdetermined system. The best balancing compensation weight setup is then usually determined by a least square method such that

$$\mathbf{v}^H \mathbf{v} = \sum_{i=1}^m |v_i|^2 = \sum_{i=1}^m \left| v_i^0 + \sum_{j=1}^n r_{ij} u_j \right|^2 = \sum_{i=1}^m \left| -v_i^0 - \sum_{j=1}^n r_{ij} u_j \right|^2 \quad (23)$$

is minimized where  $H$  means Hermitian transpose (transpose and conjugate). According to [42] (Equation 3.1 and Equation 3.2) the value of  $\mathbf{u}$  that minimizes Equation 23 is given by

$$\mathbf{u}^* = -(\mathbf{r}^H \mathbf{r})^{-1} \mathbf{r}^H \mathbf{v}^0. \quad (24)$$

What Equation (23) actually describes is a weighted sum (with weights equal to 1) of the vibration magnitudes at all defined measuring positions and rotational speeds. From Equation (23) it is easy to realize that balancing of flexible rotors over a speed range is in general a multiobjective optimization problem.

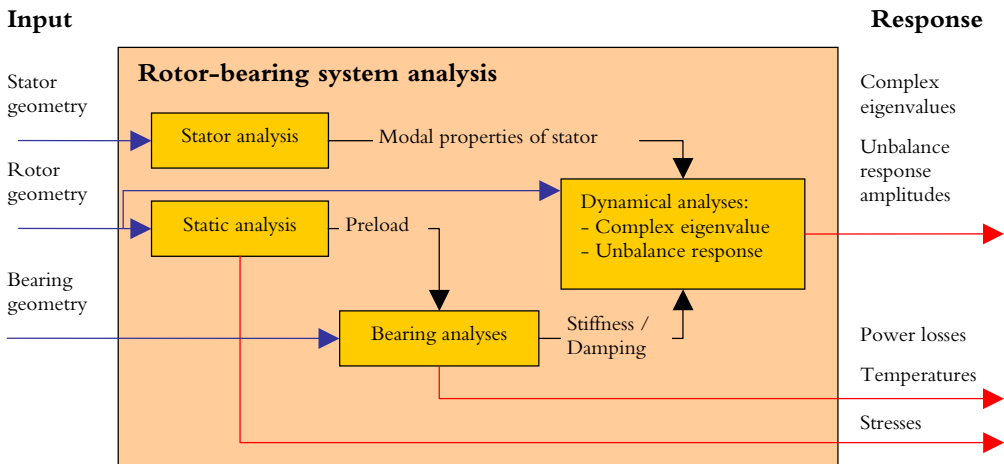
Although Paper E is not based on the influence coefficient method it presents an alternative formulation to Equation (23). The balancing of a turbo generator rotor problem is formulated as a multiobjective optimization problem with two intuitively objectives (non-linear) and solved by a Pareto based GA. This approach could also be used for balancing of flexible rotors with the influence coefficient method.

### 4.3 Design of rotor-bearing systems

Until now the analysis and balancing of rotor-bearing systems has been dealt with. In this section the practical design of these systems is discussed from rotordynamical point of view.

In general, every manufacturer has categorical machines in different segments of the market. It is unusual that a customer asks a manufacturer for an offer of an entirely new type of machine. Still almost every machine has some unique features since most customers have different demands. This implies that most of the activities spent in the design process of these systems are re-design of already known concepts. Hence, almost every machine has to be re-designed to some extent. When it comes to the rotordynamical aspects these are considered late in the re-design process. Often the rotor geometry is already constrained for other reasons. This could for example be that the active magnetic length in a turbo generator is specified or the number of compressor stages in a gas turbine is fixed. The degrees of freedom for an engineer that tries to optimize the dynamics of the machine are therefore quite limited. Often only slight modifications of the bearings or some intermediate shaft are possible at this stage of the process.

Figure 4.11 gives a rough picture of how the rotordynamical analyses are performed for these types of systems. It should be mentioned that Figure 4.11 covers only the analyses that concerns the lateral dynamics of these machines. Torsional vibrations are for example also affected by the rotor geometries in the systems but these are not considered here.



**Figure 4.11** Overview of the rotordynamical analyses of rotor-bearing systems in practical design.

Figure 4.11 shows a scheme for how the rotordynamical analyses of the systems are related and performed. The figure is valid for a system without a squeeze film damper. First a static analysis of the rotor deflection is performed. This gives the preload in the



bearings. Thereafter the bearing analyses are done. These give the linearized stiffness and damping coefficients that are required for the dynamical analyses (complex eigenvalue and unbalance response). The stator analysis is often a modal analysis that gives the modal properties of the stator. These are also required input to the dynamical analyses of the rotor-bearing systems. The inputs to the analyses are the different geometries, material properties, etc. The results for the analyses are listed as responses in Figure 4.11.

By looking at Figure 4.11 it should be clear that the rotordynamical design is about finding the optimal settings for some parameters subject to several objectives or constraints.

## **5 Summary of appended papers**

In this chapter short summaries of the appended papers are presented and finally the contribution from the author in each paper is given.

### **5.1 Paper A**

Almost all practical design optimization problems are constrained for different reasons. A possible criticism of GAs is the lack of efficient and robust generic methods to handle constraints. The most widespread approach for constrained search problems is to use penalty methods. These methods often require that some extra penalty parameters have to be specified. The settings of these parameters are problem dependent and if the penalty functions are not designed properly, false optima may be introduced.

During the last decade GAs have received increased interest in the field of multiobjective optimization. A constrained optimization problem or an unconstrained multiobjective problem may in principle be two different ways to pose the same underlying problem. Therefore it seems natural to glance at multiobjective GAs when algorithms for constrained optimization are designed. In this paper a new ranking scheme is introduced. The method is a variant of a multiobjective real coded GA inspired by the penalty approach but no extra penalty parameter is to be set. It is evaluated on six different constrained single objective problems found in the literature. The result is compared to the result for other constraint handling techniques for GAs. The results show that the proposed method performs well in terms of efficiency, and that it is robust for a majority of the test problems.

The contribution to the scientific and engineering community of this paper is a new and generic constraint handling method that can be used in many types of GAs.

### **5.2 Paper B**

This paper presents an approach on how a real coded GAs can be used in the design process of rotor-bearing systems in order to search for feasible positions of the systems complex eigenvalues. The studied application is a turbo generator rotor-bearing system.

The detailed design of a turbo generator rotor system is highly constrained by feasible regions for the complex eigenvalues of the system. A major problem for the designer is to find solutions that fulfill the design criterion for the complex eigenvalues (i.e. damping and natural frequencies). The bearing properties and geometrical parameters of the rotor are often used as the design parameters subjected to variations in order to search for feasible designs. This paper presents an alternative approach to be used at the late stage of the design process.

First the design criteria for several complex eigenvalues are reformulated in a scalar valued function that describes the degree of feasibility for a particular design. Then the search for feasible designs is formulated as an optimization problem and solved with a GA. Finally the problem is also extended to include another objective (i.e. multiobjective optimization) to show the potential of using the optimization formulation and a Pareto based GA in this rotordynamic application. The results show that the presented approach is promising as an engineering design tool.

The most important scientific contribution of this paper is how the complex eigenvalue criteria are mapped on a scalar function that easily can be used to compare different designs.

### **5.3 Paper C**

This paper is a continuation of the work done in Paper B and the constraint handling technique developed in Paper A is used.

In the design of large rotor-bearing systems such as steam turbines, gas turbines or generators, the whole rotor system should be optimal in some sense and simultaneously fulfill the design constraints. The bearing design has a crucial impact on the rotor system characteristics such as complex eigenvalues for example. In system optimizations it is therefore important to consider the bearing design. Until now the actual bearing geometry has seldom been used as design variables in system optimizations. In Paper B the uncoupled linearized bearing stiffness and damping coefficients were used as design variables. In this paper a bearing analysis code is included and the actual bearing geometry parameters are used as design variables.

First, the generic constraint handling technique for GAs developed in Paper A is briefly explained. Then the optimization problem of a generator rotor-bearing system is formulated and solved by a real coded GA and the proposed method to handle the constraints. The objective is to minimize power loss subject to design specifications for bearing temperatures and complex eigenvalue constraints for the system.

The result shows that a reduction of the power loss in the bearings may be achieved without violating the system design constraints. The result also shows how the design problem for a large rotor-bearing system can be handled. This is also the main scientific contribution in combination with the evaluation of the search method's performance on a highly constrained real-world design problem. To conclude, the paper shows a successful application of the presented search algorithm on an industrial rotor-bearing optimization problem.

### **5.4 Paper D**

This paper presents the optimization of the tilting pad bearings design in a gas turbine rotor-bearing system. The effects taken into account are power loss and limiting temperatures in the bearings. The dynamics at the system level, i. e. stability and unbalances responses, are also considered. The design variables are the bearing widths and radial clearances. A real coded GA and the constraint handling technique developed in Paper A is used. The problem formulation is specific for this particular design case but the used search method and outcome from the optimization is of general interest.

The result is compared to the result of a nominal design. The result shows that it is most likely impossible to find a design that fulfills all the constraints for the system design. Still it is possible to find a design that gives a 10.5% reduced power loss and does not violate any of the constraints more than the nominal design. The contribution to industry from this paper is that it gives a hint on how much slight modification of the bearings may improve performance of a gas turbine that is regarded as well developed and a matured design. The scientific contribution is another reference of the performance of the constraint handling method proposed in Paper A.

## 5.5 Paper E

The aim of this paper is to show how the balancing of flexible rotors in a reduced number of balancing planes can be formulated as a multiobjective optimization problem and to determine a criterion for feasibility of site balancing rewinded 2-pole turbo generator rotors.

When a 2-pole turbo generator is rewinded (the winding is replaced) an unbalance is introduced and the rotor must be balanced before taken into operation. Balancing can be performed on site in a limited number of balancing planes. However, conflicting objectives arise. Depending on the rotor geometry it may be difficult to balance the rotor at the critical speeds and operational speed simultaneously.

This paper presents estimates of the expected unbalance introduced by rewinding. These estimates are based on historical data for rewinded and balanced rotors. Then the balancing on site is formulated as a multiobjective optimization problem and a rotordynamical model is used to determine a criterion for feasibility of balancing different rotors. A multiobjective GA is used to search for the Pareto optimal solutions. The result shows that there is a trade-off to balance even short super critical rotors. The results indicate that rotors with active diameter 800–1020 mm and active length less than 3000 mm are candidates for site balancing after rewind.

The scientific contribution of this paper is how the balancing problem is formulated and solved. This formulation could directly be used in softwares for balancing flexible rotors. The industrial relevance of the paper are the values of the unbalance that should be expected by rewinding and the guidelines regarding feasibility of site balancing rewinded rotors.

## 5.6 The author's contribution

In all appended papers a main part of the work has been carried out by the author of this thesis. The second and third authors have given some guidance during the work and been more involved during the writing phase of the papers. The estimated ratio of working time spent by the authors on each paper is given in Table 2.

**Table 2.** Estimation of each authors time contribution to the appended papers.

	First author	Second author	Third author
Paper A	85 %	10 %	5 %
Paper B	85 %	15 %	–
Paper C	85 %	15 %	–
Paper D	90 %	10 %	–
Paper E	100 %	–	–

## **6 Discussion and conclusions**

In this chapter the research question is responded, conclusions and experiences of this work are summarized and an outlook for future work is given.

### **6.1 Respond to research question**

Clearly there is no unique answer to the research question. The appended papers are however attempts to give some answers to the research question. A description of how the papers are related to the research question is therefore done.

Paper A is an adaptation of a multiobjective GA to solve the constrained single objective optimization problem. The formulation of the complex eigen value constraints in Paper B is an example of how the late rotor bearing design process can be improved. Paper C and D show the magnitude of improvements that can be achieved for real rotor bearing design cases. Paper E shows how a multiobjective GA can be used in a balancing application with conflicting objectives.

### **6.2 Conclusions**

The constraint handling method for GAs introduced in Paper A is more robust (i.e. less spread in best found feasible solution) on a majority of the test problem compared to the other constraint handling techniques in the study. This feature may well be as important as efficiency in practical design optimization situations.

In Paper B it is shown that the complex eigenvalue criteria formulated as a scalar function and the used search method can be an efficient tool for an engineer working with similar kind of problems.

In Paper C the actual bearing geometry was introduced for the first time in an optimization that considers the dynamics on the system level, i.e. complex eigenvalues for the rotor-bearing system. By using the search algorithm presented in Paper A, the results show that a reduced power loss in the bearings of 19% was possible to achieve compared to the nominal design on this highly constrained problem.

In Paper D the detailed tilting pad bearing design of an existing gas turbine system is formulated as an optimization problem. A reduced power loss of 10.5% was possible to achieve compared to the existing matured design.

By using the multiobjective formulation and the GA in Paper E it is shown that there is a trade-off to balance rotors with the 2<sup>nd</sup> critical speed above the operational speed as well as longer rotors with the 3<sup>rd</sup> critical speed close to the operational speed. The result shows furthermore that rotors with diameters 800–1020 mm and active length less than 3000 mm are candidates for site balancing after rewind.

### **6.3 Experiences**

The experience from this work is that the interpretation of the design task and formulation of the optimization problem is the overriding activity looking at the total time spent. To achieve a good problem formulation, knowledge in the domain of search algorithms and the studied systems are required. This is more seldom the case in industry and the problem formulation will probably become more or less poor. The computational

time is definitely of secondary importance when optimization of rotor-bearing systems are concerned.

Since the main objective for an engineer working with design of rotor-bearing system probably is to find fairly good robust solutions within a decent time, robust global search methods are motivated.

## **6.4 Outlook**

Optima where the solution is sensitive to perturbations are not desirable. The post-optimal process has not been given priority in this thesis and is therefore an area of further work. Population based methods such as GAs has here an interesting feature since the design variables or other design parameters may be perturbed in the evolutionary process so that less robust solutions are rejected. Thereby optimal and robust solutions are searched for.

A possible area in rotordynamical design where search and optimization methods can contribute is also in inverse modeling (or parameter identification). A typical situation is that the modal properties of the stator are not well known. Inverse modeling is when some unknown parameters in a model are fit so that the model as closely as possible represents a real system. Uniqueness of solutions is however a problem that frequently appears in inverse modeling. Niche GAs may be an approach to avoid these problems since niche GAs can search several local (or global) optima simultaneously in one single search.

Another approach for further research is mixed variable problems. These arise in the design of rotor-bearing problems if different bearing types and other geometrical parameters are chosen as design variables. Binary design choices such as the possibility of a squeeze film damper or not is also imaginable. EAs are some of the more interesting methods for these types of problems since most other optimization techniques cannot handle mixed types of variables.

It should also be stressed that the complex eigenvalues (or root locus plot) is a very interesting target when optimization of rotor bearing systems is concerned. The reason is that the complex eigenvalues holds much information about the system and its performance.

A field of rotor-bearing system design where the search and optimization methods are readily applicable is in service and retrofit applications. In these contexts the design problems are quite easily formulated and the number of possible design variables is limited, for example bearing geometry parameters. Still it is of importance to find an optimal design or Pareto optimal designs.

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# Paper A



# Constrained Optimization based on a Multiobjective Evolutionary Algorithm

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**Abstract - A criticism of Evolutionary Algorithms (EAs) might be the lack of efficient and robust generic methods to handle constraints. The most widespread approach for constrained search problems is to use penalty methods. EAs have received increased interest during the last decade due to the ease of handling multiple objectives. A constrained optimization problem or an unconstrained multiobjective problem may in principle be two different ways to pose the same underlying problem. In this paper an alternative approach for the constrained optimization problem is presented. The method is a variant of a multiobjective real coded Genetic Algorithm (GA) inspired by the penalty approach. It is evaluated on six different constrained single objective problems found in the literature. The results show that the proposed method performs well in terms of efficiency, and that it is robust for a majority of the test problems.**

## 1 Introduction

During the last decades Evolutionary Algorithms (EAs) have proved to become an important tool for difficult search and optimization problems. Most real-world problems are however constrained and a possible criticism of EAs has been the lack of efficient and generic constraint handling techniques. A comprehensive survey of existing constraint handling methods for EAs is done by Coello Coello in [1]. The frequently most used methods are based on various penalty functions for which some guidelines are given in [2]. Penalty methods are generic but may however distort the cost surface and introduce false optima. Most penalty methods also require additional parameters, which are problem-dependent and increase the complexity of the problem.

The constrained optimization problem may be handled as a multiobjective optimization problem as indicated by Coello Coello in [3], Michalewicz in [4] and Fonseca and Fleming in [5]. Furthermore, EAs based on non-dominated sorting for multiobjective problems have received increased interest during the past decade. Therefore it seems natural to look upon the constrained optimization problem as a multiobjective problem. Multiobjective approaches of constrained problems based on Shaffers VEGA [6] is found in [7] and [8]. Another interesting constraint handling method based on non-domination is presented by Deb et al. in [9]. To directly apply a multiobjective EA based on non-domination on a constrained optimization problem leads to a search of the best compromises of the objective value and constraint

satisfaction. This whole set of solutions is usually not interesting since it is the optimal and feasible solution that is searched. Therefore it will not be efficient to directly apply a multiobjective EA on a constrained problem. Still the idea to handle the constrained problem with some variant of a multiobjective EA is interesting.

One of the most crucial steps in a multiobjective EA is how to rank individuals. In this paper an alternative ranking scheme for the constrained single objective problem is introduced. This ranking scheme is generic and no new parameters are introduced. The ideas of the ranking scheme are borrowed from the non-domination ranking for multiple objectives by Goldberg in [10] and penalty based methods for constrained problems.

The paper first defines the constrained optimization problem, and thereafter the proposed method is presented in more detail. Then the performance for a real coded Genetic Algorithm with the proposed ranking scheme implemented is tested on six different test problems used by Michalewicz in [11] and Deb in [12]. Finally, the result for this proposed method is compared to the result for other methods evaluated in [11] and [12].

## 2 The constrained optimization problem

In this section the constrained optimization problem and its terminology is defined. The constrained optimization problem or non-linear programming problem (NLP) with  $k$  inequality constraints and  $m$  equality constraints is formulated as

$$\begin{aligned} & \text{Minimize } f(\mathbf{x}) \\ & \text{subject to} \\ & g_i(\mathbf{x}) \leq 0 \quad i = 1, \dots, k, \\ & h_i(\mathbf{x}) = 0 \quad i = 1, \dots, m. \end{aligned} \tag{1}$$

$\mathbf{x} = [x_1, x_2, \dots, x_n]$  is a vector of the  $n$  design variables such that  $\mathbf{x} \in S \subseteq \mathcal{R}^n$ . The search space  $S$  is here defined as an  $n$ -dimensional rectangle by the upper and lower bounds for the design variables,  $x_i^l \leq x_i \leq x_i^u \quad i = 1 \dots n$ . The feasible region  $F \subseteq S$  is the region of  $S$  for which the inequality and equality constraints are satisfied. The optimal solution is denoted  $\mathbf{x}^*$ . A constraint is said to be active at the point  $\mathbf{x}^*$  if  $g_i(\mathbf{x}^*) = 0$ . By default all equality constraints are active at all points of the feasible space. Equality constraints may be transformed to inequality constraints [1] via

$$|h_i(\mathbf{x})| - \varepsilon \leq 0 \quad (2)$$

where  $\varepsilon$  is a small tolerance. Since the algorithm that will be discussed does not use gradient information it does not matter if (2) is non-differentiable.

### 3 The proposed EA approach for constrained optimization

In this section the proposed ranking scheme is introduced. The non-dominated ranking by Goldberg [10] is used in a new way to formulate a scalar valued function that is used to rank individuals in the current population. Then selection, crossover, mutation and reinsertion are used in a standard manner for a real coded GA in this paper. This is described later since the focus for this section is to define the ranking scheme.

It is first assumed that all equality constraints are transformed by (2) so the problem is now

$$\begin{aligned} & \text{Minimize } f(\mathbf{x}) \\ & \text{subject to} \\ & g_i(\mathbf{x}) \leq 0 \quad i = 1, \dots, p \end{aligned} \quad (3)$$

where  $p = k + m$ .

Now, the objective function is given index 1,  $f_1(\mathbf{x}) = f(\mathbf{x})$ . Then the constraints  $g_i(\mathbf{x})$  are reformulated into new objectives  $f_{1+i}(\mathbf{x})$ . These objectives are defined as

$$f_{1+i}(\mathbf{x}) = \max(0, g_i(\mathbf{x})), \quad i = 1, \dots, p \quad (4)$$

A natural approach would be to apply a Pareto based multiobjective GA to solve the problem. This might not be the best idea since the Pareto optimal set with respect to the new objectives  $f_1(\mathbf{x})$  to  $f_{p+1}(\mathbf{x})$  is generally not the same as the optimal solution  $\mathbf{x}^*$ . The idea here is to treat the objective  $f_1(\mathbf{x})$  and the objectives  $f_2(\mathbf{x})$  to  $f_{p+1}(\mathbf{x})$  separately. The approach is based on the following criteria

- If no feasible individual exists in the current population, the search should be directed towards the feasible region.
- If a majority of the individuals in the current population are feasible, the search should be directed towards the unconstrained optimum.
- A feasible individual closer to the optimum is always better than a feasible individual further from the optimum.
- An infeasible individual might be a better individual than a feasible individual if the number of feasible individuals is high.

From the above statements it is clear that the search direction should be dependent upon the number of feasible individuals in the current population. The reason for the fourth statement is that an infeasible individual with a good objective value ( $f_1(\mathbf{x})$ ) should not be rejected as it might guide the search towards the true optimum by improving the diversity of the population.

Now  $P$  is defined to be the population size and  $N$  the number of feasible solutions in the current population.  $\mathbf{x}_j$  is the  $j^{\text{th}}$  individual in the current population. Then,  $rank_1(\mathbf{x}_j)$  is defined as the ranking according to the first objective  $f_1(\mathbf{x})$ . The best individual gets  $rank_1 = 1$ .

$rank_2(\mathbf{x}_j)$  is defined to be the non-dominated ranking with respect to  $f_2(\mathbf{x})$  to  $f_{p+1}(\mathbf{x})$  as defined by Goldberg [10]. In the ranking the first non-dominated individuals in the population receive  $rank_2 = 1$ . Then these individuals are removed from the population and the ranking is repeated for the remaining individuals, but now the non-dominated individuals get  $rank_2 = 2$ . This is repeated until all individuals in the current population have received a value for  $rank_2$ . In [9] Deb shows a method with computational complexity  $O((p+1)P^2)$  to perform the non-dominated ranking.

Now a new objective function  $\phi(\mathbf{x}_j)$  is formulated as

$$\phi(\mathbf{x}_j) = \frac{N}{P} rank_1(\mathbf{x}_j) + \frac{P-N}{P} rank_2(\mathbf{x}_j) \quad (5)$$

Each individual is then ranked according to its value for Equation 5 and fitness is assigned in a regular manner. Note that if no feasible solution is present in the population ( $N = 0$ ), the ranking according to the objective ( $rank_1$ ) becomes inactive and the population is ranked according to the constraints ( $rank_2$ ), i.e. the search is guided towards the feasible region. On the other hand, if all individuals are feasible ( $N = P$ ), the population is ranked according to the objective ( $rank_1$ ), and the search is directed towards the unconstrained optimum. Among two feasible individuals, the most fit is the one with lower value for  $rank_1$  (the objective) since all feasible individuals receive  $rank_2 = 1$ . All these observations are consistent with the previously listed criteria.

An interesting feature for the new ranking is that the search direction depends on the number of feasible solutions. If many feasible solutions exist, the search is directed towards the unconstrained optimal solution. If now it is assumed that the unconstrained optimum is located outside the feasible region, the population may tend to oscillate over the boundary to the feasible region. This variation of the search direction gives a positive effect of the diversity in the population.

Equation 5 has a similar structure as a penalty based approach but it should be pointed out that no parameter that requires problem dependent fine-tuning is introduced. The “weights” for the two objectives in Equation 5 only depend on the population size and the number of feasible individuals in the current population.

The new ranking procedure for a NLP problem is summarized below

1. Reformulate the problem according to Equation 3 and Equation 4
2. Rank the population with respect to the objective ( $f_1(\mathbf{x})$ ) and assign it to  $rank_1$
3. Rank the population with respect to the constraints ( $f_2(\mathbf{x})$  to  $f_{p+1}(\mathbf{x})$ ) based on non-dominance according to Goldberg [10] and assign it to  $rank_2$
4. Calculate the objective ( $\phi(\mathbf{x})$ ) according to Equation 5



5. Rank the population according to the single objective  $\phi(\mathbf{x})$

Until now, only the ranking has been described. This ranking scheme may be used with any type of GA. In the rest of this paper a real coded GA with the proposed ranking scheme is used. All the GA operations and parameters are chosen as simple as possible. Therefore a more advanced algorithm, such as an adaptive GA for example, might improve the results presented in this paper. Linear fitness assignment according to the ranking for the new objective (Equation 5) is used. The selective pressure is set to 1.9. The selection method is the roulette wheel selection. The number of selected individuals are defined by the generation gap that is set to 95%. Thus 95% of the population is selected for mating and the worst parents are replaced by all the offspring. Hence, an elitist GA is used. Blend crossover, BLX, see [13], is used with a probability equal to 1. The mutation operator by Mühlenbein and Schlierkamp-Voosen [14] which produces a small mutation step with high probability and a large step with small probability is used. The mutation probability is set to  $1/n$  where  $n$  is the number of design variables. The maximum mutation step is defined in the result section.

#### 4 An illustrative example

In this section the ranking based on Equation 5 is discussed for a simple NLP problem. It should be clear that the purpose of this section is to show the important effects of the ranking and not to solve the simple NLP problem. First the result of an actual search is presented. Then the imposed search direction is discussed with the help of two hypothetical populations.

The problem is as follows. A quadratic function is to be minimized and the feasible solutions are constrained by three circles. The problem is stated as

$$\begin{aligned} & \text{Minimize } f(\mathbf{x}) = x_1^2 + x_2^2 \\ & \text{subject to} \\ & g_1(\mathbf{x}) \equiv x_1^2 + (x_2 - 3)^2 \leq 1.5^2, \\ & g_2(\mathbf{x}) \equiv (x_1 - 1)^2 + (x_2 - 1)^2 \leq 1.5^2, \\ & g_3(\mathbf{x}) \equiv (x_1 + 1)^2 + (x_2 - 1)^2 \leq 1.5^2, \\ & -5 \leq x_1 \leq 5, \\ & -3 \leq x_2 \leq 7. \end{aligned} \quad (6)$$

For the unconstrained problem the optimal solution is  $\mathbf{x}^* = [0, 0]$ . For the constrained problem (6) the optimal solution is  $\mathbf{x}^* = [0, 1.5]$ . The first constraint is active at the optimal solution. The population size is set to 10, the maximum number of generations is 50 and the maximum mutation step is set to 0.1 of the range for the design variables. The mutation probability is set to 0.2 in this case.

The initial and the final generation are shown in Figure 1. The rank of the initial generation according to Equation 5 is also given in the figure.

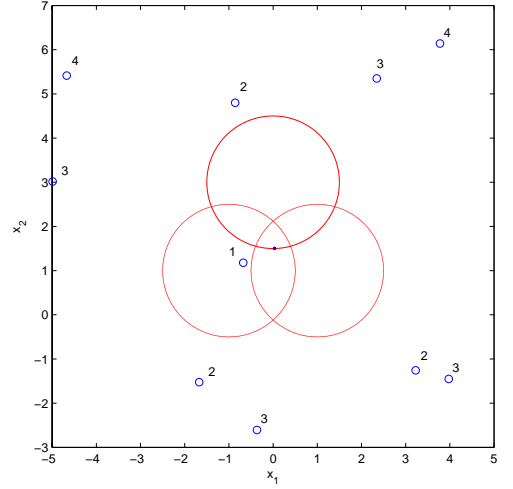


Figure 1: Initial generation (the smaller rings) and final generation (the "dot" near the optimum) shown in design variable space. The numbers indicate the rank according to Equation (5).

The best individual in the initial generation correspond to ranking 1. Figure 1 shows clearly that the search direction is towards the feasible region in the initial generation.

Figure 2 shows the ratio of feasible solutions, the mean normalized Euclidian distance and the ratio between the true optimum and the best-found feasible objective value. To avoid premature convergence it is crucial to have sufficient diversity in the population. An indication of the diversity in the population is given by the distance between the members of the population. The distance between two individuals is calculated using the normalized Euclidian distance. The mean Euclidian distance is obtained by calculating the mean distance between all individuals in the population, and hence is a measure of the diversity in the population.

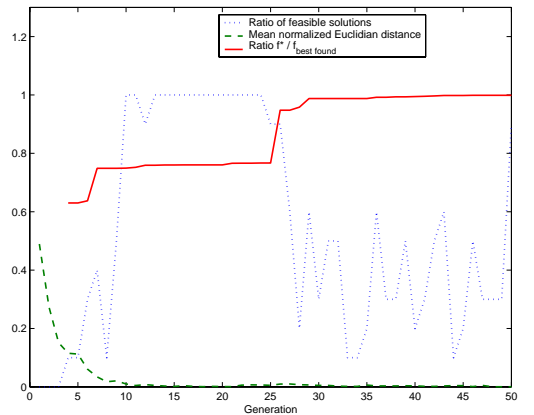


Figure 2: Search results for problem (6).

The first feasible solution is found in generation 4. In the early generations ( $\sim 5$  to 10) the number of feasible individuals increases rapidly. In generations 13 to 24 all individuals are feasible and the improvement in the

objective function is very small since the population has converged too fast. In generation 25 an infeasible individual is created by a mutation. This individual is better than all the feasible individuals in terms of the objective value,  $f(\mathbf{x})$ . Due to the “weights” in Equation 5 this infeasible individual becomes the best individual. The search is then directed out of the feasible region towards the unconstrained optimum and as a result better feasible solutions are found. To make this variation of the search direction more clear two populations with different ratio of feasible individuals are studied in Figure 3 and Figure 4.

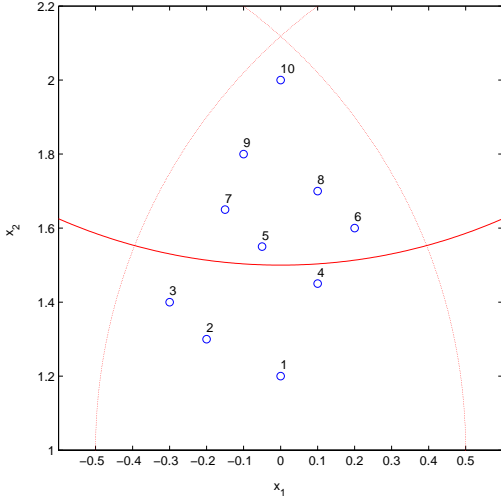


Figure 3. Rank according to Equation 5 for a population with 60% feasible individuals.

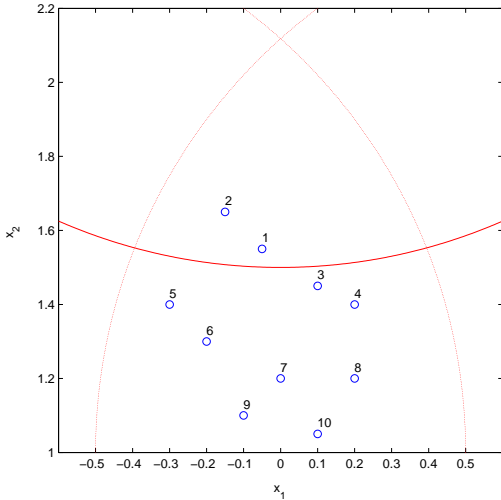


Figure 4. Rank according to Equation 5 for a population with 20% feasible individuals.

When the number of feasible individuals is high, the search is directed towards the unconstrained optimum as shown in Figure 3. In the later stage, when more infeasible

individuals occur in the population the search is directed back to the feasible region again. This explains the oscillating behaviour of the ratio of feasible individuals for generation 26 to 50 in Figure 1.

This example shows the dynamic behaviour of the search direction. The oscillation of the search direction only occurs when at least one constraint is active at the optimum. The variation of the search direction has a positive effect for the population diversity. Thus, if mutation is used no special operation to preserve the diversity in the population is required for most cases. In the next section the method is evaluated using a set of selected test problems gathered from the literature.

## 5 Constrained single objective test problems

In [15], Michalewicz et al. present a test case generator to use in tests of algorithms for constrained optimization problems. This test case generator will probably be used in future research on constrained optimization problems. The results for different constraint handling methods are yet quite limited for this test case generator. Therefore a set of test problems for which there exists results for many different algorithms is here chosen instead.

It is always difficult to make fair comparisons between different EAs. Two different strategies may well have different optimal settings for the optimization algorithm parameters on the same problem. Another difficulty is to determine how to compare different algorithms. A naive but obvious way to compare algorithms is to compare the best solution found in the same number of function evaluations. A measure of the robustness of the algorithms is indicated by the spread of the best solutions found if the optimization is run several times independently. Here it is chosen to compare the results for the proposed ranking scheme with previously reported results for other EAs by other authors on a set of problems. Six test problems are selected. Problem #2 to #6 are found in [11] and problem #1 to #6 in [12]. A short summary of the test problems is given in Table 1. The size of the feasible region is estimated by the ratio ( $\rho$ ) of feasible solutions found in a random sampling of  $10^6$  solutions in the search space<sup>1</sup>. The six test problems are described in detail in the next subsections.

Table 1: Summary of test problems. C corresponds to the number of constraints, A to the number of active constraints at the optimum and n is the number of design variables.

Problem	n	Type of $f$	$\rho$	C	A
#1	5	quadratic	52.03%	6	2
#2	13	quadratic	0.0111%	9	6
#3	8	linear	0.0010%	6	6
#4	7	polynomial	0.5121%	4	2
#5	5	nonlinear	0.0000%	3	3
#6	10	quadratic	0.0003%	8	6

In [11], Michalewicz compares the performance of six different methods on the five problems #2 to #6. Most of the methods are based on penalty functions. The result here is compared to the result for the best method found in [11]. In [12], Deb proposes a special penalty based method

<sup>1</sup> The ratio for problem #2 to problem #6 is presented in [11].

for which the following criteria are always enforced if a tournament selection operator is used:

1. Any feasible solution is preferred to any infeasible solution.
2. Among two feasible solutions, the one having better objective function value is preferred.
3. Among two infeasible solutions, the one having smaller constraint violation is preferred.

Deb tested his method on nine different problems of which test problem #1 to test problem #6 are a subset. The results obtained by the proposed method in this paper are compared to the results obtained by Deb on all these six test problems. Furthermore, Deb stated that *"In all cases, the proposed approach has been able to repeatedly find solutions closer to the true optimum solution than that reported earlier"*. Therefore a fair comparison to the results reported in [12] should give good indication of the performance of the method presented in this paper.

It is worth to notice that the effect of the ranking scheme introduced in this paper is similar to the above listed criteria only if there exist few feasible individuals in the current population. On the contrary, if there exist many feasible individuals, a good (in terms of the objective value) infeasible individual may well be preferred to a feasible individual if this is worse in terms of the objective value.

### 5.1 Test problem 1

This problem was first presented by Himmelblau in [16]. It has later been used by Coello Coello [1] and Deb [12] to evaluate the performance of various GAs for constrained optimization. The problem is stated as

$$\begin{aligned} \text{Minimize } f(\mathbf{x}) &= 5.3578547x_1^2 + 0.8356891x_1x_5 + 37.293239x_1 - 40792.141 \\ \text{subject to} \\ g_1(\mathbf{x}) &\equiv 85.334407 + 0.0056858x_2x_5 + 0.0006262x_1x_4 - 0.0022053x_3x_5 \geq 0, \\ g_2(\mathbf{x}) &\equiv 85.334407 + 0.0056858x_2x_5 + 0.0006262x_1x_4 - 0.0022053x_3x_5 \leq 92, \\ g_3(\mathbf{x}) &\equiv 80.51249 + 0.0071317x_2x_5 + 0.0029955x_1x_2 + 0.0021813x_3^2 \geq 90, \\ g_4(\mathbf{x}) &\equiv 80.51249 + 0.0071317x_2x_5 + 0.0029955x_1x_2 + 0.0021813x_3^2 \leq 110, \\ g_5(\mathbf{x}) &\equiv 9.300961 + 0.0047026x_3x_5 + 0.0012547x_1x_3 + 0.0019085x_1x_4 \geq 20, \\ g_6(\mathbf{x}) &\equiv 9.300961 + 0.0047026x_3x_5 + 0.0012547x_1x_3 + 0.0019085x_1x_4 \leq 25, \\ 78 \leq x_1 &\leq 102, \\ 33 \leq x_2 &\leq 45, \\ 27 \leq x_i &\leq 45, \quad i = 3, 4, 5. \end{aligned}$$

The best-known solution to this problem [16] is  $\mathbf{x}^* = [78, 33, 29.995, 45, 36.776]$  which gives  $f^* = -30665.5$ . At this solution the constraints  $g_2$  and  $g_5$  are active [12].

### 5.2 Test problem 2

The problem is stated as follows

$$\text{Minimize } f(\mathbf{x}) = 5 \sum_{i=1}^4 x_i - 5 \sum_{i=1}^4 x_i^2 - \sum_{i=5}^{13} x_i$$

subject to

$$\begin{aligned} g_1(\mathbf{x}) &\equiv 2x_1 + 2x_2 + x_{10} + x_{11} \leq 10, \\ g_2(\mathbf{x}) &\equiv 2x_1 + 2x_3 + x_{10} + x_{12} \leq 10, \\ g_3(\mathbf{x}) &\equiv 2x_2 + 2x_3 + x_{11} + x_{12} \leq 10, \\ g_4(\mathbf{x}) &\equiv -8x_1 + x_{10} \leq 0, \\ g_5(\mathbf{x}) &\equiv -8x_2 + x_{11} \leq 0, \\ g_6(\mathbf{x}) &\equiv -8x_3 + x_{12} \leq 0, \\ g_7(\mathbf{x}) &\equiv -2x_4 - x_5 + x_{10} \leq 0, \\ g_8(\mathbf{x}) &\equiv -2x_6 - x_7 + x_{11} \leq 0, \\ g_9(\mathbf{x}) &\equiv -2x_8 - x_9 + x_{12} \leq 0, \\ 0 \leq x_i &\leq 1, \quad i = 1, \dots, 9, \\ 0 \leq x_i &\leq 100, \quad i = 10, 11, 12, \\ 0 \leq x_{13} &\leq 1. \end{aligned}$$

The optimal objective value for this problem is  $f^* = -15$  for  $\mathbf{x}^* = [1, 1, 1, 1, 1, 1, 1, 1, 3, 3, 3, 1]$ . At this solution all constraints except  $g_4, g_5$  and  $g_6$  are active.

### 5.3 Test problem 3

The third test problem is

$$\begin{aligned} \text{Minimize } f(\mathbf{x}) &= x_1 + x_2 + x_3 \\ \text{subject to} \\ g_1(\mathbf{x}) &\equiv 1 - 0.0025(x_4 + x_6) \geq 0, \\ g_2(\mathbf{x}) &\equiv 1 - 0.0025(x_5 + x_7 - x_4) \geq 0, \\ g_3(\mathbf{x}) &\equiv 1 - 0.01(x_6 - x_5) \geq 0, \\ g_4(\mathbf{x}) &\equiv x_1x_6 - 833.33252x_4 - 100x_1 + 83333.333 \geq 0, \\ g_5(\mathbf{x}) &\equiv x_2x_7 - 1250x_5 - x_2x_4 + 1250x_4 \geq 0, \\ g_6(\mathbf{x}) &\equiv x_3x_8 - x_3x_5 + 2500x_3 - 1250000 \geq 0, \\ 100 \leq x_i &\leq 10000, \\ 1000 \leq x_i &\leq 10000, \quad i = 2, 3, \\ 10 \leq x_i &\leq 1000, \quad i = 4, \dots, 8. \end{aligned}$$

The optimum solution is  $\mathbf{x}^* = [579.3167, 1359.943, 5110.071, 182.0174, 295.5985, 217.9799, 286.4162, 395.5979]$  which gives  $f^* = 7049.330923$ . All six constraints are active at the optimal solution.

### 5.4 Test problem 4

This problem is stated as

$$\begin{aligned} \text{Minimize } f(\mathbf{x}) &= (x_1 - 10)^2 + 5(x_2 - 12)^2 + x_3^4 + \\ &\quad 3(x_4 - 11)^2 + 10x_5^6 + 7x_6^2 + x_7^4 - 4x_6x_7 \\ &\quad - 10x_6 - 8x_7 \end{aligned}$$

subject to

$$\begin{aligned} g_1(\mathbf{x}) &\equiv 127 - 2x_1^2 - 3x_2^4 - x_3 - 4x_4^2 - 5x_5 \geq 0, \\ g_2(\mathbf{x}) &\equiv 282 - 7x_1 - 3x_2 - 10x_3^2 - x_4 + x_5 \geq 0, \\ g_3(\mathbf{x}) &\equiv 196 - 23x_1 - x_2^2 - 6x_6^2 + 8x_7 \geq 0, \\ g_4(\mathbf{x}) &\equiv -4x_1^2 - x_2^2 + 3x_1x_2 - 2x_3^2 - 5x_6 + 11x_7 \geq 0, \\ -10 \leq x_i &\leq 10, \quad i = 1, \dots, 7. \end{aligned}$$

The optimal solution is  $\mathbf{x}^* = [2.330499, 1.951372, -0.4775414, 4.365726, -0.6244870, 1.038131, 1.594227]$  which gives  $f^* = 680.6300573$ . The constraints  $g_1$  and  $g_4$  are active at the optimal solution.

## 5.5 Test problem 5

The fifth test problem is stated as

$$\text{Minimize } f(\mathbf{x}) = e^{x_1 x_2 x_3 x_4 x_5}$$

subject to

$$h_1(\mathbf{x}) \equiv \sum_{i=1}^5 x_i^2 = 10,$$

$$h_2(\mathbf{x}) \equiv x_2 x_3 - 5 x_4 x_5 = 0,$$

$$h_3(\mathbf{x}) \equiv x_1^3 + x_5^3 = -1,$$

$$-2.3 \leq x_i \leq 2.3, \quad i = 1, 2,$$

$$-3.2 \leq x_i \leq 3.2, \quad i = 3, 4, 5.$$

The optimal solution is  $\mathbf{x}^* = [-1.717143, 1.595709, 1.827247, -0.7636413, -0.7636450]$ . This gives  $f^* = 0.053950$ . Since all constraints are equality type, all constraints are active at the optimal solution. The equality constraints are transformed into inequality constraints by Equation 2 and the tolerance is set to  $\varepsilon = 0.001$  for the results presented in this paper.

## 5.6 Test problem 6

The last test problem is

$$\text{Minimize } f(\mathbf{x}) = x_1^2 + x_2^2 + x_1 x_2 - 14x_1 - 16x_2 + (x_3 - 10)^2 + 4(x_4 - 5)^2 + (x_5 - 3)^2 \\ + 2(x_6 - 1)^2 + 5x_7^2 + 7(x_8 - 11)^2 + 2(x_9 - 10)^2 + (x_{10} - 7)^2 + 45$$

subject to

$$g_1(\mathbf{x}) \equiv 105 - 4x_1 - 5x_2 + 3x_7 - 9x_8 \geq 0,$$

$$g_2(\mathbf{x}) \equiv -10x_1 + 8x_2 + 17x_7 - 2x_8 \geq 0,$$

$$g_3(\mathbf{x}) \equiv 8x_1 - 2x_2 - 5x_9 + 2x_{10} + 12 \geq 0,$$

$$g_4(\mathbf{x}) \equiv -3(x_1 - 2)^2 - 4(x_2 - 3)^2 - 2x_3^2 + 7x_4 + 120 \geq 0,$$

$$g_5(\mathbf{x}) \equiv -5x_1^2 - 8x_2 - (x_3 - 6)^2 + 2x_4 + 40 \geq 0,$$

$$g_6(\mathbf{x}) \equiv -x_1^2 - 2(x_2 - 2)^2 + 2x_1 x_2 - 14x_5 + 6x_6 \geq 0,$$

$$g_7(\mathbf{x}) \equiv -0.5(x_1 - 8)^2 - 2(x_2 - 4)^2 - 3x_3^2 + x_6 + 30 \geq 0,$$

$$g_8(\mathbf{x}) \equiv 3x_1 - 6x_2 - 12(x_9 - 8)^2 + 7x_{10} \geq 0,$$

$$-10 \leq x_i \leq 10, \quad i = 1, \dots, 10.$$

The optimal solution is  $\mathbf{x}^* = [2.171996, 2.363683, 8.773926, 5.095984, 0.9906548, 1.430574, 1.321644, 9.828726, 8.280092, 8.375927]$  which gives  $f^* = 24.3062091$ . All constraints except  $g_7$  and  $g_8$  are active at the optimal solution.

## 6 Results

First some typical search results for the first three problems are presented in Figure 5 to Figure 7. These figures show the ratio of feasible solutions, the mean normalized Euclidian distance and the ratio between the optimal solution and the best-found feasible solution. The GA parameters used in this study is presented in Table 3 for each problem. Then the result for this algorithm is compared to the best result reported in [11] and the result reported in [12].

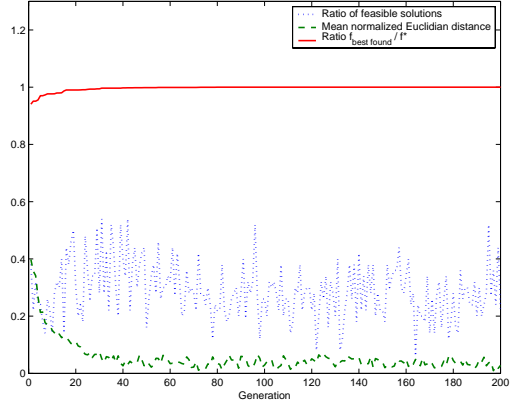


Figure 5: Typical result for problem #1.

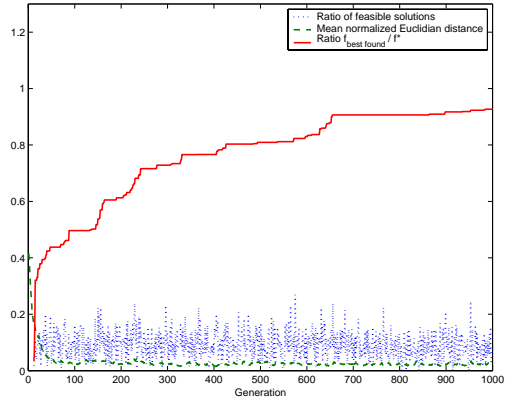


Figure 6: Typical result for problem #2.

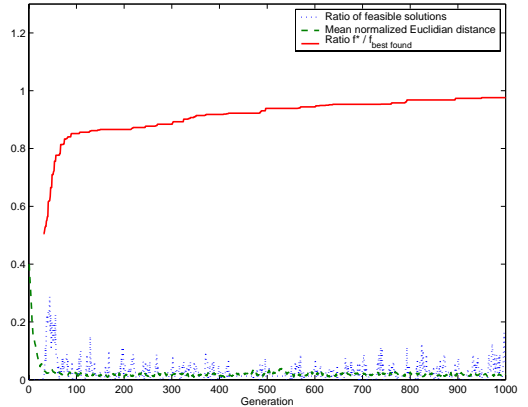


Figure 7: Typical result for problem #3.

Clearly the easiest problem is the first test problem. Near optimal solutions are found in early generations. Surprisingly the most difficult problem of these three problems for this method is test problem #2. In [11] it was reported that this was one of the easiest problem and only a few of the methods studied had any difficulties on this problem.

The results for this algorithm are now compared to the best results for all tested methods in [11] and summarized in Table 2. The population size is 70 and the maximum number of generations is 5000, both in this study and for all algorithms tested in [11]. For this algorithm the maximum mutation step is set to 0.1 of the range for the design variables. The result from [11] presented in Table 2 are the results for the method that found the best feasible solution. It should be mentioned that all the results in the coming tables correspond to feasible solutions.

Table 2: Result for this algorithm compared to best result in [11]. The number of independent runs is 10.

Problem	Study	Best	Median	Worst
#2	This study	-14.680	-14.570	-12.419
	Best in [11]	-15.000	-15.000	-15.000
#3	This study	7079.5	7107.0	7187.8
	Best in [11]	7378.0	8206.2	9653.0
#4	This study	680.636	680.640	680.646
	Best in [11]	680.642	680.718	680.955
#5	This study	0.313	0.534	0.602
	Best in [11]	0.054	0.064	0.577
#6	This study	24.519	24.600	24.735
	Best in [11]	25.486	26.905	42.358

As can be seen from Table 2 this algorithm finds better results in problem #3, problem #4 and problem #6 than all methods tested in [11]. The variation is also much less on these problems.

In Table 4 the result for this algorithm is compared to the results by Deb in [12]. Table 3 shows the GA parameters used for the results in Table 4.

Table 3: GA parameters used for the results presented in Table 4.

Problem	Study	Pop size	Max gen	Max mutation step
#1	This study	50	1000	0.1
	[12]	50	5000	-
#2	This study	130	2000	0.1
	[12]	130	N/A	-
#3	This study	80	4000	0.1
	[12]	80	4000	-
#4	This study	70	5000	0.02
	[12]	70	5000	-
#5	This study	50	7000	0.1
	[12]	50	7000	-
#6	This study	100	3500	0.02
	[12]	100	3500	-

Table 4: Result for this algorithm compared to best result in [12]. The independent number of runs is 50.

Problem	Study	Best	Median	Worst
#1	This study	-30665.5	-30665.5	-30665.4
	[12]	-30665.5	-30665.5	-29846.7
#2	This study	-14.276	-13.224	-11.963
	[12]	-15.000	-15.000	-13.000
#3	This study	7072.4	7100.2	7256.4
	[12]	7060.2	7220.0	10230.8
#4	This study	680.632	680.636	680.645
	[12]	680.634	680.642	680.651
#5	This study	0.44678	0.56967	0.83732
	[12]	0.05395	0.24129	0.50776
#6	This study	24.375	24.426	24.512
	[12]	24.372	24.409	25.075

The best found result of the 50 independent runs for this method is almost similar to the result reported by Deb for problem #1, problem #3, problem #4 and problem #6. For

these problems the variation in the best results found is less for the proposed method than that reported in [12]. For problem #2 and problem #5 the method presented by Deb performs better, both in terms of best-found solution and variation of the best-found solution.

It should be mentioned however, that the results presented by Deb are based on tournament selection with a niching method that required two extra parameters. Furthermore, the maximum number of generations for the results of test problem #2 in [12] is not known. Hence it is difficult to make a fair comparison of the results on this problem.

## 7 Conclusions

A general ranking scheme without problem specific extra parameters for constrained optimization problem has been presented. The performance for an algorithm with this ranking scheme has also been compared to the result of other algorithms on six problems previously used by other authors. The results encourage further research since the method performs better than many other algorithms for the tested constrained single objective problems. It is also shown that the robustness in terms of minimum spread in best found solutions, is better than one of the best methods on a majority of the six tested problems. It was only in the problem containing equality constraints (problem #5) that this method did not perform well. It could not match up to the results for the other algorithms on problem #2 either. The cause of this is an open question for further research. It should also be mentioned that no effort has been made to study the optimal parameter settings such as population size, generation gap, mutation probability, etc. The performance of this ranking scheme may well be better in a more advanced GA, for example an adaptive GA.

An obvious extension to the presented ranking scheme is to address constrained multiobjective problems as well. By redefining  $rank_1$  as the Pareto ranking presented by Fonseca and Fleming in [17], the presented ranking scheme could handle problems with multiple objectives. This is an area of ongoing research and the preliminary results are encouraging.

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# Paper B





# A Pareto-Based Genetic Algorithm Search Approach to Handle Damped Natural Frequency Constraints in Turbo Generator Rotor System Design

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*The detailed design of a turbo generator rotor system is highly constrained by feasible regions for the damped natural frequencies of the system. A major problem for the designer is to find a solution that fulfills the design criterion for the damped natural frequencies. The bearings and some geometrical variables of the rotor are used as the primary design variables in order to achieve a feasible design. This paper presents an alternative approach to search for feasible designs. The design problem is formulated as an optimization problem and a genetic algorithm (GA) is used to search for feasible designs. Then, the problem is extended to include another objective (i.e., multiobjective optimization) to show the potential of using the optimization formulation and a Pareto-based GA in this rotordynamic application. The results show that the presented approach is promising as an engineering design tool. [DOI: 10.1115/1.1760529]*

## Introduction

The detailed design stage of a turbo generator rotor system is an iterative process involving several conflicting objectives and constraints. One attractive approach is to look upon this design process as a multiobjective optimization problem, [1]. The generator rotor system design is highly constrained by feasible regions for the damped natural frequencies (complex eigenvalues). Therefore one of the most important tasks for the designer is to find designs that fulfill these constraints. Previous works in rotordynamic optimization applications with eigenvalue constraints are done by Chen and Wang [2] and Lee and Choi [3]. Multiple objectives with eigenvalue constraints have been studied by Shiau and Chang [4] and Shiau, Kuo, and Hwang [5]. In [2–5] other optimization methods than a genetic algorithm (GA) were used. In [6] Choi and Yang use a GA to lower the first natural frequency in a rotor-bearing system and in [7] they discuss the immune-genetic algorithm (IGA) for multiobjective rotor-bearing problems. In [8] Choi and Yang apply this to a problem with two bearings. In [2–7] constraints for maximum three eigenvalues were used and the damping of the modes were not taken into account.

The objective functions involved in the detailed rotor system design are expected to have complex shapes. Depending on the chosen design variables, the objective functions may even be discontinuous. During the past two decades GAs have successfully been used for optimization of difficult problems. The interest of application of GAs in the area of rotordynamics has recently started, [6–8]. GAs are, however, yet only applicable on problems with computationally cheap objective function evaluations. Foundations of GA are found in Goldberg [9] and a comprehensive survey with engineering applications is given by Gen and Cheng [10]. In [11] Andersson gives a survey of different methods for multiobjective problems. GAs are well suited for multiobjective problems since the search is done from a set of solutions. A common goal for multiobjective GAs is to evolve the Pareto optimal

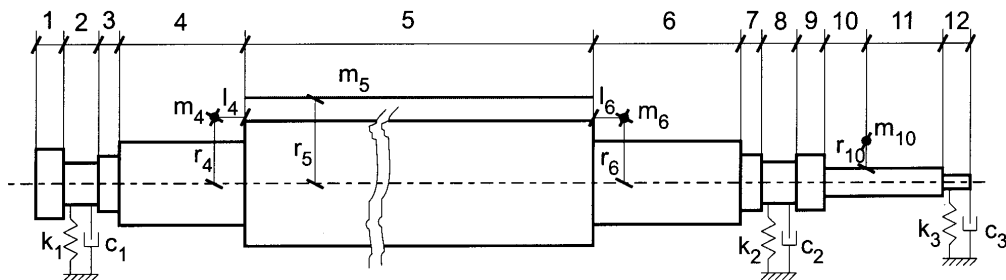
tradeoff surface for the objectives, [1]. To achieve a robust algorithm a Pareto-based ranking scheme is preferable since it is insensitive to the shape of the Pareto optimal front, [12].

A drawback of GAs has been the lack of robust methods to incorporate constraints in the search algorithms. A frequently used approach is to add a penalty term to the objective value, [13,14]. Another interesting way to handle a constrained problem is to transform the constraints into objectives. The problem is then a multiobjective unconstrained problem. This approach is addressed as one method by Coello in [15], Michalewicz in [16], and Fonseca and Fleming in [17].

In this paper this approach is applied to the damped natural frequency constraints. The constraints are mapped on a scalar function and formulated as a design objective. Not only the frequencies, but also the damping ratio for each mode is taken into account. Furthermore, there is no restriction in the number of modes involved in the search. First a real-number encoded GA is applied to search for a feasible solution. Once a feasible solution is found, a second objective is included and a Pareto-based GA, [12], is applied to extend the search, i.e. multiobjective optimization. A Pareto-based GA has not been used earlier in optimization of rotor-bearing systems. The used rotordynamic system model is kept relatively simple in this study. The bearings are here described by linear, isotropic coefficients without cross coupling effects. In a real situation, a more complex model of the rotor system is needed.

In the first section a description of the design problem is given. Factorial experiments are then conducted in section two in order to get some knowledge of the behavior of the damped natural frequencies. In section three a single objective optimization is performed for the new function describing the degree of feasibility with respect to the damped natural frequency constraints. The optimization is also extended to include minimum rotor length as a design objective in the last section. The length of the rotor is related to costs and may, as well as the dynamical objectives, be an important objective at the system level in generator applications.

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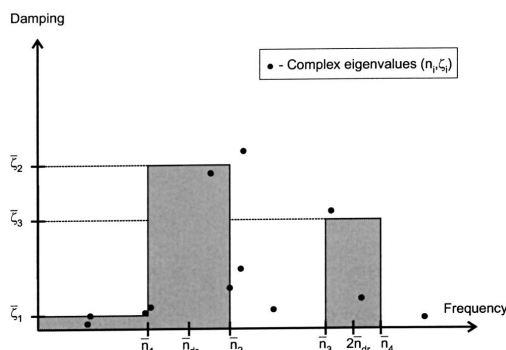


Fig. 2 Damped natural frequency constraints for a generator rotor system. The colored regions are infeasible regions for the damped natural frequencies.

constraints for a 50 Hz generator (operating speed 3000 rpm) used in this paper are based on experience and given in Table 2.

The resulting damped natural frequencies (marked "x") with frequency less than 7500 rpm and damping less than 20% for the nominal design are plotted in Fig. 3. The corresponding eigenmodes are given in Fig. 4.

With this design, the task for the designer would be to increase the damping for the first two modes and change the frequency for mode 3 and 4.

## Factorial Experiments

Factorial experiments are performed in order to study the parameters effect on the damped natural frequencies. Since no single response variable exist it is not possible to perform a factor effect estimation. The purpose with the factorial experiments rather is to attain some knowledge of the objective space for the damped natural frequencies. The parameters that possibly can be changed at this stage of the design process are chosen as factors. The factors with corresponding levels are given in Table 3.

The high and low levels of the bearing factors are set to the vertical and horizontal properties of two existing cylindrical journal bearings. The low level of the geometrical factors are set to their nominal values and the high levels are set with respect to practical limitations.

Since the computational cost is not of major concern full factorial experiments are performed. In the first experiment the bearing factors effect on the damped natural frequencies are studied. The geometrical factors are kept at their nominal levels (low). An intermediate level is also introduced only to get a more dense distribution of the damped natural frequencies in the frequency-damping plane. The results for the  $3^6$  full factorial experiment are found in Fig. 5. The effect of changes in all factors is tested in a second  $2^{11}$  experiment and the result is found in Fig. 6.

From Fig. 5 it is clear that changes in the bearings mainly affects the damping ratio, when the geometry factors are constant. Figure 6 indicates that a single damped natural frequency can be placed in a feasible region. However, no design is found which simultaneously fulfills the design criterion for all the damped natural frequencies. The design indicated by rings in Fig. 5 and

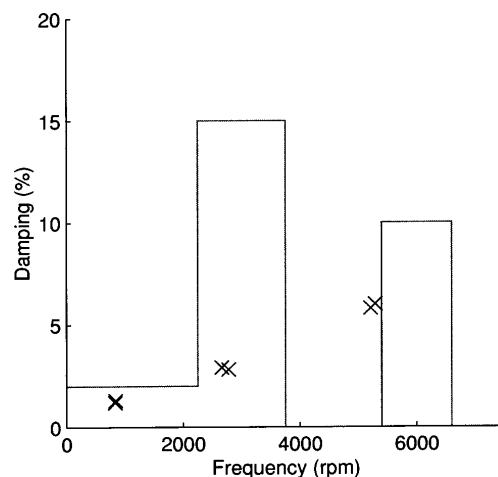


Fig. 3 Damped natural frequencies for nominal rotor-bearing system design. The design criterion is marked by the lines.

Fig. 6 is the best design found with respect to a function defined in the next section about optimization. Still remains the important question whether feasible designs exist if all the factors are chosen as design variables. This is the main issue for the next section.

## Optimization

The damped natural frequency constraints may be expressed by a number of inequality constraints for the modal damping. The number of constraints as well as the damping level for each constraint is, however, dependent on the problem. A function  $D(\mathbf{X})$  is then formulated to describe the degree of feasibility of the damped natural frequency constraints. The basic idea behind this function is that it should decrease if the infeasible damped natural frequencies are moved in the directions indicated by the arrows in Fig. 7. With all damped natural frequencies outside the shaded region, which corresponds to a feasible design, the function is zero. Hence,  $D(\mathbf{X}) \geq 0$ . The function is explained in the Appendix. The function is basically a sum of terms where each term describes an infeasible damped natural frequency. No weights are used since no preference between the damped natural frequencies is done. Each term can take a value within the range  $[0,1]$  depending on the actual position within the infeasible area. The damped natural frequency constraints are fulfilled for a design that satisfies  $D(\mathbf{X})=0$ . The search for feasible designs with respect to the damped natural frequency constraints is now the problem of minimizing  $D(\mathbf{X})$ .

A first search with the design variables  $\mathbf{X} = [k_1, c_1, k_2, c_2, k_3, c_3]$  is performed to find what possibly can be achieved if only changes in the bearings are allowed. The next search is performed with  $\mathbf{X} = [k_1, c_1, k_2, c_2, k_3, c_3, L_3, L_7, L_9, L_{10}, L_{11}]$  as design variables. The additional geometry variables are chosen since these are the only geometry variables that may be changed at the current stage of the design process.

Table 2 Numerical values of damped natural frequency constraints

$\bar{\zeta}_1$	$\bar{\zeta}_2$	$\bar{\zeta}_3$	$\bar{n}_1$	$\bar{n}_{dr}$	$\bar{n}_2$	$\bar{n}_3$	$\bar{n}_4$
2.0%	15.0%	10.0%	2250 rpm	3000 rpm	3750 rpm	5400 rpm	6600 rpm

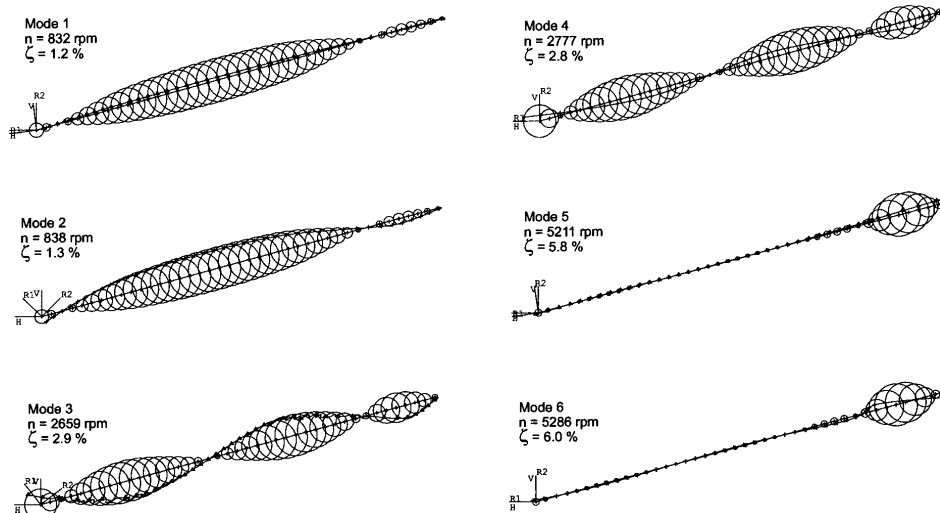


Fig. 4 Eigenmodes for nominal rotor-bearing system design

Due to the electrical requirements the other geometry variables are fixed. The side constraints for each design variable are set to the low and high factor levels found in Table 3.

The chosen search algorithm is a GA with real-number encoding. The GA parameters are set to values that should result in a stable algorithm for the global search. Fine-tuning of all parameters is necessary in order to achieve an efficient algorithm. For definition of GA parameters and methods see Gen and Cheng [10]. Roulette-wheel selection is used with linear ranking and the selection pressure 1.9. Extended intermediate crossover is used to produce new offsprings, [20]. The mutation probability is the same in every generation and selected such that on average one variable per individual is mutated. The used mutation operator is explained in [20]. This operator produces a small mutation step with high probability and large mutation step with low probability. The maximum mutation step size is 10% of the range given by the side constraints for the actual design variable. 90% of the population is replaced in each generation. Hence the reinsertion scheme is of elitist type. The code used is the GEATbx 3.30, which is a toolbox for MATLAB written by Hartmut Pohlheim [21].

Table 3 Factor levels

Factor	Bearing Factors	
	Low level	High level
$k_1$	$1.0 \cdot 10^9$ N/m	$3.0 \cdot 10^9$ N/m
$c_1$	$2.0 \cdot 10^6$ Ns/m	$14.0 \cdot 10^6$ Ns/m
$k_2$	$1.0 \cdot 10^9$ N/m	$3.0 \cdot 10^9$ N/m
$c_2$	$2.0 \cdot 10^6$ Ns/m	$14.0 \cdot 10^6$ Ns/m
$k_3$	$1.2 \cdot 10^8$ N/m	$2.4 \cdot 10^8$ N/m
$c_3$	$4.0 \cdot 10^5$ Ns/m	$14.0 \cdot 10^5$ Ns/m
Factor	Geometrical Factors	
	Low level	High level
$L_3$	310 mm	910 mm
$L_7$	310 mm	910 mm
$L_9$	350 mm	650 mm
$L_{10}$	490 mm	790 mm
$L_{11}$	1200 mm	1500 mm

The search result is found in Fig. 8 and Fig. 9. The population is run for a predefined number of generations hence no convergence criterion is used.

Figure 8 shows that the best design found results in  $D(\mathbf{X}) = 0.593$ . This is found in generation 169. Figure 9 shows that a feasible design ( $D(\mathbf{X}) = 0$ ) is found in the 65th generation for the used design variables and side constraints.

It is now known that at least one feasible design exists for  $\mathbf{X} = [k_1, c_1, k_2, c_2, k_3, c_3, L_3, L_7, L_9, L_{10}, L_{11}]$ . An important question to raise at this point is therefore if other feasible designs exist. A natural continuation of the study would be to tune the search

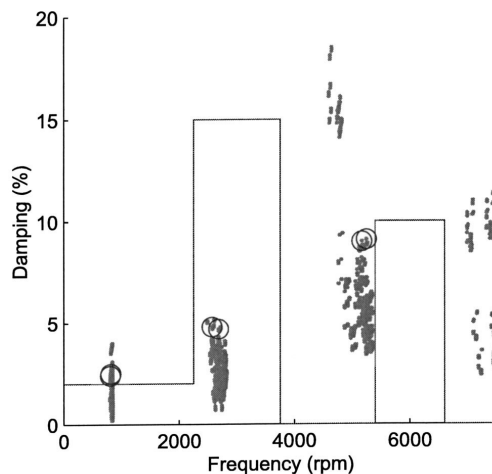


Fig. 5 Damped natural frequency results for  $3^6$  full factorial experiment for bearing factors. The geometrical factors are kept at their nominal values.

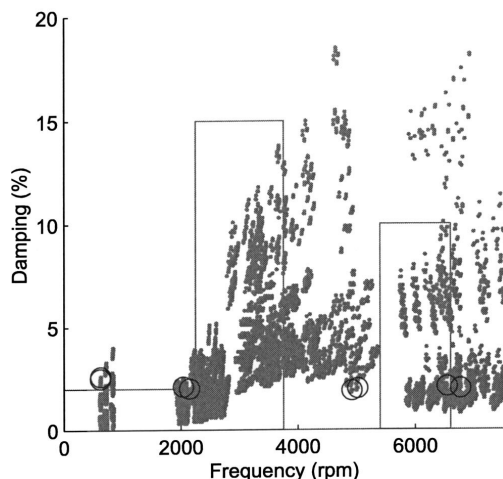


Fig. 6 Damped natural frequency results for  $2^{11}$  full factorial experiment for all factors

algorithm so that the population evolves the whole set of feasible designs. All feasible designs may, however, not be of interests since other objectives also exist. One interesting objective is the increase of rotor length which is necessary in order to achieve a feasible design. The increase of rotor length from the nominal value is formulated as

$$L(\mathbf{X}) = L_3 - L_{3,\text{nom}} + L_7 - L_{7,\text{nom}} + L_9 - L_{9,\text{nom}} + L_{10} - L_{10,\text{nom}} + L_{11} - L_{11,\text{nom}} \quad (1)$$

The nominal values for the lengths are indicated by “nom” as extension of the index in Eq. (1). These nominal values are the same as the low levels of the factors in Table 3. Since an increased rotor length is associated with additional costs, it is of interests to keep the increase at a minimum. The search for interesting designs is now the problem of minimizing  $D(\mathbf{X})$  and  $L(\mathbf{X})$ .

The most interesting design in this case is clearly a design that satisfies  $D(\mathbf{X})=0$  and minimizes  $L(\mathbf{X})$ . Infeasible designs may still contain important information and it is of interest to see the tradeoff (Pareto optimal) curve for these objectives. This would be

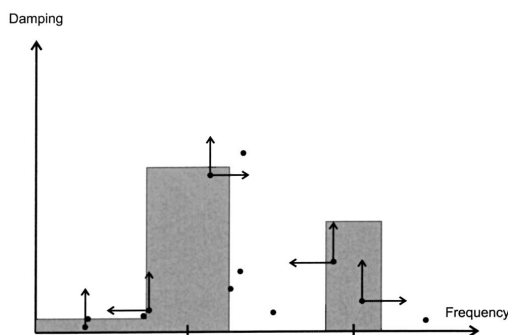


Fig. 7 Directions for movements of infeasible damped natural frequencies that results in a better design with respect to the damped natural frequency constraints

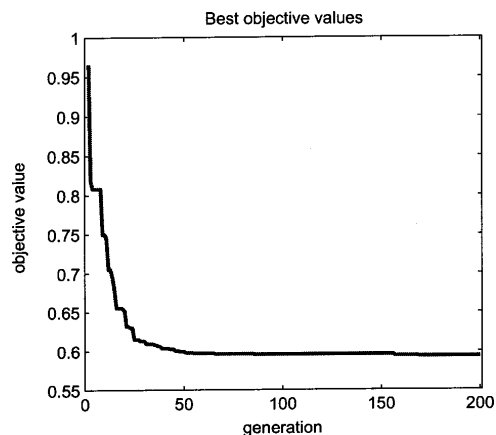


Fig. 8 Search result for  $\min(D(\mathbf{X}))$  as objective and  $\mathbf{X} = [k_1, c_1, k_2, c_2, k_3, c_3]$  as design variables. The population size is 30.

especially important if economic objectives are included and other types of design variables are used. This could for example be bearing types, i.e., categorical variables.

A search for the Pareto optimal front between the both objectives with  $\mathbf{X} = [k_1, c_1, k_2, c_2, k_3, c_3, L_3, L_7, L_9, L_{10}, L_{11}]$  as design variables is done. The used population size is 200 and 20% of the population is replaced in each generation. The algorithm is therefore more of steady-state type. For definition of a steady state GA see p. 108 in [22]. The high elitism is used since no valuable genetic information about the Pareto optimal front should get lost. The ranking scheme by Fleming and Fonseca [12] is used but solutions with exactly the same values in one objective are ranked equally if they are not dominated by other solutions. This makes it possible to evolve several feasible designs. The sharing method described by Gen and Cheng at p. 111 in [10] is used. The sharing

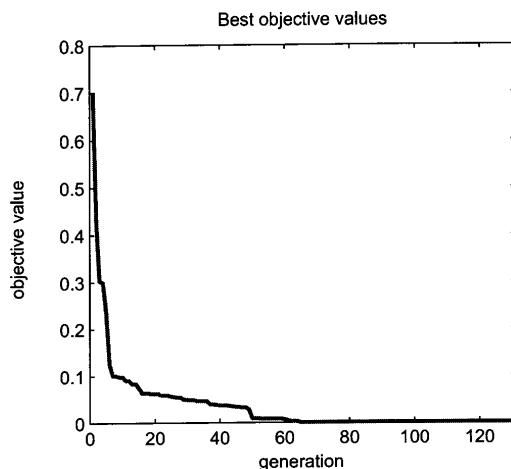


Fig. 9 Search result for  $\min(D(\mathbf{X}))$  as objective and  $\mathbf{X} = [k_1, c_1, k_2, c_2, k_3, c_3, L_3, L_7, L_9, L_{10}, L_{11}]$  as design variables. The population size is 60.

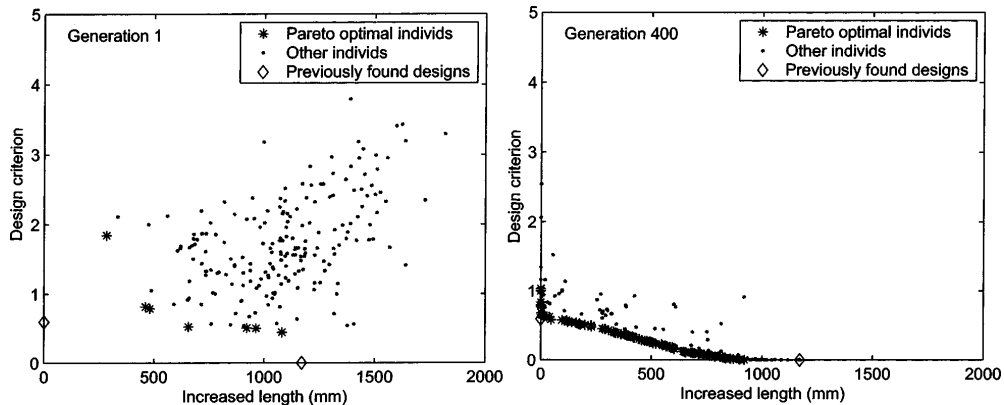


Fig. 10 The initial and 400th generation in the objective space

is based on the distance between designs in the phenotype space (design variable space). All the design variables are linearly scaled on the range  $[0,1]$ . The sharing parameters are set to the GEATbx default values. In this particular case they become  $\sigma_{share}=1.02$  and  $\alpha=1$ . It is however shown that it can be better to base the sharing on the distance between pair wise designs in the objective space, [23]. Sharing on the objective space attempts to achieve a more uniform distribution of solutions at the global Pareto optimal front. For more information on Pareto based GAs, see [1,12].

The initial and 400th generation are given in Fig. 10. The previously best found design and feasible design for the single objective optimization problem  $\min(D(\mathbf{X}))$  are also found in Fig. 10. Twelve feasible designs ( $D(\mathbf{X})=0$ ) are found in the 400th generation. The best found feasible design with respect to  $\min(L(\mathbf{X}))$  results in  $L(\mathbf{X})=918$  mm which is better than the previously found feasible design that resulted in  $L(\mathbf{X})=1171$  mm. The multiobjective approach results in a better design than the single objective approach.

## Results and Conclusions

A new approach to search for feasible designs in a turbo generator rotor system has been shown. In this case a feasible design is when the damped natural frequencies satisfies a design criterion according to Fig. 2. In Table 4 the results from the two factorial experiments and three optimizations are summarized. Each optimization is evaluated against the objective function  $D(\mathbf{X})$  where  $D(\mathbf{X})=0$  corresponds to a feasible design. Two sets of factors or design variables are evaluated namely; bearing properties ( $k_1, c_1, k_2, c_2, k_3, c_3$ ) and bearing properties together with some lengths of the rotor ( $k_1, c_1, k_2, c_2, k_3, c_3, L_3, L_7, L_9, L_{10}, L_{11}$ ). In the table,  $L(\mathbf{X})$  is the increased length of the rotor with respect to the original design. It is normally an advantage if changes are only made in the bearings since changes in the rotor geometry will imply higher cost.

Table 4 shows that no feasible design was found in the factorial experiment. However, if all design variables are included a fea-

**Table 4 Summary of best designs with respect to  $\min(D(\mathbf{X}))$  found in factorial experiments and single objective search. For the multiobjective search the best design found is defined by  $D(\mathbf{X})=0$  and  $\min(L(\mathbf{X}))$ . Factors or design variables within parentheses are held constant in the actual experiment or optimization.**

	Factorial Experiments		Optimization		
	Full 3 <sup>6</sup> with bearing factors	Full 2 <sup>11</sup> with all factors	Single objective with bearing design variables	Single objective with all design variables	Multiobjective with all design variables
$k_1/10^9$ (N/m)	1.0	1.0	1.0	1.4	1.0
$c_1/10^6$ (Ns/m)	2.0	14.0	2.4	8.9	3.8
$k_2/10^9$ (N/m)	1.0	1.0	1.0	1.0	1.0
$c_2/10^6$ (Ns/m)	8.0	14.0	4.4	11.0	12.0
$k_3/10^8$ (N/m)	1.2	1.2	1.2	1.7	2.4
$c_3/10^5$ (Ns/m)	4.0	14.0	5.5	13.0	13.0
$L_3$ (mm)	(310)	910	(310)	727	770
$L_7$ (mm)	(310)	910	(310)	841	762
$L_9$ (mm)	(350)	350	(350)	371	351
$L_{10}$ (mm)	(490)	490	(490)	500	492
$L_{11}$ (mm)	(1200)	1200	(1200)	1392	1203
$D(\mathbf{X})$	0.686	0.0724	0.593	0	0
$L(\mathbf{X})$ (mm)	(0)	1200	(0)	1171	918
Objective function evaluations	729	2048	4566	3516	16,200



sible design could be achieved in the single objective optimization. By including the  $\min(L(\mathbf{X}))$  objective another feasible design with shorter rotor length was found.

The results show that the method of using a Pareto-based GA and multiobjective optimization with the damped natural frequency constraints as one objective is interesting in this rotordynamic application. Normally the design of a turbo generator is based on simple linear dynamical analyses. A serious problem for the designer is, however, how to handle multiple conflicting objectives. With the knowledge of the Pareto optimal tradeoff surface, for the objectives, the designer is better equipped to make a rational choice for the final design.

The presented way to handle the damped natural frequency constraints may also be applicable in other rotordynamic systems. It is also shown that the search easily can be extended to include more objectives. The computational effort will however increase if the dimension of the objective space is increased. Mating restrictions may be needed in order to zoom in on the most important ranges of the objectives. In this paper a linear and isotropic bearing model is used without cross coupling effects. Therefore, a better bearing-pedestal model is required in order to achieve results of more practical interests. The direction for the author's further research is therefore to investigate the possibility to include a bearing code in the search algorithm.

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## Appendix

The function,  $D(\mathbf{X})$ , describing the degree of feasibility with respect to the damped natural frequency constraints is shown by these rows of coding.

```

D(X)=0
for i=1 to N
  if 0<ni≤n̄1 and ζi<ζ̄1
    D(X)=D(X)+(ζ̄1-ζi)/ζ̄1
  elseif n̄1<ni≤n̄dr and ζi<ζ̄2
    D(X)=D(X)+((ζ̄2-ζi)(ni-n̄1))/((ζ̄2(n̄dr-n̄1))
  elseif n̄dr<ni<n̄2 and ζi<ζ̄2
    D(X)=D(X)+((ζ̄2-ζi)(n̄2-ni))/((ζ̄2(n̄2-n̄dr))
  elseif n̄2<ni≤2n̄dr and ζi<ζ̄2
    D(X)=D(X)+((ζ̄3-ζi)(ni-n̄2))/((ζ̄3(2n̄dr-n̄2))
  elseif 2n̄dr<ni<n̄4 and ζi<ζ̄3
    D(X)=D(X)+((ζ̄3-ζi)(n̄4-ni))/((ζ̄3(n̄4-2n̄dr))
  end
end

```

$N$  is the number of damped natural frequencies with frequency less than  $n̄_4$ , see Fig. 2 for definition of the other variables. Only infeasible damped natural frequencies will give a contribution to  $D(\mathbf{X})$ . The magnitude of an infeasible eigenvalues contribution to  $D(\mathbf{X})$  depends on its location within the infeasible region.

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# Paper C



## OPTIMIZATION OF A ROTOR-BEARING SYSTEM WITH AN EVOLUTIONARY ALGORITHM

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### ABSTRACT

In the design of large rotor-bearing systems such as steam turbines, gas turbines or generators, the whole rotor system should be optimal in some sense and simultaneously fulfill the design constraints. The bearing design has a crucial impact on the rotor system characteristics such as complex eigenvalues for example. In system optimizations it is therefore important to consider the bearing design. Until now the actual bearing geometry has seldom been used as design variables in system optimizations.

In this paper the optimization of a rotor-bearing system is discussed. Then, a generic search algorithm suitable to apply in many rotor-bearing design cases is presented. The algorithm is based on an Evolutionary Algorithm. Finally, the optimization of a generator rotor-bearing system with the presented method is shown. The chosen design variables are bearing geometry parameters. The objective is to minimize power loss subject to design specifications for bearing temperatures and complex eigenvalue constraints for the system.

The result shows that a reduction of the power loss in the bearings may be achieved without violation of the system design constraints. The result also shows how the design problem for a specific rotor-bearing system can be handled. The search method is however general and therefore it may be of interests in other similar applications. To conclude, the paper shows a successful application of the presented search algorithm on an industrial rotor-bearing optimization problem.

### INTRODUCTION

In Rajan et al. (1987) it is stated that "The design of a rotor-bearing system is an iterative process in which the parameters that influence the design are modified until the desired design objectives are achieved". The detailed design stage of large rotor-bearing systems is clearly an iterative process involving several conflicting objectives

and constraints. Therefore it is natural to formulate the design of rotor-bearing systems as optimization problems.

Optimization of journal bearing design has recently been performed by for example Hashimoto (2001). The design variables were some geometrical parameters and two of the objectives were minimum friction loss and minimization of maximum oil film temperature rise. At system level several other objectives that are affected by the bearing design are also important. The complex eigenvalues of a rotor-bearing system holds information of stability and damped natural frequencies, Lund (1974). Hence the complex eigenvalues are important objectives. Rotordynamic optimizations with eigenvalue constraints have been performed by Chen and Wang (1993) and Lee and Choi (2000). Multiple objectives with eigenvalue constraints have been studied by Shiao and Chang (1993) and Shiao et al. (1994). Choi and Yang (2000) used a Genetic Algorithm (GA) to lower the first natural frequency in a rotor bearing system. In Choi and Yang (2001 a) they discuss the Immune-Genetic Algorithm (IGA) for multiobjective rotor-bearing problems. In Choi and Yang (2001 b) bearing parameters are included as design variables and the IGA is applied to a rotor-bearing problem with two journal bearings. Bearing geometry parameters were also used in an optimization with another GA by Choi et al. (2002). Nevertheless, nothing was said about the bearing model in Choi and Yang (2001 b) and Choi et al. (2002). In all cited references constraints for maximum three eigenvalues were used and the damping of the modes were not taken into account. Furthermore, the actual bearing geometry parameters were not used as design variables yet than in Choi and Yang (2001 b) and Choi et al. (2002).

The current trend to use Evolutionary Algorithms (EAs) in these kinds of optimizations is natural since EAs can be applied to a wide range of problems and difficult objective functions may be handled. EAs are based on evolution and propagation of good information from parents to offsprings in every generation. The drawback

is the high computational cost required in terms of many objective function evaluations. The fundamental principles of GAs, which is one class of EAs, are given by Goldberg (1989). A good survey with engineering applications is given by Gen and Cheng (2000).

The objective for this paper is to discuss and show how rotor-bearing system optimizations can be performed with the actual bearing geometry parameters as design variables. First, the detailed design of journal bearings and the analysis of a rotor-bearing system are discussed in general. Special emphasis is put on the impact on the systems complex eigenvalues. In the next section a search algorithm, based on a GA, which can handle constrained problems is presented. Finally, the detailed design of the bearings in a generator rotor-bearing system is formulated as an optimization problem. The presented search method is used as the optimization algorithm. The design variables are width and radial clearance for the cylindrical journal bearings. The objective is to minimize the total power loss in the bearings. The constraints are maximum allowed bearing temperatures and a design criterion for the complex eigenvalues of the system.

## NOMENCLATURE

$\lambda_j$	- $j^{\text{th}}$ complex eigenvalue
$\alpha_j$	- real part of $j^{\text{th}}$ complex eigenvalue
$\omega_j$	- imaginary part of $j^{\text{th}}$ complex eigenvalue
$\zeta_j$	- damping ratio of $j^{\text{th}}$ mode
$k$	- number of inequality constraints
$m$	- number of equality constraints
$\mathbf{x}$	- vector of design variables
$n$	- number of design variables
$x_i^l$	- lower bound of design variable $i$
$x_i^u$	- upper bound of design variable $i$
$S$	- search space
$F$	- feasible region
$P$	- number of individuals
$N$	- number of feasible individuals
$W$	- bearing width
$\Delta R$	- bearing clearance
$L_i$	- length of rotor section $i$
$D_i$	- outer diameter of rotor section $i$
$d_i$	- inner diameter of rotor section $i$
$m_i$	- additional mass to rotor section $i$
$r_i$	- radius to additional mass
$l_i$	- distance to additional mass
$\bar{\zeta}_i$	- damping ratio constraint $i$
$\bar{n}_i$	- frequency constraint $i$
$\bar{n}_{dr}$	- rotational frequency
$M$	- number of complex eigenvalues

## ROTOR-BEARING SYSTEM ANALYSIS

The analysis and detailed design of a rotor-bearing system with several hydrodynamic journal bearings is briefly discussed in this section. The design of the rotor has a strong impact on the system characteristics. The rotor design is often constrained by other factors in the late design stage. The bearing design is however easy to

change even in the late design stage. Therefore the discussion in this section is limited to consider only the bearing design since this is an important industrial design problem.

In this paper the steady state bearing analyses are performed with a solver based on ALP3T, Mittwollen and Glienicke (1990). The solver handles a temperature dependent viscosity model. Hence, Reynolds equation and the energy equation are nonlinearly coupled. The results from a bearing analysis are e.g. linearized stiffness and damping coefficients, eccentricity, altitude angle, temperature distribution, power loss, etc.

It is well known that different types of bearings have different dynamical properties. Tilting pad bearings have zero cross coupling stiffness coefficients for example, Vance (1988). Cylindrical bearings may carry higher load but may also cause instability due to asymmetry of the stiffness coefficients. When several bearings are put into a system with a flexible rotor it becomes quite difficult to predict how changes in the bearing geometries affects the system characteristics. This motivates the use of search and optimization methods in this area.

The system dynamic characteristics may be unbalance response, complex eigenvalues, etc. The resulting unbalance response is difficult to predict accurately since the unbalance mass distribution along the rotor is generally not known. The complex eigenvalues are however easily computed and gives much information about the system. The complex eigenvalues appears as conjugate pairs

$$\lambda_j = \alpha_j \pm i\omega_j \quad (1)$$

$\omega_j$  is the damped natural frequency for the  $j^{\text{th}}$  eigenvalue.  $\alpha_j$  is the growth factor and  $\alpha_j > 0$  implies an unstable mode. Stability may also be expressed, Choi and Yang (2001 b), by the damping ratio for the  $j^{\text{th}}$  mode as

$$\zeta_j = -\frac{\alpha_j}{\sqrt{\alpha_j^2 + \omega_j^2}} \quad (2)$$

In the rest of this paper the damping ratio (2) is used instead of the growth factor. The damped natural frequencies are also given in rpm. The complex eigenvalue analyses are in this paper done with an in-house code specially developed for rotordynamic analyses. This is based on the transfer matrix method and a substructure method.

If the bearing analyses and rotordynamic analyses are run in series within an optimization loop some difficulties will occur. For some bearing conditions the bearing analyses will not converge. Then the analyses have to be restarted with other initial guesses for some parameters, for example eccentricity and altitude angle. For this reason it will become quite complicated to perform bearing analyses automatically within an optimization loop. Therefore a bearing database is first created for all the conditions spanned by the chosen design variables. Interpolated values of the stiffness and damping from the

bearing database are then used in the rotordynamic analyses of the system.

One may argue whether geometrical bearing design variables are continuous or discrete variables. In most cases the bearings are chosen from standard dimensions. If this is the case and no other continuous design variables exists, the optimization problem is a pure combinatorial problem. If there is a possibility to affect the bearing design, the geometrical bearing design variables will be of continuous type. In this case the bearing design variables are therefore said to be of continuous type. It is also a common practice to map discrete variables onto corresponding continuous variables if some gradient-based optimization method is used.

It has been indicated that the bearing design has a large impact on the systems complex eigenvalues. Nothing has yet been said about other objectives or constraints. In fact an optimization problem has actually not been formulated yet. This will not be done in this section since the formulation of an optimization problem strongly depends of the particular design case. In the last section an optimization problem for a generator is formulated on the basis of the discussion in this section. The purpose with this case is that it should serve as an example of how these types of problems may be posed. In the next section a generic search algorithm that can handle constrained optimization problems is presented.

### SEARCH ALGORITHM

The purpose of this section is to give the reader a brief introduction to the search algorithm used in the generator optimization application in the next section. The used constraint handling technique will be described in more detail. The generality of the search algorithm makes it interesting for other similar rotor-bearing applications. Before the search algorithm is explained, a definition of the optimization problem is done.

In most rotor-bearing applications several objectives exist. In many cases the objectives may be formulated as constraints. Then the problem is the general nonlinear programming problem (NLP). The NLP-problem with  $k$  inequality constraints and  $m$  equality constraints is formulated as

$$\begin{aligned} & \text{Minimize } f(\mathbf{x}) \\ & \text{subject to} \\ & g_i(\mathbf{x}) \leq 0 \quad i=1,\dots,k, \\ & h_i(\mathbf{x}) = 0 \quad i=1,\dots,m. \end{aligned} \quad (3)$$

$\mathbf{x} = [x_1, x_2, \dots, x_n]$  is a vector of the  $n$  design variables such that  $\mathbf{x} \in S \subseteq \mathbb{R}^n$ . The search space  $S$  is here defined as an  $n$ -dimensional rectangle by the upper ( $x_i^u$ ) and lower ( $x_i^l$ ) bounds for the real valued design variables ( $x_i$ ),  $x_i^l \leq x_i \leq x_i^u \quad i=1..n$ . The feasible region  $F \subseteq S$  is the region of  $S$  for which the inequality and equality constraints are satisfied. Many engineering design problems may be formulated as problem 3. The generator application problem in the next section is of this type.

It has already been indicated that a GA is used. GA research is a large field itself. Therefore only a brief introduction can be done here. The basic operations in the algorithm are shown in Figure 1.

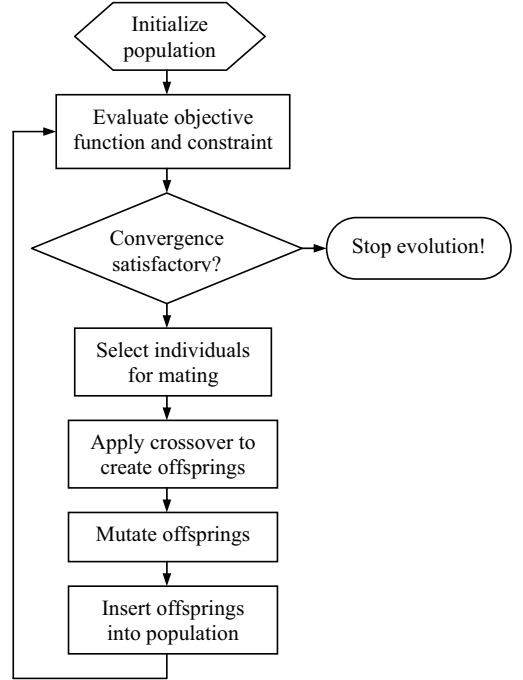


Figure 1. Schematic sketch of the search algorithm.

First, a set of solutions is randomly initialized. No single starting point is required. This set of solutions is called the initial population. Hence, each individual in the population corresponds to a solution of the problem. Parents are selected for mating based on their fitness. Crossover is applied to create new offsprings. These are possibly mutated and inserted into the population again.

If a constrained problem is to be solved (3), a crucial step in a GA is how to rank and select individuals for mating. A drawback of GAs has been the lack of robust and generic methods to handle constraints. Often constraints are formulated as penalty functions that are added to the objective function. Some guidelines for penalty functions are given by Richardson et al. (1989). The success in the use of penalty functions is however often determined of some problem dependent penalty parameters. Angantyr et al. (2003) proposed and evaluated an alternative generic constraint handling technique. Equality constraints are first transformed into inequality constraints as  $|h_i(\mathbf{x})| - \varepsilon \leq 0$  where  $\varepsilon$  is a small tolerance. Then, new objectives are defined as

$$f_i(\mathbf{x}) = \max(0, g_i(\mathbf{x})), \quad i=1,\dots,p \quad (4)$$

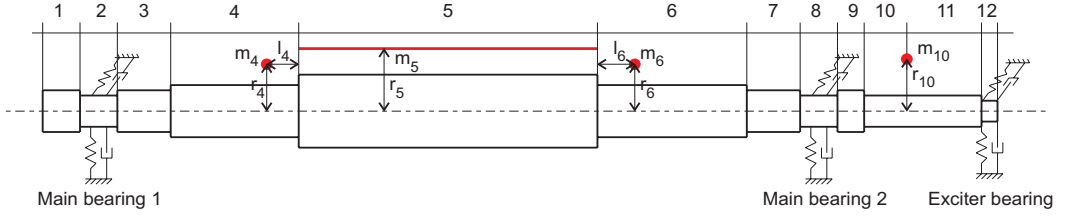


Figure 2. Rotor geometry.

where  $p = k + m$ . Now,  $rank_2$  is defined to be the non-dominated ranking with respect to the objectives (4) as defined by Goldberg (1989).  $rank_1$  is defined as the ranking according to the original objective  $f(\mathbf{x})$  in (3). Each individual is then ranked according to the equation

$$\phi(\mathbf{x}_j) = \frac{N}{P} rank_1(\mathbf{x}_j) + \frac{P-N}{P} rank_2(\mathbf{x}_j). \quad (5)$$

$\phi(\mathbf{x}_j)$  is the new objective value for the  $j^{\text{th}}$  individual in the current population.  $P$  is the number of individuals in the population and  $N$  is the number of feasible individuals in the current population.

This ranking scheme may be used with any type of GA. In the generator application in the next section a real coded GA is used. Linear fitness assignment according to the ranking for the new objective (5) is used. The selective pressure is set to 1.9. The selection method is the roulette wheel selection. See Gen and Cheng (2000) for the just mentioned GA terminology. The number of selected individuals is defined by the generation gap that is set to 95%. Thus 95% of the population is selected for mating and the worst parents are replaced by all the offspring. Hence, an elitist GA is used. Blend crossover, BLX, Eshelman and Schaffer (1993), is used with a probability equal to 1. The mutation operator by Mühlenbein and Schlierkamp-Voosen (1993) which produces a small mutation step with high probability and a large step with small probability is used. The mutation probability is set to  $1/n$  where  $n$  is the number of design variables. The maximum mutation step is set to 10% of the range for the actual design variable. In the next section this search algorithm is applied to a constrained generator rotor-bearing problem.

## A GENERATOR APPLICATION

In this section an optimization of the bearing geometry for a generator is presented. The objective is to minimize the total power loss in the bearings at operational speed (3000 rpm). The bearing surface temperature should not exceed a specified limit and a design criterion for the complex eigenvalues should be fulfilled. This criterion is discussed later in this section. The easiest bearing geometry variables to change are the bearing width (W) and radial clearance ( $\Delta R$ ). Therefore these variables are chosen as design variables in this case.

## Model

The rotor geometry is constant and described in Figure 2. The used values are listed in Table 1.

Table 1. Rotor geometry dimensions.  $L_i$  is the length of section  $i$ .  $D_i$  is the outer diameter and  $d_i$  is the inner diameter of section  $i$ .

Section $i$	$L_i$	$D_i$	$d_i$	Additional mass
1	490 mm	425 mm	0	-
2	540 mm	340 mm	0	-
3	770 mm	425 mm	0	-
4	1800 mm	590 mm	0	$m_4=1670$ kg, $r_4=445$ mm, $l_4=300$ mm
5	4300 mm	890 mm	0	$m_5=1.11$ kg/mm, $r_5=500$ mm
6	2100 mm	590 mm	110 mm	$m_6=1670$ kg, $r_6=445$ mm, $l_6=300$ mm
7	770 mm	425 mm	110 mm	-
8	540 mm	340 mm	110 mm	-
9	350 mm	425 mm	110 mm	-
10	490 mm	245 mm	110 mm	$m_{10}=825$ kg, $r_{10}=225$ mm
11	1200 mm	245 mm	110 mm	-
12	190 mm	180 mm	110 mm	-

The additional masses in Table 1 are added to model various rotor details that do not affect the lateral stiffness of the rotor. The retaining rings in sections 4 and 6 are modeled as ring masses ( $m_4$  and  $m_6$ ) rigidly attached to the rotor. An additional mass ( $m_5$ ) is uniformly applied to section 5 to model the weak copper conductors. The exciter at the end of section 10 is also modeled as a ring mass ( $m_{10}$ ) rigidly attached to the rotor. The rotor is discretized into 200 cylindrical beam elements. The rotor material is steel.

The bearings are of cylindrical type with diameter 340 mm for the main bearings and 180 mm for the exciter bearing. The bearing width and radial clearance are given in Table 2.

Table 2. Nominal values and limits for bearing geometry design variables.

	Bearing 1 and 2			Bearing 3		
	Min	Nom	Max	Min	Nom	Max
W	140 mm	300 mm	340 mm	70 mm	90 mm	180 mm
$\Delta R$	190 $\mu$ m	310 mm	340 $\mu$ m	100 $\mu$ m	150 $\mu$ m	180 $\mu$ m

The exciter bearing is displaced vertically such that equal static bearing load due to gravity is achieved for the main bearings. The static bearing load is in this case 204 kN for the main bearings and 4.37 kN for the

exciter bearing. The bearing characteristics are calculated for an ISO VG 32 oil. It should also be mentioned that the pedestals (bearing supports) are assumed rigid in this case. Before an optimization is performed, the complex eigenvalues of the system are discussed.

### Complex eigenvalue constraints

Obviously, it is not desirable to have modes with damped natural frequencies close to the rotational frequency. Modes with the double rotational frequency are excited by gravity due to the anisotropy of the bending stiffness for the real rotor. These modes should therefore also be avoided. Furthermore, all modes should have some margin to the stability limit. The infeasible regions for the complex eigenvalues are schematically shown in Figure 3.

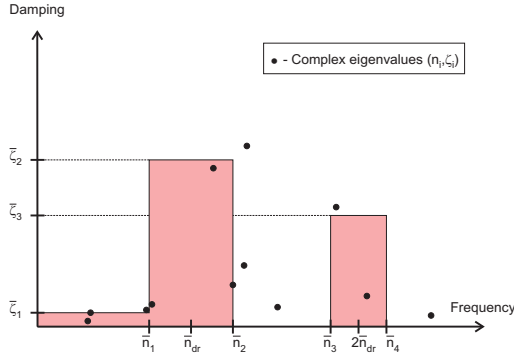


Figure 3. Complex eigenvalue constraints. The colored regions are infeasible regions.

The chosen numerical values of the damped natural frequency constraints for the 50 Hz generator (operating speed 3000 rpm) used in this paper are based on experience and given in Table 3.

Table 3. Numerical values for the complex eigenvalue constraints.

$\bar{\zeta}_1$	$\bar{\zeta}_2$	$\bar{\zeta}_3$	$\bar{n}_1$	$\bar{n}_{dr}$	$\bar{n}_2$	$\bar{n}_3$	$\bar{n}_4$
0.1	15.0	10.0	2250	3000	3750	5400	6600
%	%	%	rpm	rpm	rpm	rpm	rpm

The complex eigenvalue constraints are here described with a scalar function  $D(\mathbf{x})$ . If  $M$  is the number of complex eigenvalues with damped natural frequency less than  $\bar{n}_4$ ,  $D(\mathbf{x})$  is best described by the rows of pseudo code in (6).

The function is a sum of terms where each term in principle describes the normalized distance to the nearest boundary of the feasible region. The value of the function decreases as the infeasible complex eigenvalues approach the nearest boundary of feasibility. A feasible solution with respect to  $D(\mathbf{x})$  satisfies  $D(\mathbf{x}) = 0$

$$D(\mathbf{x}) = 0$$

for  $i = 1$  to  $M$

if  $0 < n_i < \bar{n}_1$  and  $\zeta_i < \bar{\zeta}_1$

$$D(\mathbf{x}) = D(\mathbf{x}) + (\bar{\zeta}_1 - \zeta_i) / \bar{\zeta}_1$$

elseif  $\bar{n}_1 < n_i \leq \bar{n}_{dr}$  and  $\zeta_i < \bar{\zeta}_2$

$$D(\mathbf{x}) = D(\mathbf{x}) + ((\bar{\zeta}_2 - \zeta_i)(n_i - \bar{n}_1)) / (\bar{\zeta}_2(\bar{n}_{dr} - \bar{n}_1))$$

elseif  $\bar{n}_{dr} < n_i < \bar{n}_2$  and  $\zeta_i < \bar{\zeta}_2$

$$D(\mathbf{x}) = D(\mathbf{x}) + ((\bar{\zeta}_2 - \zeta_i)(\bar{n}_2 - n_i)) / (\bar{\zeta}_2(\bar{n}_2 - \bar{n}_{dr}))$$

elseif  $\bar{n}_3 < n_i \leq 2\bar{n}_{dr}$  and  $\zeta_i < \bar{\zeta}_3$

$$D(\mathbf{x}) = D(\mathbf{x}) + ((\bar{\zeta}_3 - \zeta_i)(n_i - \bar{n}_3)) / (\bar{\zeta}_3(2\bar{n}_{dr} - \bar{n}_3))$$

elseif  $2\bar{n}_{dr} < n_i < \bar{n}_4$  and  $\zeta_i < \bar{\zeta}_3$

$$D(\mathbf{x}) = D(\mathbf{x}) + ((\bar{\zeta}_3 - \zeta_i)(\bar{n}_4 - n_i)) / (\bar{\zeta}_3(\bar{n}_4 - 2\bar{n}_{dr}))$$

end

end.

(6)

### Optimization

Now it is time to formulate the optimization problem. The chosen design variables are the bearing variables width and radial clearance for each bearing. Hence,  $\mathbf{x} = [W_1, \Delta R_1, W_2, \Delta R_2, W_3, \Delta R_3]$ . The side constraints for the design variables are given by the minimum and maximum limits in Table 2. The total power loss in the bearings, which should be minimized, is  $P_{loss}(\mathbf{x})$ . The maximum temperatures of the journal surface in each bearing are  $T_1(\mathbf{x})$ ,  $T_2(\mathbf{x})$  and  $T_3(\mathbf{x})$ . The temperature should not exceed  $T_{max}$ . The optimization problem is formulated as

$$\text{Minimize } P_{loss}(\mathbf{x})$$

subject to

$$T_1(\mathbf{x}) \leq T_{max},$$

$$T_2(\mathbf{x}) \leq T_{max},$$

$$T_3(\mathbf{x}) \leq T_{max},$$

$$D(\mathbf{x}) \leq 0.$$

(7)

The maximum allowed bearing surface temperature is set to  $T_{max} = 100^\circ\text{C}$ . The optimization result it given in the next sub section.

### Results

First, the complex eigenmodes for the nominal design are shown in Figure 4. The corresponding damped natural frequencies and damping ratios are also shown in Figure 4. As can be seen from Figure 4, the first mode is unstable for the nominal rotor-bearing design.

In order to get a hint of how the bearing widths and lengths affect the complex eigenvalues, a numerical experiment was set up.  $10^4$  randomly chosen designs within the limits given in Table 2 were evaluated. The results for all the evaluated designs are plotted in terms of their damped natural frequencies and damping ratios in Figure 5. The chosen design constraints (Table 3) are also shown in Figure 5 with the line.

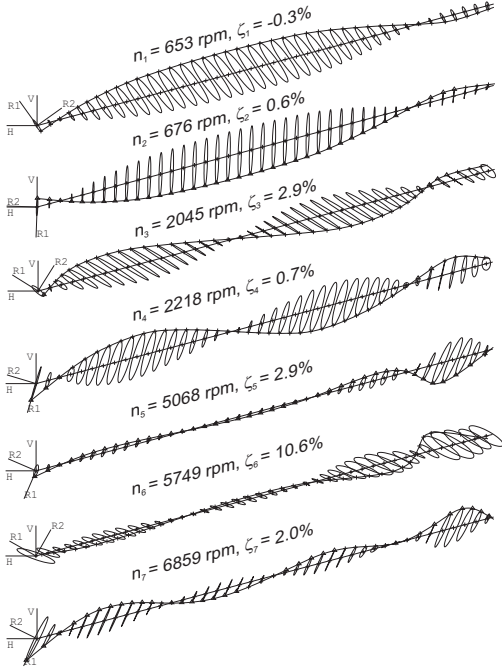


Figure 4. Complex eigenmodes for nominal design.

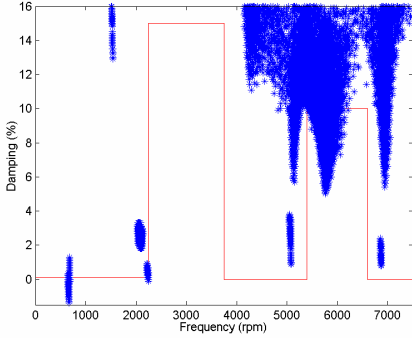


Figure 5. Complex eigenvalues for  $10^4$  randomly generated designs.

Possible infeasible modes found in the experiment are the first two modes, the mode around 2200 rpm and the mode around 5700 rpm. The best-found feasible design in the numerical experiment, with respect to (7), was  $\mathbf{x} = [216 \text{ mm}, 327 \mu\text{m}, 278 \text{ mm}, 331 \mu\text{m}, 78 \text{ mm}, 154 \mu\text{m}]$ .

The optimization was done with a population size of 60 individuals. The search was run for 50 generations and the result is shown in Figure 6.

Figure 6 shows the objective value for the best-found feasible solution, the ratio of feasible solutions in the population and the mean normalized Euclidian distance in the population. The mean normalized Euclidian distance is a measure of how diverse the population is. The first feasible solution is found in generation 3. The best-

found feasible design is  $\mathbf{x}^* = [213 \text{ mm}, 320 \mu\text{m}, 219 \text{ mm}, 286 \mu\text{m}, 71 \text{ mm}, 115 \mu\text{m}]$ . This is found in generation 45. The damping of the two lowest complex eigenmodes for this design is 0.2% and 0.1%.

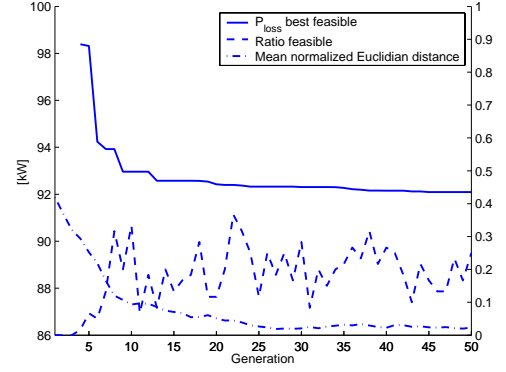


Figure 6. Search history. The left axis corresponds to the power loss for the best-found feasible solution. The right axis corresponds to the ratio of feasible solutions and the mean normalized Euclidian distance between the solutions in the population.

## DISCUSSION AND CONCLUSIONS

Some important aspects in the optimization of the journal bearings in a rotor-bearing system have been discussed. A generic search algorithm suitable to apply for example in constrained rotor-bearing problems was also presented. Since this is based on a GA it is only applicable to problems with reasonably short objective and constraint evaluation times. The success of using the proposed constraint handling method depends (as with other methods) on how the optimization problem is formulated in each specific case.

The presented generator application was a highly constrained problem since the ratio of feasible solutions, i.e. solutions that fulfill the constraints in (7), found in the numerical experiment was only 0.17 %. Still feasible and near optimum designs are found quite early in the search as one can see from Figure 6. Feasible designs with  $P_{loss}(\mathbf{x}) < 93 \text{ kW}$  was found in generation 12 (after less than 700 objective function evaluations).

A comparison between the nominal design, the best found feasible in the numerical experiment and the best found feasible in the optimization is done in Table 4.

Table 4. Comparison of different designs.

Design	$P_{loss}(\mathbf{x})$	$T_1(\mathbf{x})$	$T_2(\mathbf{x})$	$T_3(\mathbf{x})$	$D(\mathbf{x})$
Nominal	113.9 kW	89.4 °C	89.4 °C	65.5 °C	4.0
Best in $10^4$ random trials	101.3 kW	99.2 °C	91.1 °C	66.9 °C	0
Best-found in optimization	92.1 kW	99.9 °C	99.9 °C	75.2 °C	0

The obtained power loss in the bearings for the optimal design was 19% lower than compared to the nominal design. Furthermore, the found optimal solution does



not violate any of the design constraints. It should also be mentioned that the optimization have been performed several times. Solutions close to the presented optimum were found in every optimization.

To conclude, an important industrial rotor-bearing system design problem has been formulated as an optimization problem and successfully solved by a constrained Evolutionary Algorithm.

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# Paper D



# **Constrained Optimization of Gas Turbine Tilting Pad Bearing Designs**

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## **Abstract**

This paper presents the constrained optimization of the tilting pad bearing design on a gas turbine rotor system. A real coded Genetic Algorithm with a robust constraint handling technique is used as the optimization method. The objective is to develop a formulation of the optimization problem for the late bearing design of a complex rotor-bearing system. Furthermore, the usefulness of the search method is evaluated on a difficult problem. The effects considered are power loss and limiting temperatures in the bearings as well as the dynamics at the system level, i. e. stability and unbalance responses. The design variables are the bearing widths and radial clearances.

A nominal design is the basis for comparison of the optimal solution found. An initial numerical experiment shows that it is likely impossible of finding a solution that fulfills all the constraints for the system design. Still, the optimization shows the possibility to find a solution resulting in a reduced power loss while not violating any of the constraints more than the nominal design. Furthermore, the result also shows that the used search method and constraint handling technique works on this difficult problem.

# 1 Introduction

During a late design stage, much of the dynamics of a rotor-bearing system are already determined by earlier design decisions. At this point it may, for example, be difficult to change the rotor geometry since this affects the function of the machine. However, the bearing design is easily changed even at a late stage. The bearing design has an important impact on the system dynamics, e. g. stability. Its design is often an iterative process and opens the possibilities for efficient use of optimization methods.

Optimizations of a single bearing design have been performed in many papers, see for example [1]. Optimizations of the dynamics of rotor-bearing systems with rotor dimension parameters and bearing stiffness coefficients as design variables have also been studied, e. g. [2] and [3]. The authors of this paper believe that optimizing bearing design and system dynamics separately will not result in optimal system performance. This is due to the fact that the dynamics of the whole system is closely coupled to the bearing design. Recent studies with bearing design parameters as design variables and system dynamics as the target have been done in [4-6]. In [4,5], the optimization was performed on a single rotor supported on two bearings (two lobe type), with no information being provided on the bearing model. The rotor weight, natural frequency and damping of a single mode were considered. In [6] a generator with three bearings was studied. A bearing database was used and several modes were considered.

This paper presents the optimization of four titling pad bearings in a rotor system, including gas turbine, intermediate shaft and gear. The objective is to minimize power loss with constraints on bearing temperatures and stability. The amplitudes for several unbalance response cases are also introduced as constraints. The bearing widths and radial clearances are chosen as design variables since these may be subject to changes in the late stage of the design. Non-linear bearing analyses are performed within the optimization loop. This is a highly constrained problem where nothing is known about the shape of the objective function or the constraints. The global optimum is of interest and local search efficiency is of secondary importance. Therefore, a real coded Genetic Algorithm (GA) [7] with the constraint handling method proposed by Angantyr et al. [8] is used. In [4-6,9,10], GAs were used in the design of simpler rotor-bearing systems. The main objective of this paper is to develop an optimization problem formulation for a case with an industrial degree of complexity. The second objective is to evaluate usefulness of the search method.

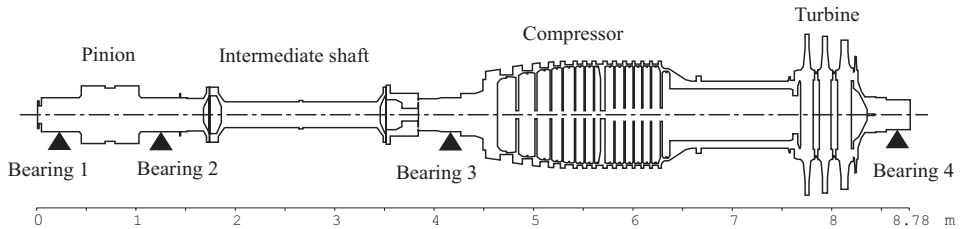
A well-developed and mature nominal design of the rotor-bearing system exists. However, the nominal design is not feasible since some of the constraints are unfulfilled. By definition, a feasible design is a design that satisfies all constraints. If a feasible design cannot be found, the paper should answer

whether a better design than the nominal at least exists. The problem is generally speaking, to be regarded as difficult. The problem formulation is specific for this particular case. However, it might inspire others working on similar problems. The used search method and constraint handling technique are generally and widely applicable and should therefore be of interest to a broad audience.

Bearing and system dynamics analyses are presented in the next section. The objectives and constraints for the system design are defined in section 3. An initial numerical experiment is explained in section 4. The purpose of this experiment is to gain a better insight of the parameters possible effect on the system. The actual optimization problem is formulated in section 5 and the results of the numerical experiment and optimization are given in section 6. Finally, some concluding remarks are given.

## 2 Model and analysis approach

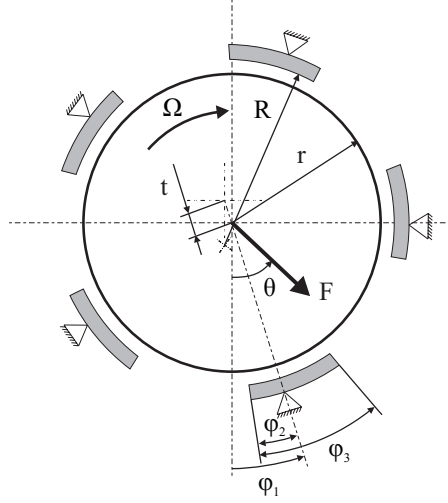
This section describes the dynamical analyses and models. A schematic sketch of the 43 MW gas turbine rotor-bearing system is found in Fig. 1. The compressor shaft is coupled via an intermediate shaft to the pinion wheel in the gear (left side in Fig. 1). The total weight of the rotor is 8800 kg and the total polar inertia is  $778 \text{ kgm}^2$ . The gas turbine normally operates at 6608 rpm.



**Figure 1. Schematic sketch of rotor.**

The bearing stiffness and damping properties are nonlinear, but a linear bearing model is used in the dynamical analyses of the rotor-bearing system. Hence, the bearings are described by eight stiffness and damping coefficients [11]. The rotor is discretized into 366 elements. For the dynamical analyses an in-house code based on the transfer matrix method, Genta [12], and a sub-structuring method is used. The bearing supports 1 and 2 are described by simple mass-spring models. The gas turbine stator is described by its modal properties resulting from an FEM modal analysis of the complete stator. Hence, there is a coupling between bearing supports 3 and 4 in the dynamical analysis model. A modal damping ratio of 3% is assumed for the stator modes. Complex eigenvalue analyses (stability) and steady-state unbalance response analyses are performed in the optimization.

The bearings are hydrodynamical journal bearings of the tilting pad type. The bearings are analyzed under stationary conditions. For this purpose, a code based on ALP3T [13] is used. Typical output from the bearing analyses are eccentricity and attitude angle that define the center of the shaft. Other results are the linear stiffness and damping coefficients, power loss, maximum pad surface temperature, etc. In these analyses a temperature dependent viscosity model is used with similar fluid properties to an ISO VG32 oil. Figure 2 defines the bearing geometry.



**Figure 2. Tilting pad bearing geometry.**

The rotational direction is indicated by  $\Omega$ .  $R$  defines the radius of each lobe;  $r$  defines the radius of the shaft. The preset distance for each lobe is  $t$ . Hence, the radial clearance is defined as  $\Delta R = R - r - t$ . The width of the bearing is  $B$ . The geometry of the lobes is defined by the angles  $\phi_1$ ,  $\phi_2$  and  $\phi_3$ . The applied static load is  $F$  and  $\theta$  defines the direction of the force. For gravitational load only, the angle  $\theta$  is zero. Observe that this is not the case if the gas turbine transfers a torque load. The contact forces in the gear then give rise to static loads in other directions. The geometry for the nominal design of the five bearings is given in Table 1.

**Table 1. Bearing geometry for nominal design.**

Bearing	$r$ [mm]	$B$ [mm]	$\Delta R$ [ $\mu\text{m}$ ]	$t$ [ $\mu\text{m}$ ]	$\phi_1$ [ $^\circ$ ]	$\phi_2$ [ $^\circ$ ]	$\phi_3$ [ $^\circ$ ]
1 and 2	125	250	233	150	20	27	60
3	125	140	291	22	-36	24	60
4	110	154	255	23	-36	24	60



The dynamical analyses of the system are linear but the bearing analyses are non-linear. The computational time required at the present date on a standard PC for the linear dynamical analyses is a few seconds, whereas the time required for a single bearing analysis is tens of seconds.

### 3 Design objectives and constraints

The objectives and constraints for the design of the rotor-bearing system are defined in this section. Clearly there exist several objectives for the system design that will not be considered here. The focus for this work is the optimization of the bearing designs. Therefore, only the objectives and constraints affected by the bearing designs will be defined.

#### 3.1 Bearing design

A target for the overall design of the gas turbine is to achieve a high efficiency. The bearings should obviously be designed for minimal power losses. This will be the objective of the problem formulation in section 5.

It is important that the white metal on the pad surfaces in the bearings does not become too warm. Therefore the bearing designs are constrained by the maximum allowed pad surface temperature. A function describing the feasibility with respect to the bearing surface temperature constraints is formulated as

$$T(\mathbf{x}) = \sum_{p=1}^4 \max \left( 0, \frac{T_p - T_p^L}{T_p^L} \right). \quad (1)$$

$T_p$  is the maximum bearing surface temperature for bearing No.  $p$  at normal operating conditions.  $T_p^L$  is the maximum allowed bearing surface temperature for bearing No.  $p$ . Equation (1) gives a zero value for a design  $\mathbf{x}$  having all temperatures below the specified limits. The vector of design variables  $\mathbf{x}$  and maximum allowed bearing temperature  $T_p^L$  are defined later.

#### 3.2 Stability

A necessary condition is for the system to be stable. Tilting pad bearings are known to have good stabilizing properties due to the small cross-coupling coefficients in the bearing stiffness matrix. Hence, stability should probably not be difficult to achieve. Still, this important criterion must be checked since an unstable system will certainly lead to failure and possibly severely damage the rotor-bearing system. Stability is determined by the complex eigenvalues of the system. These appear as conjugate pairs

$$\lambda_j = \alpha_j \pm i\omega_j . \quad (2)$$

$\omega_j$  is the damped natural frequency for the  $j^{\text{th}}$  eigenvalue.  $\alpha_j$  is the growth factor and  $\alpha_j > 0$  implies an unstable mode. Stability may also be expressed [4] by the damping ratio for the  $j^{\text{th}}$  mode as

$$\zeta_j = -\frac{\alpha_j}{\sqrt{\alpha_j^2 + \omega_j^2}} . \quad (3)$$

For a stable operation, the theoretical criterion is now  $\zeta_j > 0 \ \forall j$ . This must be ensured for all operating conditions. Therefore, the complex eigenvalues of the system must be stable for running up and normal operation. Since it is practically impossible to check stability conditions for the whole range up to normal speed, discrete cases  $k$  are defined and summarized in Table 2.

**Table 2. Definition of the different stability cases.**

Case ( $k$ )	Operating conditions
1	40 % speed and no torque load
2	60 % speed and no torque load
3	80 % speed and no torque load
4	100 % speed and no torque load
5	100 % speed and full torque load (normal operating conditions)

For 100% torque load,  $F = 154 \text{ kN}$  and  $\theta = -160^\circ$  for bearing 1 and  $F = 148 \text{ kN}$  and  $\theta = -157^\circ$  for bearing 2. The bearings at the gear are therefore subject to high load during normal operation.

For the real rotor there may exist destabilizing effects such as internal damping and fluid induced forces, though these effects are not considered in the analysis. Therefore a minimum required damping ratio is preferable. This is set to  $\zeta^L$ . A function describing the feasibility regarding the operating case  $k$  is formulated as

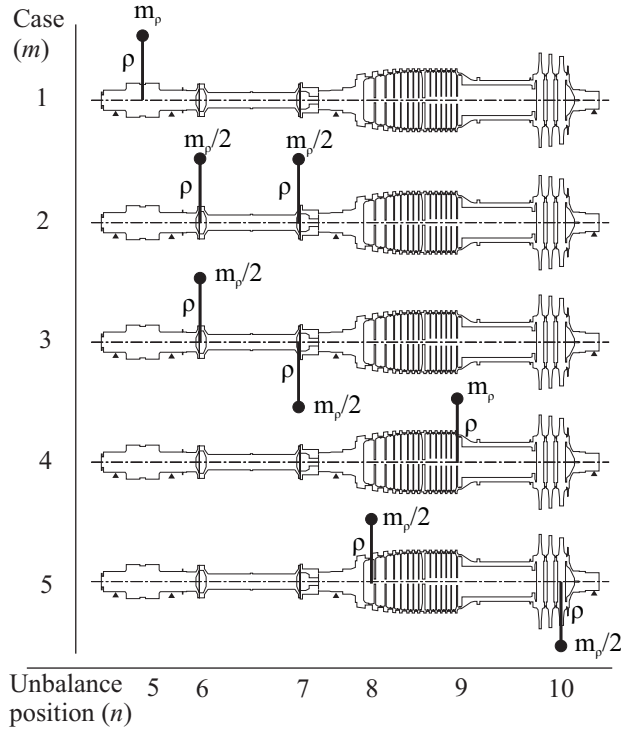
$$C_k(\mathbf{x}) = \sum_{j=1}^N \max \left( 0, \frac{\zeta^L - \zeta_{jk}}{\zeta^L} \right) . \quad (4)$$

The number of complex eigenvalues in the frequency range of interest is  $N$ . The modal damping for the mode  $j$  and case  $k$  is  $\zeta_{jk}$ . With this formulation a feasible design  $\mathbf{x}$  regarding the case  $k$  satisfies  $C_k(\mathbf{x}) = 0$ . For an infeasible design  $C_k(\mathbf{x}) > 0$  holds.

### 3.3 Unbalance response

For practical design purposes the unbalance response analysis of a distributed real rotor system can be problematic due to the actual unbalance mass distribution along the rotor being generally unknown. The system should still be designed to have the unbalance response below standard values. In [14] an interesting approach based on singular value decomposition is proposed. This overcomes the problem with the unknown unbalance mass distribution and defines a general and conservative criterion that could be used for design purposes.

Another approach is to study the modal unbalance sensitivity [15]. When doing so, the interaction between modes is neglected. In this paper a third approach is used. A set of different point mass unbalance cases is defined, as schematically shown in Fig. 3.



**Figure 3. Unbalance cases.**

The rotor mass between two consecutive bearings is the unbalance mass  $m_p$ . This is 790 kg in case 1, 1050 kg in cases 2 and 3 and 6820 kg in cases 4 and 5. The radius to the unbalance mass is  $\rho$  and defined to fulfill the balancing grade G2.5 according to ISO 1904/1. In case 3 and 5 the unbalance masses are shifted

180° in phase. The unbalance response is calculated for excitation frequencies from 85% to 120% of the operating speed. The bearings are analyzed for seven different speeds in this range and linear interpolation of the bearing coefficients is done for intermediate excitation frequencies.

The maximum amplitudes are checked at the bearing positions (1 to 4) and unbalance mass positions (5 to 10). Clearly this approach would generate 50 constraints, i.e.  $5 \times (4+6)$ . To get a more manageable problem constraint functions based on a weighted sum approach are formulated for each unbalance case.

Now let  $A_{mn}$  be the vibration amplitude (major axis of elliptical whirling orbit) for the unbalance case  $m$  and position defined by  $n$ . The bearing positions correspond to  $n = 1$  to 4. The unbalance positions indicated in Fig. 3 correspond to  $n = 5$  to 10. For  $n = 1$  to 4,  $A_{mn}$  is the maximum relative amplitude between the shaft and the bearing journals. Also let the maximum allowed vibration amplitude for unbalance  $m$  case and position  $n$  be  $A_{mn}^L$ . The function that describes the degree of feasibility with respect to the unbalance case  $m$  is then formulated as

$$U_m(\mathbf{x}) = \sum_{n=1}^{10} \max \left( 0, \frac{A_{mn} - A_{mn}^L}{A_{mn}^L} \right). \quad (5)$$

If  $U_m(\mathbf{x}) > 0$  the unbalance case  $m$  is violated for the design  $\mathbf{x}$ . A feasible design  $\mathbf{x}$  satisfies  $U_m(\mathbf{x}) = 0$ .

### 3.4 Constraint limits

The limits for the constraints are defined in this section. The maximum allowed temperature in bearings 1 and 2 is 110°C and 100°C for bearings 3 and 4, i. e.  $\forall p \in \{1,2\}: T_p^L = 110^\circ\text{C}$  and  $\forall p \in \{3,4\}: T_p^L = 100^\circ\text{C}$  in Eq. (1). The minimum allowed damping ratio for the complex eigenmodes is  $\zeta^L = 2\%$ . For the unbalance response, the relative amplitude in the bearings should not exceed 22  $\mu\text{m}$ , i.e.  $\forall m \in \{1,..,5\}$  and  $n \in \{1,..,4\}: A_{mn}^L = 22 \mu\text{m}$ . The amplitude at the unbalance positions should not exceed 33  $\mu\text{m}$ , i.e.  $\forall m \in \{1,..,5\}$  and  $n \in \{5,..,10\}: A_{mn}^L = 33 \mu\text{m}$ .

The temperature and unbalance constraint are the most difficult to satisfy. The nominal design does not fulfill the constraints Eq. (1) and Eq. (5) with the above given limits. However, if the constraint limits are set to the values as in Table 3, the nominal design becomes feasible.

**Table 3. Limits required for feasibility of the nominal design.**

$T_1^L$ [°C]	$T_2^L$ [°C]	$A_{26}^L$ [μm]	$A_{27}^L$ [μm]	$A_{43}^L$ [μm]	$A_{46}^L$ [μm]	$A_{47}^L$ [μm]
115	111	41.4	58.6	35.3	43.2	41.2

## 4 Numerical experiment

To gain some insight about the parameters effect on the studied rotor-bearing system, an initial numerical experiment is performed. Although there are a wide variety of methods on how to design experiments, Montgomery [16], a simple but informative method is chosen. Solutions are generated randomly with uniform distribution within the limits specified in Eq. (6). The chosen factors for variation are the bearing widths and radial clearances. The ranges for the variation in the factors are

$$\begin{aligned}
100 \text{ mm} &\leq B_1, B_2, B_3 \leq 250 \text{ mm}, \\
88 \text{ mm} &\leq B_4 \leq 220 \text{ mm}, \\
138 \text{ μm} &\leq \Delta R_1, \Delta R_2 \leq 250 \text{ μm}, \\
138 \text{ μm} &\leq \Delta R_3 \leq 310 \text{ μm}, \\
121 \text{ μm} &\leq \Delta R_4 \leq 270 \text{ μm}.
\end{aligned} \tag{6}$$

The indices correspond to the bearing position. The choice of factors for the numerical experiment is based on the fact that these are the only possible factors to change at a late design stage. Furthermore, it is known from experience that these factors (at various amounts) affect the objectives and constraints discussed in the previous section. The result of this experiment is discussed in section 6.

## 5 Optimization problem

The subject of optimization is the design of the bearings. Since these are parts of the system, each bearing cannot be optimized separately. The objectives and constraints for the whole system design must be considered. One possibility may be to divide the overall problem into a top-down hierarchy of sub problems. The Target Cascading method [17] is an example of how to formulate and solve problems with this approach. In this case, the sub-problems (bearing designs) are tightly coupled via numerous implicit constraints. Although the objectives for the sub-system level are quite obvious, they would be difficult to achieve. Therefore, a classical formulation of the optimization problem is used here. The optimization problem is stated as

$$\begin{aligned}
& \text{Minimize } P(\mathbf{x}) \\
& \text{subject to} \\
& T(\mathbf{x}) = 0, \\
& C_k(\mathbf{x}) = 0 \quad \text{for } k = 1 \text{ to } 5, \\
& U_m(\mathbf{x}) = 0 \quad \text{for } m = 1 \text{ to } 5.
\end{aligned} \tag{7}$$

$P(\mathbf{x})$  is the total power loss in the bearings under operating conditions. The functions in the constraints are defined by Eq. (1), Eq. (4) and Eq. (5). The design variables are the same as the factors in the initial numerical experiment. Hence, the vector of design variables is  $\mathbf{x} = [B_1, \Delta R_1, B_2, \Delta R_2, B_3, \Delta R_3, B_4, \Delta R_4]$ . The side constraints of the design variables (upper and lower limits) are given in Eq. (6).

Nothing is known about the shapes of the objective and constraint functions. Since the global optimum is of interest, a robust global search method is a preferable optimization algorithm. Therefore, a real-coded GA is chosen. A good introduction to real coded GAs is the book by Gen and Cheng [7]. A frequently used method to handle constraints is by penalty functions [18]. A drawback of these methods is often that problem dependent penalty coefficients have to be specified. In [8], Angantyr et al. proposed a robust and generic constraint handling method requiring no extra parameters to be set, i. e. penalty coefficients. This method is used to handle the constraints. Table 4 shows the other GA parameter settings.

**Table 4. GA parameter settings.**

Representation	real number
Population size	80
Crossover operator	BLX- $\alpha$ [19] ( $\alpha = 0.25$ )
Crossover probability	1
Mutation operator	[20]
Mutation probability	0.13
Generation gap	95 %

The generation gap is the ratio of individuals replaced in each generation. Since this is set to 95% an elitist GA is used. In addition to elitism the population size and mutation probability are the most important parameters for the convergence of the GA. There is always a contradiction between computational effort and the risk of premature convergence when using a GA. The population size is here chosen to be  $10 \times 8$  since the dimensionality of the optimization problem is 8. The mutation probability is set to  $1/8$ . These values should give a reasonable

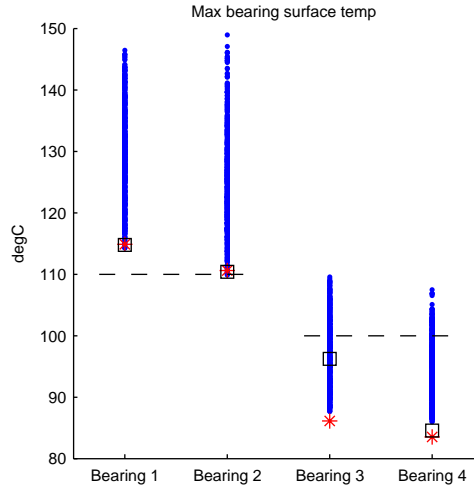
trade-off between convergence and computational time. It should also be mentioned that a linear ranking scheme is used with selective pressure 1.6.

The computational time required for a single evaluation of the objective function and the constraints is approximately 5 min. This rather long time depends on the 44 required non-linear bearing analyses (i.e. 4 bearings and 4 cases at 0% load and 7 cases at 100% load). Since a considerable amount of evaluations is necessary in the GA, a parallel implementation is used. The computations are done on a cluster of standard PCs running under Linux with four nodes in the cluster being used in the computations. The results from the optimization are summarized in the next section.

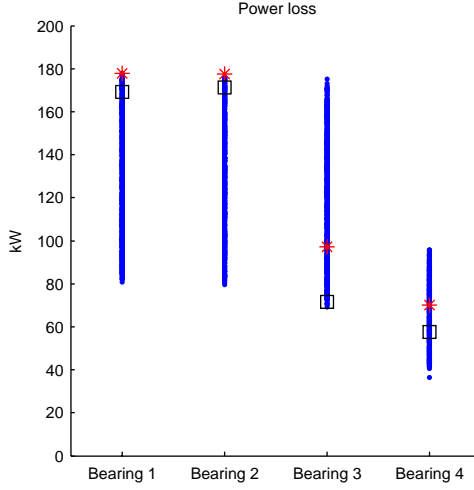
## 6 Results

The nominal design, given in Table 5 and indicated by the stars in Fig. 4 and Fig. 5, gives a total power loss of 523 kW in the bearings. As can be seen from Fig. 4, the temperature constraints for bearings 1 and 2 are not satisfied by the nominal design. The temperatures for bearings 3 and 4 are below the constraint limits. Furthermore, the unbalance constraint cases 2 and 4 (Eq. (5)) are not satisfied by the nominal design. The weak intermediate shaft gives rise to the high unbalance responses in these cases. The nominal design satisfies all the constraints (Eq. (4)) for the different stability cases.

The result from the numerical experiment is shown as dots in Fig. 4 and Fig. 5. The dots show the result for 600 randomly generated designs (due to the dense spacing the result almost appears as solid lines in Fig. 4 and Fig. 5).



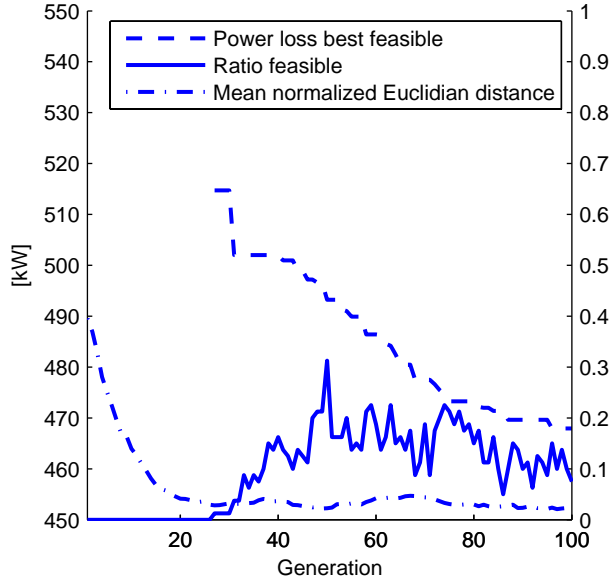
**Figure 4. Bearing temperature for randomly generated designs (dots), nominal design (star) and optimal design (square).**



**Figure 5. Power loss for randomly generated designs (dots), nominal design (star) and optimal design (square).**

Figure 4 shows that it seems unlikely that the bearing temperature constraint is possible to satisfy. Furthermore, none of the 600 randomly generated designs fulfilled the limits for the second and fourth unbalance cases. Hence, finding a feasible solution for the constraints given in Eq. (7) seems very unlikely. Still it is interesting to know if there exist better designs than the nominal. Therefore the constraint limits that were violated by the nominal design are reformulated and set to the values as in Table 3. Thereby the existence of at least one feasible design for the reformulated constraints is known. Now it is possible to search for a design with a lower power loss that is not worse in the violated constraints than the nominal design. The search result from the optimization with the reformulated constraint limits is shown in Fig. 6.





**Figure 6. Search history in optimization.**

Figure 6 shows the mean normalized Euclidian distance in the design variable space between the individuals in the population (right axis). This can be seen as a measure of the convergence in the population. Figure 6 also shows the ratio of feasible solutions (right axis) in the population. This is zero until generation 27 when the first feasible individual appears. The best design (optimal), given in Table 5, is found in generation 96. The power loss for this design is 468 kW. To facilitate comparison, the nominal design is also shown in Table 5. The optimal design is indicated by the squares in Fig. 4 and Fig. 5.

**Table 5. Nominal and optimal design.**

Design	$B_1$ [mm]	$\Delta R_1$ [ $\mu\text{m}$ ]	$B_2$ [mm]	$\Delta R_2$ [ $\mu\text{m}$ ]	$B_3$ [mm]	$\Delta R_3$ [ $\mu\text{m}$ ]	$B_4$ [mm]	$\Delta R_4$ [ $\mu\text{m}$ ]
Nominal	250	233	250	233	140	291	154	255
Optimal	239	243	241	248	101	256	124	266

The widths of the bearings for the optimal design are smaller than for the nominal design. Furthermore, all the radial clearances except for the third bearing are larger for the optimal design than the nominal design. The radial clearances for the first, second and fourth bearing are the only design variables for the optimal design that are close to the side constraints Eq. (6).

## 7 Conclusions

In this paper a GA-optimization is performed on a gas turbine rotor system with four tilting pad bearings. A real design situation has been formulated as an optimization problem. Non-linear bearing analyses and several load cases are included. According to the numerical experiment, it is likely impossible to find a solution that satisfies the bearing temperature and unbalance response constraints. Still the optimization shows that it is possible with minor modifications of the bearings to find a better design than the nominal design. This design does not violate any of the constraints more than the nominal design and gives 10.5 % reduced power loss. Hence, a significant improvement was possible to achieve on a rotor bearing system that is regarded in the industry as well developed. Practically, it would be difficult to find this solution without the use of a search method.

Another conclusion of the work is that the constraint handling method [8] is able to locate the feasible region even for this highly constrained problem. Hence, it seems to be robust and work well. In this case only a single optimization run is performed which shows the strength of the used constraint handling method. It works in the first shot since no problem dependent parameters must be specified.

Finally, an important aspect that not yet has been addressed is the robustness of the found optimal solution. This is however the matter for further research.

## 8 Acknowledgements

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# Paper E



**PWR2006-88093**

## **FEASIBILITY OF SITE BALANCING REWIRED 2-POLE TURBO GENERATORS**

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### **ABSTRACT**

At power plants with large distances to workshops and balancing facilities the outage time for rewinding of a generator rotor may be considerably reduced if the work can be carried out on site. However a problem arises when balancing is concerned. If the rotor is balanced on site, i.e. in the stator and driven by the turbine, the balancing weights at the rotor body are not accessible. This constraint and the critical speeds of the rotor determine the feasibility to achieve an acceptable balancing state.

This paper first presents estimates of the expected unbalance introduced by rewinding based on the balancing weight distribution for a set of rewired rotors. These estimates are then applied to a rotordynamical model and a search algorithm is used to see what can be achieved by balancing in the accessible balancing planes. Several numerical examples are studied. Finally, some guidelines for feasibility of site balancing rewired turbo generator rotors are defined based on the numerical results.

### **INTRODUCTION**

Balancing of flexible rotors is a field of rotordynamics that is of extraordinary importance for the power industry. In normal circumstances field balancing or balancing in a balancing pit is routine work today. The two most important balancing methods are the influence coefficient method [1] and the modal balancing method [2]. In practice the influence coefficient method is the

dominating one. The current research trend within this field is towards methods that do not need too many time consuming trial runs, see for example [3-5]. A review of different rotor balancing methods is given in [6].

Regardless which balancing method is chosen the possibility to achieve a well balanced rotor depends on the number of critical speeds, near or in the speed range for the rotor, as well as the positions for the balancing planes. In some situations only a limited number of balancing planes are accessible for practical reasons, such as site balancing of turbo generators when the balancing planes at the rotor body cannot be accessed with the rotor positioned in the stator.

If the rotor is rigid (i.e. the 1<sup>st</sup> bending mode is well above the operational speed), clearly a 2-plane balancing is enough. However if the rotor is super critical and influenced by the 3<sup>rd</sup> bending mode at the operational speed, it is probably not possible to balance the rotor in only two balancing planes. The question for this paper therefore is: Where is the limit for feasibility of site balancing rewired rotors to an acceptable degree if only two non-optimal balancing planes are accessible? It should be noted that a rotor should be balanced to an acceptable state, not necessarily a perfect balancing state.

This paper therefore first presents estimate values of the introduced unbalance by rewinding. These estimates are based on known balancing data (i.e. balancing weight distribution) for a set of rewired and balanced 2-pole turbo generator rotors.

Then a formulation useful when judging whether a rewinded rotor can be balanced to a satisfactory level or not is presented. The expected introduced unbalance by rewinding is applied to a rotordynamical model and a search algorithm is used to see what can be achieved by balancing in the accessible non-optimal balancing planes.

A set of numerical examples is therefore studied with the presented search method. Finally, some guidelines of the feasibility for site balancing rewinded 2-pole turbo generators are presented based on the results from the numerical examples.

### UNBALANCE INTRODUCED BY REWINDING

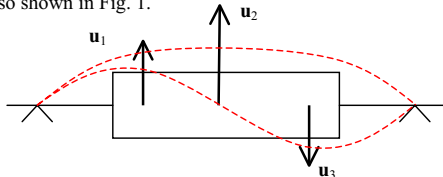
The unbalance introduced by rewinding is estimated based on the difference in balancing state for rotors that have been rewinded and balanced in balancing pits. Air-cooled rotors in the range 20-200 MVA are studied. An overview of these rotors is given in Table 1.

**Table 1. Summary of studied rotors.**

Rotor	Speed [rpm]	Power [MVA]	Active length [mm]	Active diameter [mm]	Rotor weight [ton]	Critical speeds [rpm]	
						1 <sup>st</sup> mode	2 <sup>nd</sup> mode
1	3000	150	3783	1025	36.3	1400-1450	3400-3700
2 - 4	3600	126	2900	1020	28	1350-1700	2900-4000
5	3600	200	5000	1020	45	1000-1100	2700-2900
6	3000	96.5	2650	1020	25.4	~1800	~5000
7	3600	140	4450	890	27	950-1100	2300-2800
8 - 9	3000	22	2040	788	11	1400-1500	3900-4400

The ranges for the critical speeds given in Table 1 are calculated for the conditions on site. By 1<sup>st</sup> mode means the first bending mode and by 2<sup>nd</sup> mode means the second bending mode. Due to anisotropy in the bearing and pedestal stiffness there exists often vertical and horizontal bending modes.

Now we consider a typical generator rotor with three balancing planes at the rotor body indicated by the vectors  $\mathbf{u}_1$ ,  $\mathbf{u}_2$  and  $\mathbf{u}_3$  in Fig. 1. The approximate mode shapes of the first two modes are also shown in Fig. 1.



**Figure 1. Typical generator rotor with three rotor body balancing planes.**

Now we assume that the introduced unbalance due to rewinding is the difference in balancing state before and after rewinding.

Hence we let the found differences in the each balancing plane define the vectors  $\mathbf{u}_1$ ,  $\mathbf{u}_2$  and  $\mathbf{u}_3$ . An estimate of the modal unbalance for the first mode then is

$$\mathbf{u}_{\text{mode } 1} = \mathbf{u}_1 + \mathbf{u}_2 + \mathbf{u}_3 \quad (1)$$

which is reasonable since the curvature of the first mode is small along the rotor body.

The balancing plane in the middle of the rotor does not affect the second mode. Therefore an estimate of the modal unbalance for the second mode is

$$\mathbf{u}_{\text{mode } 2} = \mathbf{u}_3 - \mathbf{u}_1 \quad (2)$$

The estimated introduced modal unbalances due to rewinding are calculated according to Eq. (1) and Eq. (2) and the resulting magnitudes (normalized to the rotor mass) are given in Table 2.

**Table 2. Estimated and normalized modal unbalance introduced by rewinding for the rotors given in Table 1.**

Rotor	$ \mathbf{u}_{\text{mode } 1} $	$ \mathbf{u}_{\text{mode } 2} $
	[ $\mu\text{m}$ ]	[ $\mu\text{m}$ ]
1	3.1	4.7
2	18.4	43.3
3	5.2	19.4
4	22.5	30.8
5	55.3	11.3
6	22.2	10.0
7	31.3	12.8
8	0	0
9	26.6	11.0

The mean values for the modal unbalances are 21  $\mu\text{m}$  for the 1<sup>st</sup> mode and 15  $\mu\text{m}$  for the 2<sup>nd</sup> mode. The 95% confidence interval for the estimated mean 1<sup>st</sup> modal unbalance is  $7.9\mu\text{m} < |\mathbf{u}_{\text{mode } 1}| < 33.2\mu\text{m}$  and  $6.5\mu\text{m} < |\mathbf{u}_{\text{mode } 2}| < 23.3\mu\text{m}$  for the 2<sup>nd</sup> modal unbalance. This means that with 95% confidence the expected mean value for the modal unbalances of all similar rewinded rotors are within the given ranges.

A rotor influenced by the 1<sup>st</sup> and the 2<sup>nd</sup> mode should according to ISO 11342 have an equivalent modal residual unbalance for the 1<sup>st</sup> and 2<sup>nd</sup> mode less than 60% of the residual unbalance for an equivalent rigid rotor in ISO 1940/1 based upon the highest service speed. For turbo generators balancing grade G2.5 applies. This means that the residual modal unbalance (normalized to the rotor mass) should not exceed 4.8  $\mu\text{m}$  for a 50 Hz 2-pole generator and 4.0  $\mu\text{m}$  for a 60 Hz 2-pole generator. Hence, it should be expected that the introduced unbalance by rewinding is larger than allowed according to ISO 11342 and ISO 1940/1.

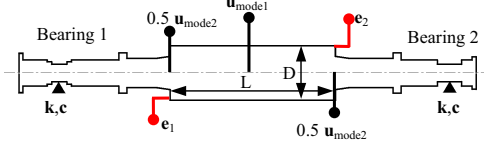
### ROTORDYNAMICAL MODEL

The estimated modal unbalances by rewinding are applied on different rotordynamic models and unbalance response analyses are performed. It is studied how balancing in the retaining ring balancing planes affects the unbalance response. (The reason for



choosing these balancing planes is that they are accessible on site.) To search for the optimum solutions a search algorithm is used. This is described in the next section. In this section the rotordynamical model is described.

Figure 2 shows a schematic sketch of the rotordynamical model with the most important dimensions active length ( $L$ ) and active diameter ( $D$ ) defined.



**Figure 2. Schematic sketch of rotordynamical model.**

For the rotordynamical analyses, the rotor is discretized into beam elements and a special purpose code is used which can handle non-symmetric matrices. Gyroscopic effect is included. The bearing stiffness matrix ( $\mathbf{k}$ ) and damping matrix ( $\mathbf{c}$ ) are included in the rotordynamical model but the pedestals are assumed rigid.

The bearings are analyzed for stationary conditions. For this purpose, a code based on ALP3T [7] is used. Offset halves bearings with static load of 2 MPa and width to diameter ratio  $w/d = 0.6$  are assumed. The bearing relative clearance is assumed to 1.7‰ of the lobe radius. The lobe pre-set ratio is assumed to be 0.35. In the numerical examples, the bearings are analyzed for different static bearing loads and speeds. Hence, the bearing stiffness ( $\mathbf{k}$ ) and damping ( $\mathbf{c}$ ) is a function of static load (i.e. rotor weight) and speed.

Complex eigenvalue and unbalance response analyses are performed and the result is given in the result section. In the unbalance response analyses the modal unbalances are applied according to Fig. 2 with the masses for the 2<sup>nd</sup> modal unbalance applied 180° out of phase. The magnitudes of the applied modal unbalance corresponds to the upper limit of the 95% confidence interval for the mean modal unbalances given in the previous section. The vectors  $\mathbf{e}_1$  and  $\mathbf{e}_2$  represents the correction weights in the retaining rings.

The numerical study is done for a set of 50 Hz rotors with different active diameters and different active lengths. The studied range of rotors is indicated with gray color in Table 3.

**Table 3. Summary of numerical examples studied ( $D$  and  $L$  in mm).**

$D \backslash L$	1500	2000	2500	3000	3500	4000	4500	5000
800						-	-	-
900	-						-	-
1020	-	-						

The range of rotors given in Table 3 should cover most air-cooled rotors in the range 20-200 MVA that is the focus for this work.

## OPTIMIZATION FORMULATION AND SEARCH ALGORITHM

Generally, the balancing of flexible rotors is a challenge with several objectives if the rotor should perform well over the whole speed range. The fundamental idea behind this formulation is that vibrations during passage of critical speeds and vibrations at operational speed should be minimized simultaneously.

First we define  $A_1(\omega)$  and  $A_2(\omega)$  to be the relative amplitude of vibration (for major semi axis in this case) at bearing 1 and bearing 2. (Since we use a stiff pedestal assumption, there is no difference between the absolute or relative vibrations in this case.) For the rotational speed it is assumed that  $\omega \in [0, \omega_{op}]$ .

Hence, the upper limit is the operational speed, denoted  $\omega_{op}$ . Now two objective functions are formulated as

$$f_1(\mathbf{x}) = \max[A_1(\omega), \max(A_2(\omega))] \quad (3)$$

and

$$f_2(\mathbf{x}) = \max[A_1(\omega_{op}), A_2(\omega_{op})]. \quad (4)$$

The first objective function describes the maximum vibration amplitude for both bearings during run down. The second objective is the maximum amplitude of vibration for both bearings at the operational speed. The optimization problem is now formulated as

$$\begin{aligned} &\text{Minimize} \quad \mathbf{F}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x})] \\ &\text{subject to} \quad \mathbf{x} \in S \end{aligned} \quad (5)$$

The vector of design variables is defined as  $\mathbf{x} = [\mathbf{e}_1, \mathbf{e}_2]$ . The search space  $S$  is defined assuming that maximum balancing weight is 2 kg in each balancing plane. The objective to minimize is the vector  $\mathbf{F}$ . Hence, a multi-objective optimization problem with two objectives is formulated.

Probably the most common approach to handle this kind of problem is to formulate a weighted sum and minimize it. This means that one have to decide the preference of the goals. Another approach used here is to search for the whole Pareto optimal set of solutions. This set is dependent on the number of critical speeds passed during running up.

The search for the Pareto optimal set can be done with population based Evolutionary Algorithms. Here the NSGA-II [8] with search parameters according to Table 4 is used. It should be mentioned that there today exist a vast number of multi-objective evolutionary algorithms [10] of which the NSGA-II is one of the more cited.

**Table 4. Search parameters for the NSGA-II.**

Population size	50
Max generations	100
Crossover probability	0.9
SBX-crossover [9] with distribution index	10
Mutation probability	0.1
Mutation distribution index	20

## RESULTS

This section summarizes the numerical results. First results from the complex eigenvalue and unbalance response analyses are given then the search results from the optimizations are presented. All the results are then discussed in the next section.

### Complex eigenvalues and unbalance response

The complex eigenvalues are calculated for the rotational speed 3000 rpm. The damping vs. the natural frequencies for all eigenvalues (i.e. root locus) are plotted for the different cases in Fig. 3 to Fig. 5. The corresponding mode shapes are given for the case  $D = 900$  mm in Fig. 9.

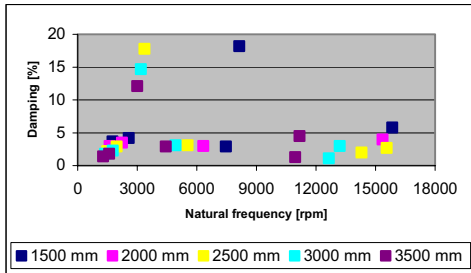


Figure 3. Root locus plot for cases with  $D = 800$  mm.

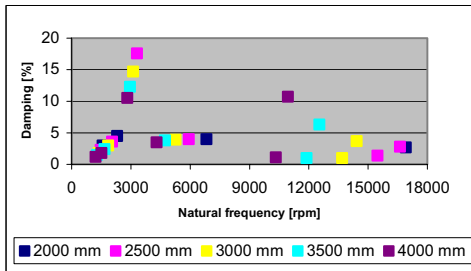


Figure 4. Root locus plot for cases with  $D = 900$  mm.

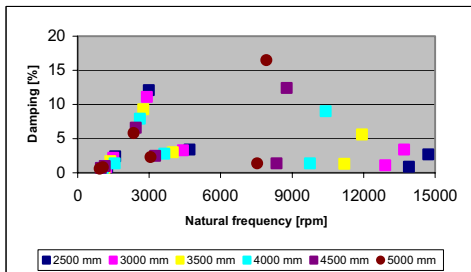


Figure 5. Root locus plot for cases with  $D = 1020$  mm.

The calculated unbalance response (0-p, major axis at the bearing positions) for the case without correction weights in the retaining rings is given for the different cases in Fig. 6 to Fig. 8.

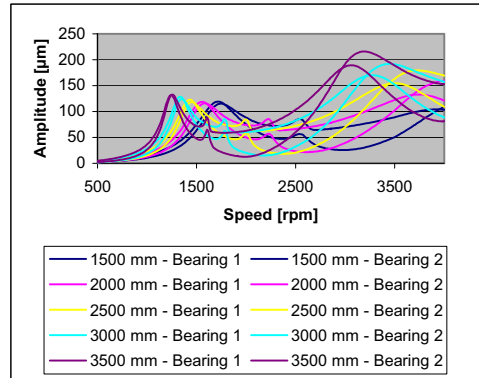


Figure 6. Calculated unbalance response for the case with  $D = 800$  mm.

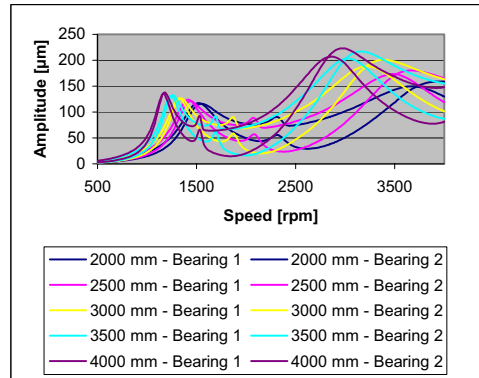


Figure 7. Calculated unbalance response for the case with  $D = 900$  mm.

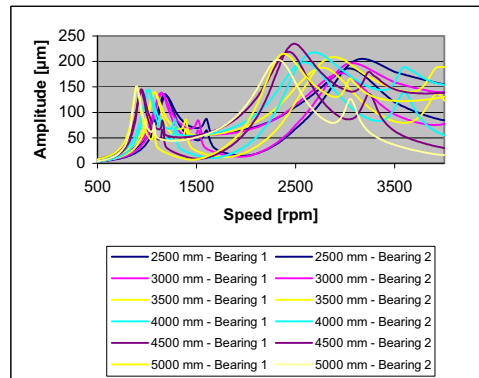


Figure 8. Calculated unbalance response for the case with  $D = 1020$  mm.

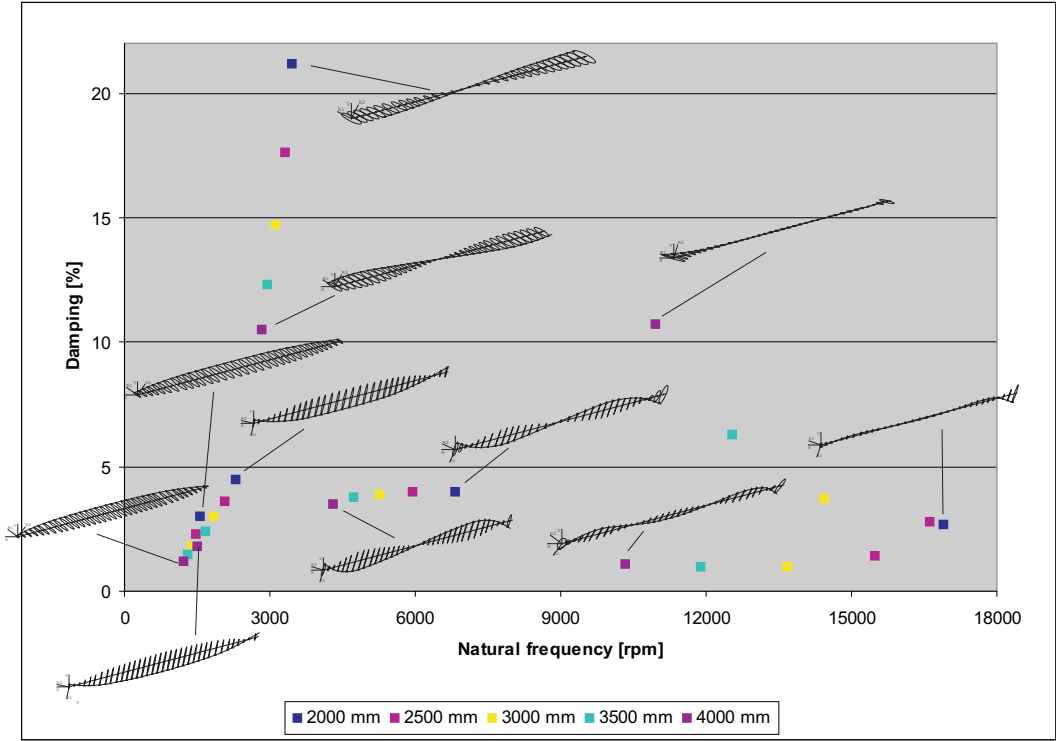


Figure 9. Root locus plot with corresponding complex mode shapes for the case with  $D = 900$  mm and different active lengths.

### Optimization results

The final generations are plotted in the objective space for the different cases in Fig. 10 to Fig. 12. The final generation is an estimate of the Pareto optimal set of solution for the two objectives, i.e. the optimal trade-off solutions. Observe that the scale is different in Fig. 10 to Fig. 12.

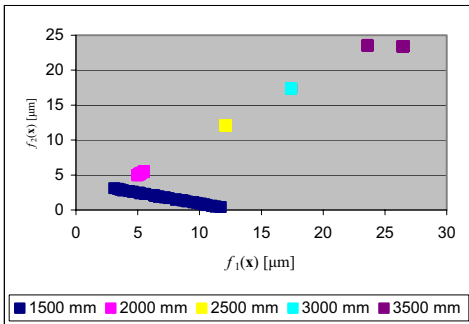


Figure 10. Final generations for the cases with  $D = 800$  mm.

The unbalance response for the two extreme Pareto optimal solutions indicated in Fig. 12 is shown in Fig. 13. The vibrational shapes for these two solutions are given in Fig. 14 and Fig. 15.

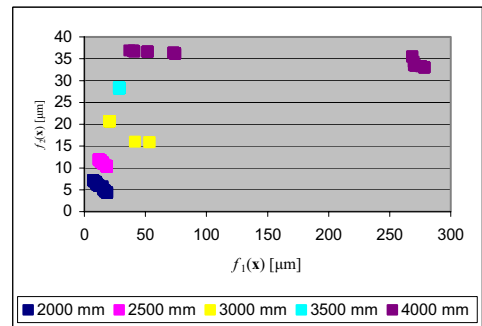


Figure 11. Final generations for the cases with  $D = 900$  mm.

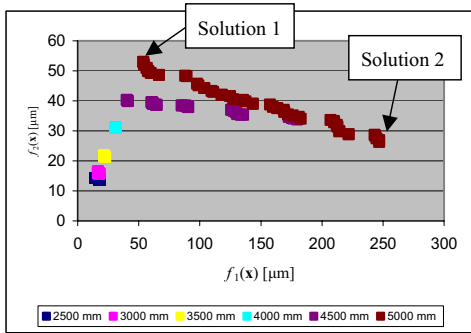


Figure 12. Final generations for the cases with  $D = 1020$  mm.

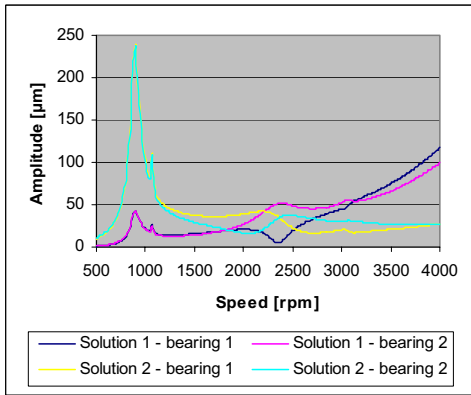


Figure 13. Unbalance response in the bearings for the two extreme Pareto optimal solutions indicated in Fig. 12.

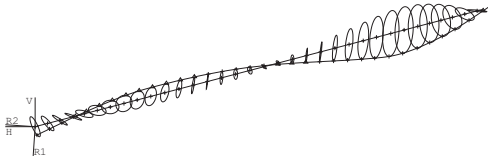


Figure 14. Vibrational shape at 3000 rpm for solution 1 indicated in Fig. 12 (max amplitude is  $119 \mu\text{m}$ ).

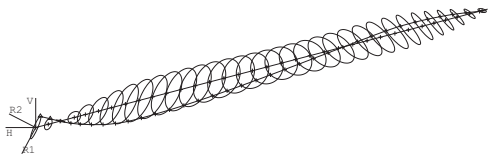


Figure 15. Vibrational shape at 3000 rpm for solution 2 indicated in Fig. 12 (max amplitude is  $101 \mu\text{m}$ ).

## DISCUSSION

The root locus plots Fig. 3 to Fig. 5 and Fig. 9 show how the eigenvalues change as rotor active length is varied. Figure 9 also shows how the modes shapes change when the active length is changed. It can be seen that several modes that are not pure rigid body modes nor pure flexible bending modes are important for the response.

The smallest rotor in the study with  $D = 800$  mm and  $L = 1500$  mm has the 3<sup>rd</sup> bending mode above 16000 rpm. Hence, it should not be significantly affected by the 3<sup>rd</sup> bending mode at the operational speed 3000 rpm. Nevertheless Fig. 10 shows that there is a contradiction to minimize the response during rundown and at operational speed simultaneously. The reasonable explanation for this is that there are actually four modes of importance for the response at the operational speed and during rundown. Two of these are similar to (but not pure) the 1<sup>st</sup> bending mode. The other two are similar to (but not pure) the 2<sup>nd</sup> bending mode.

For the longest rotor in the study with  $D = 1020$  mm and  $L = 5000$  mm it can be seen from Fig. 12 that there is a clear trade-off between the objectives. This rotor has a mode similar to the 3<sup>rd</sup> bending mode at 7500 rpm that influence the response at the operational speed and above. This can be seen for solution 1 in Fig. 13.

According to ISO 7919-2 the maximum relative bearing vibration is  $80 \mu\text{m}$  (p-p) at the operational speed for a newly commissioned 50 Hz machine. During rundown the corresponding limit is  $390 \mu\text{m}$  (p-p). If a safety factor of 2 is assumed this gives  $20 \mu\text{m}$  (0-p) at the operational speed and  $98 \mu\text{m}$  during rundown. If these limits are adopted Fig. 10 to Fig. 12 show that rotors with active diameters 800-1020 mm and an active length less than 3000 mm should be possible to balance in only the retaining ring balancing planes after rewind.

## CONCLUSIONS

By 95% confidence the expected mean modal unbalance introduced by rewinding for the 1<sup>st</sup> mode is  $7.9 \mu\text{m} < \overline{u_{\text{mode1}}} < 33.2 \mu\text{m}$  and  $6.5 \mu\text{m} < \overline{u_{\text{mode2}}} < 23.3 \mu\text{m}$  for the 2<sup>nd</sup> mode. This is larger than allowed according to ISO 11342 and ISO 1940/1, G2.5.

The numerical result shows that there is a trade-off to balance also short rotors with the 3<sup>rd</sup> bending mode well above the operational speed in only the retaining ring balancing planes. The reason is that the anisotropic bearings give several modes with shapes similar to the 1<sup>st</sup> and 2<sup>nd</sup> bending modes and rigid body modes.

The feasibility of site balancing a rewinded rotor depends on its geometry and to what degree it should be balanced. If vibration limits according to ISO 7919-2 (zone A/B) and a safety margin of 2 is assumed, the numerical results show that 50 Hz rotors with active diameters 800-1020 mm and active lengths less than 3000 mm are candidates for site balancing after rewind.

## ACKNOWLEDGEMENTS

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