APPENDIX E

PROBABILISTIC BASIS FOR SAC/FEMA GUIDELINES

This appendix outlines the probabilistic framework for seismic design and assessment of steel frame buildings in the SAC/FEMA guidelines. The following material is basically excerpted from Cornell et al. (2002).

E.1 Basic approach

Seismic structural behavior in SAC/FEMA guidelines is represented explicitly by nonlinear, dynamic, displacement-based quantities. Both demand *D* and capacity *C* are measured in terms of maximum interstory drift ratios. From the total probability theorem, the probability of a performance level not being met can be evaluated as

$$\begin{aligned} P_{PL} = P\left[C \le D\right] &= \int P\left[C \le D \middle| D = d\right] \left|dH_D\left(d\right)\right| \\ &\approx \int P\left[C \le d\right] \left|dH_D\left(d\right)\right| \end{aligned} \tag{E.1}$$

where $H_D\left(d\right)$ is the (structure-specific) drift hazard curve that provides the (mean) probability of the drift demand *D* exceeding any specified value *d*. It is obtained from

$$H_D\left(d\right) = P\left[D \ge d\right] = \int P\left[D \ge d \middle| S_a = s_a\right] \left|dH\left(s_a\right)\right| \tag{E.2}$$

where $\left|dH\left(s_a\right)\right|$ is the absolute value of the derivative of the site's spectral acceleration hazard curve times *dx*, which is approximately the likelihood of $S_a = s_a$.

Closed-form formulas can be derived based on a series of assumptions. First, by assuming that conditional median drift demand $\hat{D}$ can be approximated by

$$\hat{D} = a\left(S_a\right)^b \tag{E.3}$$

and the drift demand D follows a lognormal distribution about its median value $\hat{D}$ with a standard deviation of $\beta_{D|S_a}$, the integrand of Equation E.2 becomes

$$P\left[D \geq d \middle| S_a = s_a\right] = 1 - \Phi\left(\frac{\ln\left[d/a{s_a}^b\right]}{\beta_{D|S_a}}\right) \tag{E.4}$$

in which Φ is the standardized Gaussian distribution function.

Further assuming that the site hazard curve can be approximated by

$$H\left(s_a\right) = P\left[S_a \geq s_a\right] = k_o s_a^{-k} \tag{E.5}$$

Equation E.2 can then be derived as

$$H_D\left(d\right) = H\left(s_a^d\right)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\beta_{D|S_a}^2\right] \tag{E.6}$$

in which $s_a^d = \left(d/a\right)^{1/b}$ the spectral acceleration associated with the drift demand d.

Assuming the drift capacity C also follows a lognormal distribution about its median value $\hat{C}$ with a standard deviation of β_C, the integrand of Equation E.2 becomes

$$P\left[C \leq d\right] = 1 - \Phi\left(\frac{\ln\left[d/\hat{C}\right]}{\beta_C}\right) \tag{E.7}$$

Substituting Equations E.6 and E.7 into E.1 leads to

$$P_{PL} = H\left(S_a^{\hat{C}}\right)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\left(\beta_{D|S_a}^2 + \beta_C^2\right)\right] \tag{E.8}$$

in which $s_a^{\hat{C}} = \left(\hat{C}/a\right)^{1/b}$ the spectral acceleration associated with the median drift capacity $\hat{C}$.

E.2 Uncertainty treatment

By considering the (epistemic) uncertainties in both drift demand and capacity, P_{PL} itself becomes a random variable. The mean estimate of P_{PL} is

$$\begin{aligned} \bar{P}_{PL} &= \hat{H}\left(s_a^{\hat{C}}\right)\exp\left[\frac{1}{2}\beta_H^2\right]\exp\left[\frac{1}{2}\frac{k^2}{b^2}\left(\beta_{DR}^2+\beta_{DU}^2+\beta_{CR}^2+\beta_{CU}^2\right)\right] \\ &= \bar{H}\left(s_a^{\hat{C}}\right)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\left(\beta_{DR}^2+\beta_{DU}^2+\beta_{CR}^2+\beta_{CU}^2\right)\right] \end{aligned} \tag{E.9}$$

where $\hat{H}, \bar{H}$ = median and mean estimates of spectral acceleration hazard, respectively; β_H = dispersion measure for hazard; β_{DR}, β_{CR} = dispersion measures for randomness in drift demand and capacity, respectively; β_{DU}, β_{CU} = dispersion measures for uncertainty in drift demand and capacity, respectively.

The median or 50% confidence estimate of P_{PL} is

$$\hat{P}_{PL} = \bar{H}\left(s_a^{\hat{C}}\right)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\left(\beta_{DR}^2+\beta_{CR}^2\right)\right] \tag{E.10}$$

An (epistemic) uncertainty for P_{PL} can be calculated by

$$\beta_{P_{PL}} = \left[\frac{k^2}{b^2}\left(\beta_{DU}^2+\beta_{CU}^2\right)\right]^{1/2} \tag{E.11}$$

Hence the confidence level estimate of P_{PL} is obtained from

$$P_{PL}^x = \hat{P}_{PL}\exp\left[K_x\beta_{P_{PL}}\right] \tag{E.12}$$

where K_x = standardized Gaussian variate associated with probability x of not being exceeded.

E.3 Codified safety/performance checking schemes

Using the mean estimate of probability as the objective, one rearranges Equation E.9 with a performance level P_{obj} to obtain

$$\left\{\exp\left[-\frac{1}{2}\frac{k}{b}\left(\beta_{CR}^2+\beta_{CU}^2\right)\right]\right\}\hat{C}\geq\left\{\exp\left[\frac{1}{2}\frac{k}{b}\left(\beta_{DR}^2+\beta_{DU}^2\right)\right]\right\}\hat{D}^{P_{obj}} \quad \text{(E.13)}$$

or in a codified form

$$\phi\,\hat{C}\geq\gamma\,\hat{D}^{P_{obj}} \quad \text{(E.14)}$$

with

$$\begin{cases}\phi=\exp\left[-\frac{1}{2}\frac{k}{b}\left(\beta_{CR}^2+\beta_{CU}^2\right)\right]\\ \gamma=\exp\left[\frac{1}{2}\frac{k}{b}\left(\beta_{DR}^2+\beta_{DU}^2\right)\right]\end{cases} \quad \text{(E.15)}$$

To obtain the associated confidence level, one calculates the factored-demand to factored-capacity ratio

$$\lambda_{con}=\gamma\,\hat{D}^{P_{obj}}/\phi\,\hat{C}=\exp\left[-K_x\beta_{UT}+\frac{1}{2}\frac{k}{b}\beta_{UT}^2\right] \quad \text{(E.16)}$$

where $\beta_{UT}=\left(\beta_{DR}^2+\beta_{DU}^2+\beta_{CR}^2+\beta_{CU}^2\right)^{1/2}$.

The Gaussian variate K_x is then solved for as

$$K_x=\frac{\left[\ln\left(\lambda_{con}\right)+\frac{1}{2}\frac{k}{b}\beta_{UT}^2\right]}{\beta_{UT}} \quad \text{(E.17)}$$