

Chapter 7

Creature Complexity

¹ Complexity has been and will remain a debatable concept. We all know it when we see it, yet when we need to provide a functional definition of complexity, it becomes a mythical entity. Although the study of complex systems has attracted much interest over the last decade and a half, the definition of what makes a system complex is still the subject of much debate among researchers (Adami 1998; Feldman and Crutchfield 1998b; Standish 2001). What is complexity? Is there a universal measure of complexity? Does complexity arise when a system reaches a critical point or is there a phase transition between simple and complex systems? Are adaptability, adaptation, emergence, hierarchy, bifurcation, and self-organization evidence for complexity? These are all common concerns raised by researchers when speaking of complex systems and complexity itself.

Although there are numerous methods available in the literature for measuring complexity (Badii and Politi 1997; Edmonds 1999), it has been argued however that such complexity measures are typically too difficult to compute to be of use for any practical purpose or intent (Shalizi 2001). This chapter attempts to unfold some mysteries about complexity and to pose EMO as a simple and highly accessible methodology for characterizing the complexity of artificially evolved creatures using a multi-objective methodology. One of the main objectives of evolving

¹Some of the material presented in this chapter have been previously published in Teo, Nguyen, and Abbass (2003).

artificial creatures is to synthesize increasingly complex behaviors and/or morphologies either through evolutionary or lifetime learning (Pfeifer and Scheier 1999; Nolfi and Floreano 2000; Hornby and Pollack 2001a; Komosinski and Rotaru-Varga 2001; Bongard 2002b). Needless to say, the term “*complex*” is generally used very loosely since there is currently no general method for comparing between the complexities of these evolved artificial creatures’ behaviors and morphologies. As such, without a quantitative measure for behavioral or morphological complexity, an objective evaluation between these artificial evolutionary systems becomes very hard and typically ends up being some sort of a subjective argument.

We first present attempts at defining complexity followed by a comprehensive review of the different views of complexity from the social sciences to concrete measures in information systems and the physical sciences. Then, we propose a characterization of the notion of complexity in embodied cognition using multi-objectivity as a natural and theoretically-founded paradigm in mathematics. Specifically, we will attempt to characterize the behavioral and morphological complexities of different artificial creatures using the multi-objective controller evolution approach introduced in Chapter 5.

7.1 Complexity Defined?

The following list provides some suggested definitions of complexity and complex systems:

- “The complexity of a system S is a contingent property, depending upon the nature of the observables describing S , and their mutual interactions.” — Casti (1986, p.155)
- “Complexity is the study of the behavior of macroscopic collections of such units that are endowed with the potential to evolve over time.” — Coveney and Highfield (1995, p.7)
- “... a “theory of complexity” could be viewed as a theory of modelling, en-

compassing various reduction schemes (elimination or aggregation of variables, separation of weak from strong couplings, averaging over subsystems, evaluating their efficiency and, possibly, suggesting novel representations of natural phenomena.” — Badii and Politi (1997, p.6)

- “I use *complex* and *complexity* intuitively to describe self-organized systems that have many components and many characteristic aspects, exhibit many structures in various scales, undergo many processes in various rates, and have the capabilities to change abruptly and adapt to external environments.” — Auyang (1998, p.13)
- “Complexity is that property of a model which makes it difficult to formulate its overall behavior in a given language, even when given reasonably complete information about its atomic components and their inter-relations.” — Edmonds (1999, p.72)
- “...in defining complexity we need to consider both functions of perception and analysis. For what we want to know is not whether a simple or short description can be found of every detail of something, but merely whether such a description can be found of those features in which we happen to be interested.” — Wolfram (2002, p.557).

It is surprising to note that although there is a large body of literature that discusses issues relating to complexity, few actually provide a definition to complexity as used in their respective contexts (Feldman and Crutchfield 1998b). As pointed out in the introduction to this chapter, the task of defining complexity is difficult in itself, which may explain why the term “complexity” is so commonly used without qualification. A number of books authored about complexity theory confirms this observation, where an enormous range of views were drawn about what complexity means to different researchers and to different disciplines (Lewin 1993; Waldrop 1994; Mainzer 1997).

In the social sciences, complex systems typically refer to social phenomena that exhibit some form of dynamic nonlinear behavior that are difficult to explain

using basic linear models. Examples of complex systems described in the social sciences include fluctuations of stock markets and exchange rates, impacts of economic policies, human population growth and migration, societal organization, political revolutions, organizational cooperation and conflict, human interactions and their communication structures (Coveney and Highfield 1995). With reference to complex systems and the evolution of human society, Mainzer (1997) states that

“The crucial point of the complex system approach is that from a macroscopic point of view the development of political, social, or cultural order is not only the sum of single intentions, but the collective result of non-linear interactions.” (p.253)

Biological organisms and processes associated with living things and life in general typically correspond to what we intuitively know as objects and systems that exhibit the highest degree of complexity (Badii and Politi 1997). Examples of complex systems in the biological sciences include genomic evolution, genetic regulatory networks, population ecology, morphogenesis and biological neural networks to name but a few (Mainzer 1997). More specific studies of biological complex systems have been conducted on the self-replication of bio-molecules, in-vitro RNA evolution, self-regulation of the glycolytic process, self-organization of slime moulds, pattern formation of the hydra, oscillations of the heart, spatio-temporal organization of *hymenoptera* colonies, punctuated equilibrium as catastrophe theory, planetary self-regulation and *Gaia* theory (Coveney and Highfield 1995).

In chemical and physical systems, complexity is often attributed to processes that exhibit some form of aperiodicity or possess high degrees of freedom (Badii and Politi 1997). Terms commonly associated with complex physical and chemical systems include chaos, phase transitions, bifurcation, self-organized criticality and percolation. Examples of such complex phenomena include cement formation caused by the percolation of water through irregularly shaped particles in cement powder, criticality of sand-piles that produce avalanches beyond a certain geometric configuration, fluid instabilities that produce turbulence, optical instabilities in lasers that produce quasi-periodic light patterns and fluctuations of spin-glasses

caused by the disorderly arrangements of spinning electrons (Coveney and Highfield 1995; Badii and Politi 1997; Adami 1998).

As can be seen from the diverse interpretations and applications to social, biological and physical systems, complexity provides very different meanings depending on the context it is being referred to. Perhaps Mainzer (1997) provides the best summary of what complex systems encompasses:

“...the theory of nonlinear complex systems cannot be reduced to special natural laws of physics ...it is an interdisciplinary methodology to explain the emergence of certain macroscopic phenomena via the nonlinear interactions of microscopic elements ...” (p.1)

In the next section, we will provide more concrete examples of complex systems and specifically what types of measures have been formulated to capture complexity across the different disciplines.

7.2 Measures of Complexity

In this section, we present a review of existing measures for defining or simply characterizing complexity as viewed from the social, physical and biological sciences’ perspectives. A summary of the literature surveyed on general reviews and specific applications of complexity measures is given in Table 7.1. Here we give a high-level survey of the more significant measures from these diverse fields. Our intention is simply to provide an indication of the wide spectrum of efforts in trying to capture complexity into something mathematical or formal so as to assist with the characterization or comparison of different systems. A more comprehensive and detailed survey of existing complexity measures is available elsewhere (Edmonds 1999). Nonetheless, our shorter survey will show that such measures are often highly specific, being specially designed or formulated for application in a particular domain or area of research not readily transferable to another application domain. A discussion of the advantages and disadvantages associated with these different methodologies for measuring complexity will be also highlighted. We will also

Category	Reference
General Reviews	Casti (1986)
	Lewin (1993)
	Waldrop (1994)
	Coveney and Highfield (1995)
	Mainzer (1997)
	Auyang (1998)
Biological Sciences	Cavalier-Smith (1985)
	Bonner (1988)
	Atlan and Koppel (1990)
	Smith (1994)
	Maynard Smith and Szathmary (1995)
	Nehaniv (2000a), Nehaniv (2000b)
	Szathmary, Jordan, and Pal (2001)
Social Sciences	Kelly (1955)
	Albin (1980)
	Cooper (1993)
	Holm (1993)
	Lyon (1993)
	Gibson (1998)
	Halford, Wilson, and Phillips (1998)
	Clement (1999)
	Neyman and Okada (1999)
	Butts (2001)
	Andrews and Halford (2002)
	DeShazo and Fermo (2002)
	Warren and Gibson (2002)
Physical Sciences	Shannon (1948)
	Kolmogorov (1965)
	Bennett (1988)
	Badii and Politi (1997)
	Wolpert and MacReady (1997)
	Adami (1998)
	Feldman and Crutchfield (1998a), Feldman and Crutchfield (1998b)
	Edmonds (1999)
	Shalizi (2001)
	Standish (2001)
	Wolfram (2002)

Table 7.1: Summary of literature survey on reviews of complexity measures and their applications in the biological, social and physical sciences.

cover the more recent complexity measures suggested since the review conducted by Edmonds (1999) especially for biological organisms as they may provide critical insights to the measurement of complexity in their artificial counterparts.

7.2.1 Social Sciences

An early attempt to capture complexity in a numerical form exists in the psychology literature. It is called cognitive complexity and is used to describe the complexity of mental constructions of the world possessed by an individual (Kelly 1955). Cognitive complexity here is estimated numerically by counting the number of different relationships constructed by the subject from given object attributes. In this sense, a person who sees the world in more dimensions would be considered to have a higher cognitive complexity. A related and more recent technique for measuring cognitive complexity called Relational Complexity (RC) theory was proposed by Halford, Wilson, and Phillips (1998). It is defined as the number (*arity*) of relations between entities or arguments in a given decision task. For example, an unary relation would have one entity, such as *woman(Jane)* and a binary relation would have two entities such as *married(Jane,Dean)*. Hence, each entity corresponds to a variable or attribute and an n-ary relation maps to a set of points in an n-dimensional cognitive space. A recent study has tested the validity of this metric and was found to be effective in measuring the cognitive development of young children (3–8 years) (Andrews and Halford 2002).

In studies of group and organizational behavior, an early conceptualization of measuring the complexity of social interactions was proposed by Albin (1980). It applies graph theory to participating individuals within an interacting group and measures the level of complexity of social actions based on the connectivity of actions between individuals. In a more recent application of complexity to the social sciences, Butts (2001) proposed the use of algorithmic complexity to measure the complexity of social networks. Again based on graph theory, the interaction of roles representative of human social structures are first represented as directed graphs, the complexity of which is then measured according to the amount of “reducibility” or “compressibility” that can be achieved on the network. In this case, a social network with higher compressibility would be considered to have higher reducibility and the converse in a social network with low reducibility. Interestingly, complexity has also been applied to command and control theory associated with military operations

in an attempt to unify difficulties encountered with such systems (Cooper 1993). Specifically, three types of complexity were identified: dimensional, uncertainty and computational, although no actual method of measuring these complexities within the military setting was given.

Another area where complexity has been applied numerically is the study of human language processing. In a theory called Syntactic Prediction Locality Theory (SPLT) proposed by Gibson (1998), the complexity of a sentence can be predicted according to the memory cost associated with keeping a partial sentence in memory and integration cost associated with integrating new words into existing syntactical structures built thus far. Memory cost is quantified according to the number of syntactic categories necessary to complete the current input string as a grammatical sentence. Integration cost is quantified according to the distance between an incoming input and the nearest syntactic component it attaches to. This technique has been used to empirically measure how different localizations of noun phrases affected sentence complexity (Warren and Gibson 2002).

Complexity in economics typically refers to simply the relaxation of assumptions made on the behavior of market agents (Edmonds 1997). More specific applications of complexity can be found in game theory where the number of agent states is used as a measure of economic complexity (Holm 1993). An entropy-based measure has been formulated to capture the complexity of agent strategies in a repeated games environment (Neyman and Okada 1999). Complexity measures for gauging consumer demand and preferences have also been developed based on the quantity of information and configuration of information present in a given choice set (DeShazo and Fermo 2002). In a study that looked at the accuracy of market earnings forecasts, the portfolio complexity of research analysts was defined simply as the number of firms and industries being tracked in their market analysis (Clement 1999).

Measures of complexity have largely been applied at only a very superficial level in the social sciences, typically taking size as a simplistic basis for describing or capturing complexity. There are obvious deficiencies associated with size-based

complexity measures, the most evident being that not all large systems are complex (Edmonds 1999). It has been argued that the application of complex systems theory to the social sciences results in reductionist view of complexity when applied to the social domain (Lyon 1993). The important point raised is that contextual relationships such as political and moral issues are lost during the transformation into a mathematical or metaphoric model for complexity analysis.

7.2.2 Biological Sciences

It is especially difficult to define or measure biological complexity (Maynard Smith and Szathmary 1995; Szathmary, Jordan, and Pal 2001). An obvious measure would be the size of an organism's genome in terms of the number of base pairs (BP) present in the DNA, which can be thought of in the sense that a more complex organism would require lengthier instructions for making the organism (Cavalier-Smith 1985; Maynard Smith and Szathmary 1995). However, a total DNA count would place the complexity of humans (3.5×10^9 BP) an order of magnitude below a newt (19.0×10^9 BP) and two orders of magnitude below a lungfish (140.0×10^9 BP) and a lily (130.0×10^9 BP) (Maynard Smith and Szathmary 1995). An alternate measure of biological complexity based on DNA is that of counting only parts of the DNA that actually code for proteins that are expressed (Cavalier-Smith 1985). This complexity measure would then make more sense in that eukaryotes would have more coding DNA than prokaryotes, multi-celled organisms have more coding DNA than single-celled organisms, and that vertebrates have more coding DNA than invertebrates. However, this is an extremely coarse-grained classification that tells us very little about the structural and functional complexity between different organisms.

Another suggested measure of complexity for biological organisms based on genomic information is the number of genes present in the DNA (Szathmary, Jordan, and Pal 2001). However, humans, previously thought to have an order of magnitude more genes, are now estimated to have only around 20,000–35,000 genes and have the same order of magnitude of genes as the flowering plant *A. thaliana*

(25,498 genes), the nematode worm *C. elegans* (18,424 genes) and the fruit fly *D. melanogaster* (13,601 genes) (Szathmary, Jordan, and Pal 2001). As such, merely counting the number of genes as a measure of biological complexity may not be very insightful.

There are a number of other suggested methods of measuring biological complexity. Focusing on multicellular organism, the number of cell types present can be used to define the complexity of such an organism (Bonner 1988). A problem with this approach is that what constitute a distinct cell type as opposed to another cell type depends on our current understanding of molecular biology and biochemistry and may vary significantly between different groups of researchers (Nehaniv 2000a; Szathmary, Jordan, and Pal 2001).

The ability to measure the complexity of brains has been critically analyzed by Smith (1994). He laments

“...how very far we are at present from being able to give a numerical estimate.” (Smith 1994, p.93)

Showing the inadequacies of “borrowed” complexity measures from the physical sciences, he argues that *organization* as well as *levels of organization* need to be considered when attempting to capture the complexity of brains. A numerical measure based on the columnar organization of the neocortex was suggested citing a quantitative example that estimates the complexity of human brains, with roughly 300,000 such columns, to be 375 times greater than that of mouse brains, with only 800 columns.

A more formal approach based on algorithmic complexity to measuring biological complexity has been suggested by considering the number of developmental steps required to produce the organism from its DNA (Atlan and Koppel 1990). However, as critically pointed out by Szathmary, Jordan, and Pal (2001),

“The snag here is that evolution is not an engineer but a tinkerer, so that there is no reason to expect that, for example, elephants have developed according to a minimalist program.” (p.1315)

Another formal measure for biological complexity was suggested by Nehaniv (2000a) based on the notion of hierarchical complexity. In this measure, biological systems are assigned an integer value which gives the least number of hierarchical organized computing levels needed to construct an automata model of the biological system. Although powerful in terms of generalization since it does not require the actual knowledge of how a biological system is built nor its components, it does however have the requirement that the system can first be adequately modelled using finite automata (Nehaniv 2000a). The process of transforming biological systems into such automata is highly subjective and can be executed in a myriad of ways depending on how the system is viewed by the transformer. This measure of complexity was later applied to the measurement of evolvability in a later study and argued that open-ended evolutionary systems should show unbounded complexity increase over time (Nehaniv 2000b).

Szathmary, Jordan, and Pal (2001) more recently proposed the measurement of biological complexity by considering the connectivity of networks of transcription factors and the genes that regulate rather than direct counting of genes or the interactions among genes. They argue that biological complexity normally thought of in terms of morphological and behavioral complexity correlates better with the connectivity of gene-networks than direct measurements such as gene numbers since the former will correctly account for the presence of so-called *delegated information processing systems* in the form of vertebrate nervous and immune systems. However, current artificial evolutionary systems lack the level of sophistication in terms of such gene-regulatory networks and as such, do not readily lend themselves to such an analysis. Nonetheless, work has begun to imbue artificial evolutionary systems with some form of genetic regulation (Bongard 2002b) in the hope of evolving more sophisticated artificial organisms and will conceivably in the future allow for such a measure to be applied.

7.2.3 Physical Sciences

In the physical sciences literature, there are generally two widely-accepted views of measuring complexity. The first is an information-theoretic approach based on Shannon's entropy (Shannon 1948) and is commonly referred to as statistical complexity due to its formulation based on probability. Shannon's entropy measure $H(X)$ of a random variable X , where the outcomes x_i occur with probability p_i , is given by

$$H(X) = - C \sum_i^N p_i \log p_i \quad (7.1)$$

where C is the constant related to the base chosen to express the logarithm. It is a probabilistic measure of disorder present in a system and thus gives an indication of how much we do not know about a particular system's structure. Shannon's entropy is used to measure the amount of information content present within a given message or more generally any system of interest. Thus a more complex system would be expected to give a much higher information content than a less complex system. In other words, a more complex system would require more bits to describe compared to a less complex system. However, a sequence of random numbers will lead to the highest entropy and hence give a false indication of the system being complex when it is really just random. In this sense, complexity is somehow a measure of order or disorder that does not give a true indication of the information value present in the system, which in turn leads to an inaccurate characterization of complexity. Furthermore, an entropic measure does not take into account the semantic nature of the system. Consider for example a simple behavior such as walking. Let us assume that we are interested in measuring the complexity of walking in different environments and the walking itself is undertaken by an ANN. From Shannon's perspective, the complexity can be measured using the entropy of the data structure holding the neural network. Obviously a drawback for this view is its ignorance of the context and the concepts of embodiment and situatedness. The complexity of walking on a flat landscape is entirely different from walking on a rough landscape. Two neural networks may be represented using the same number of bits but exhibit entirely different behaviors. Using the outputs from the neural

networks as a measure of entropy is similarly problematic. Consider the case where a particular neural network optimized to perform robotic control has two of its output nodes swapped. The entropy as measured from the outputs of both the original and modified networks will remain the same but the behavior of the robot will change dramatically since the signals being sent to the individual actuators connected to these swapped output nodes have been disrupted. Hence, the change in the robot's behavior cannot be captured using this form of entropic measure.

The other approach to measuring complexity is a computation-theoretic approach based on Kolmogorov's application of universal Turing machines (Kolmogorov 1965) and is commonly known as Kolmogorov complexity or algorithmic complexity. It is a deterministic measure concerned with finding the shortest possible computer program or any abstract automaton that is capable of reproducing a given string. The Kolmogorov complexity $K(s)$ of a string s is given by

$$K(s) = \min\{|p| \mid s = C_T(p)\} \quad (7.2)$$

where $|p|$ represents the length of program p and $C_T(p)$ represents the result of running program p on Turing machine T . A more complex string would thus require a longer program while a simpler string would require a much shorter program. In essence, the complexity of a particular system is measured by the minimum amount of computation required to recreate the system in question. A well-known theoretical shortcoming of Kolmogorov complexity is that it is effectively uncomputable since by virtue of the halting problem (Turing 1936), it cannot be determined with certainty that the absolute shortest program or description has been found (Badii and Politi 1997; Edmonds 1999; Shalizi 2001). On an empirical level, the following example will show the limitations of Kolmogorov complexity. Assume we have a sequence of random numbers. Obviously the shortest program which is able to reproduce this sequence is the sequence itself. Consequently, it is somehow also a measure of order or disorder, thereby endowing it with highly similar properties to that of Shannon's entropy (Badii and Politi 1997; Edmonds 1999). In addition, let us re-visit the neural network example. Assume that the robot is not using a fixed neural network but some form of evolvable hardware (which may be an evolutionary

neural network). If the fitness landscape for the problem at hand is monotonically increasing, a hill climber will simply be the shortest program which guarantees to re-produce the behavior. However, if the landscape is rugged, reproducing the behavior is only achievable if we know the seed. Otherwise, the problem will require complete enumeration to recreate the behavior. Unlike Shannon's entropy measure, Kolmogorov complexity is both a syntactic and semantic measure of complexity but ignores the pragmatic nature of the system. Furthermore, Kolmogorov complexity has been shown to be a poor measure for biological complexity (Smith 1994; Szathmary, Jordan, and Pal 2001).

A measure of complexity commonly discussed in computer science and software engineering literature is computational complexity, which is the time and storage space required by actual algorithms to solve a given problem (Badii and Politi 1997). Normally, it is referred to by the big- O notation which is a worst-case complexity measure that is defined as the order of the rate of growth of the resources required to compute the output to a problem as compared to the size of its input (Edmonds 1999). For example, an algorithm with computational complexity $O(n^2)$ would be expected to have a quadratic increase in computational resources with each linear increase in its input while an algorithm with computational complexity of $O(n^3)$ would be expected to have a cubic increase with each linear increase in its input. The analysis of computational complexity has important implications in the study of NP-completeness (Garey and Johnson 1979). The problem is to ascertain whether or not a particular problem is tractable or intractable, or more accurately to determine whether or not a polynomial time algorithm exists that can solve the problem on a von Neumann architecture. However, computational complexity provides only a rough approximation as it is measured only according to the order of the polynomial associated with the increase required in computational resources when there is an increase in input. Furthermore, this measure of complexity specifically looks at the construction of program code and how the computational cost is affected by this code. Again, it measures complexity at the syntactic level and thus is unable to accommodate notions of environments or interactions which would be

paramount in a study of embodied organisms.

Bennett (1988) proposed a measure of complexity called logical depth by combining the notions of Kolmogorov complexity and computational complexity. The logical depth $D_s(x)$ of a string s at level x is defined as

$$D_x(s) = \min\{T(p) \mid |p| - |p^*| < x \wedge U(p) = s\} \quad (7.3)$$

where p is the range of programs, $T(p)$ is the run-time required by program p , p^* is the smallest such program and U is a Turing machine. It essentially states that the logical depth of a string is based on the running time of the shortest algorithm that will reproduce a given string. It is poised between Kolmogorov complexity and computational complexity in that it considers the size of the shortest program as well as the run-time of the program respectively (Badii and Politi 1997). Logical depth was proposed as a measure of the value of information as reflected by the degree to which that information has been organized in a particular object. However, as logical depth is defined based on Kolmogorov complexity, it too is essentially uncomputable (Badii and Politi 1997).

A starkly contrasting measure of complexity based on self-dissimilarity properties was recently proposed by Wolpert and MacReady (1997). Incorporating statistical inference and information theory, this complexity measure based on self-dissimilarity argues that the spatio-temporal signatures of complex systems vary markedly at different scales whereas the spatio-temporal signatures of simple systems do not differ significantly between different scales. Furthermore, the variation in a complex system's patterns over different space and time scales are considered to be the very essence of complexity rather than just an aberration of the modelling or measurement process. The spatio-temporal patterns in terms of the internal structure of a complex biological system for example, differs greatly when the observation scale is changed from the molecular level, to the cellular level, to the level of organs and to the level of the organisms itself. On the other hand, this self-dissimilarity measure argues that the spatio-temporal patterns of simple systems such as crystals, gases and even fractals do not change very much as one changes the scale of observation. However, as stated by the authors themselves, this notion of complexity has

only been formulated at the theoretical level and its real worth will only be proven when it is finally applied to real-world data (Wolpert and MacReady 1997).

In another research field known as computational mechanics, which is concerned with the dynamics of automata behavior, a mathematically-based entity that captures statistical complexity called the ϵ -machine was proposed by Feldman and Crutchfield (1998a). The ϵ -machine acts as a model for capturing the ensembles allowable configurations of a state machine. In other words, it is an object which allows for the inference of causal architecture from observed behavior (Shalizi 2001). As such, it allows for the definition and calculation of the global and macroscopic properties that reflect the average information processing capabilities of the system. The ϵ -machine has been applied empirically to measure the amount of self-organization achieved by four increasingly sophisticated types of process: memoryless transducers, time series, transducers with memory, and cellular automata (Shalizi 2001). Although the ϵ -machine has been shown to be effective and useful in capturing the increase in statistical complexity of such self-organized systems, this complexity measure is again based on automata theory and as such requires that the system being studied readily transforms into some form of state machine. As with other automata-based methods (Nehaniv 2000a), the question of how these transformations should be undertaken and what effects these transformations ultimately have when applied to less readily transformable systems such as creature behaviors and morphologies remains unanswered. More importantly, it provides a one dimensional view of complexity through the reduction of complex processes into a finite state machine and as such, does not leave any room for the interpretation of interactions between the system and its environment, for example. Hence, such automata-based complexity measures may not be a suitable methodology to apply to areas such as embodied cognition in terms of usefulness and pragmatic value where the essence of complexity lies in the system operating as a fully-interacting, adaptable and reactive whole.

7.3 Proposed EMO-Based Complexity Measure

We will now introduce the use of EMO as a convenient platform which researchers can utilize practically in attempting to define, measure, or simply characterize the complexity of everyday problems in a useful and purposeful manner. We first explain why a Pareto view to complexity is advantageous and then proceed to present our proposed method of measuring complexity using EMO. Finally, we discuss the assumptions associated with our proposed EMO-based complexity measure.

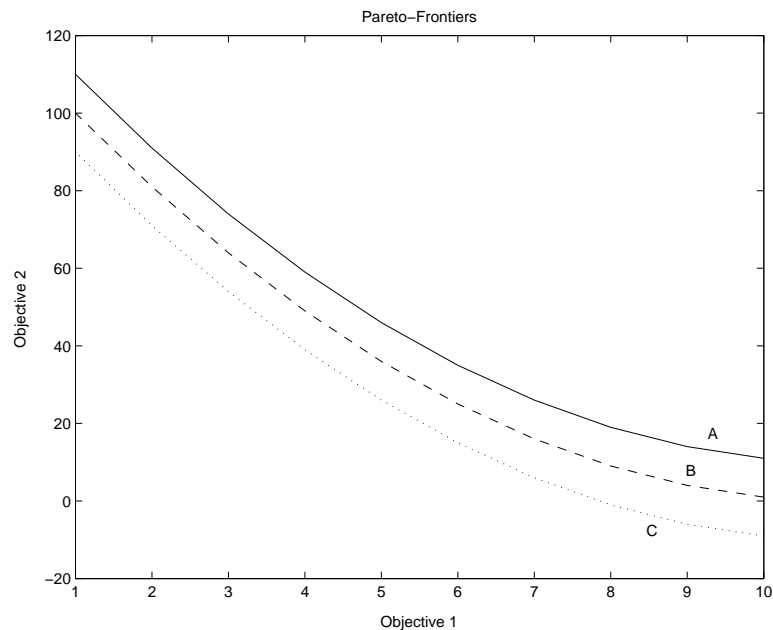


Figure 7.1: Diagram illustrating three different Pareto-frontiers for a problem with multiple objectives. X-axis: Objective 1, Y-axis: Objective 2.

Figure 7.1 provides an illustration of three layers of a potential Pareto-front of a particular multi-objective optimization problem. These layers can be viewed as providing three different levels of optimality. If we consider the problem to be maximization of both objectives, then the Pareto-front that dominates is curve A. All solutions along this front will dominate all other solutions in B and C because the solutions of A are more optimal along both objectives. If the problem involves minimization of both objectives, then the Pareto-front that dominates in this case

will be the curve C. We will show in the section that follows how this dominance ordering can provide useful insights for characterizing complexities from more than one perspective.

7.3.1 A Pareto View to Complexity

Here, we are interested in finding a binary relation which is able to say that one object “*is more complex than*” another object. There are a number of characteristics in which the elements in a set are related to one another and we shall visit each in turn to see which are desirable to have in such a binary relation. Assume a set A and a binary relation R on A . R is

- **Reflexive** if $(a, a) \in R \forall a \in A$. This is undesirable since an object should not be more complex than itself.
- **Irreflexive** if $(a, a) \notin R \forall a \in A$. This is desirable since an object should not be more complex than itself.
- **Symmetric** if $\forall a, b \in A, (a, b) \in R \rightarrow (b, a) \in R$. This is undesirable since if a is more complex than b , then b should not be more complex than a .
- **Asymmetric** if $\forall a, b \in A, (a, b) \in R \rightarrow (b, a) \notin R$. This is desirable since if a is more complex than b , then b should not be more complex than a .
- **Antisymmetric** if $\forall a, b \in A, (a, b) \in R$ and $(b, a) \in R \rightarrow a = b$. This is undesirable since if a and b are identical objects, then it should not hold true that a is more complex than b and b is more complex than a .
- **Transitive** if $\forall a, b, c \in A, (a, b) \in R$ and $(b, c) \in R \rightarrow (a, c) \in R$. This is desirable since if a is more complex than b , and b is more complex than c , then a should be more complex than c .
- **Negatively Transitive** if $\forall a, b, c \in A, (a, b) \notin R$ and $(b, c) \notin R \rightarrow (a, c) \notin R$. This is undesirable since if a is not more complex than b and b is not more complex than c , it does not imply that a is not more complex than c , which

we will show through contradiction. Assume two complexity measures 1 and 2 with three objects a , b , and c having the complexity values of $(20,30)$, $(30,10)$, and $(10,20)$ respectively with reference to the complexity measures 1 and 2 in that order. In this case, a is not more complex than b since b has a higher value than a in terms of complexity measure 1. Similarly, b is not more complex than c since c has a higher value than b in terms of complexity measure 2. If the complexity relation R is negatively transitive, then this implies that a is not more complex than c . However, this is a contradiction as a is actually more complex than c since a has higher values in terms of both complexity measures. Therefore, this axiom is undesirable for the complexity binary relation R .

- **Connected** if $\forall a, b \in A, a \neq b \rightarrow (a, b) \in R$ or $(b, a) \in R$. This is undesirable since some pairs of objects may share the same complexity class and hence not all pairs of objects are necessarily connected through the relation that one object is more complex than the other. We will show that connectedness is an undesirable axiom using the example described above. a has higher values in terms of both complexity measures than c , hence a is more complex than c and therefore is connected to c through the complexity relation R . However, a is not more complex than b and thus shares the same complexity class as b , thus a is not connected to b through the complexity relation R . Similarly, b is not more complex than c and therefore b is also not connected to c . Thus, this axiom is undesirable for the complexity binary relation R since some objects may share the same complexity class.
- **Strongly Connected** if $\forall a, b \in A, (a, b) \in R$ or $(b, a) \in R$. This is undesirable since if the connectedness axiom does not hold true, then this axiom cannot hold true.

Therefore, the binary relation “is more complex than”, R , should satisfy the irreflexivity, asymmetry and transitivity axioms.

It is important to point out that our purpose here is not to introduce another measure of complexity that can supposedly overcome all previous limitations

associated with existing measures neither claiming that it is an all-encompassing technique which will be able to calculate a definitive complexity value for complex systems. Our objective here is simply to propose and demonstrate that the Pareto set of solutions arising from an EMO process can be highly beneficial for characterizing and comparing between the complexities of different systems and at the same time satisfy the axioms desirable in a complexity binary relation. Furthermore, we will show through our experiments that the Pareto approach is a *useful* complexity measure. A complexity measure is said to be *useful* when it is able to capture what we intuitively regard as complex (Edmonds 1999).

There are two major advantages associated with using an EMO-based approach for capturing complexity. Firstly, it measures complexity of a particular system as seen from an observer's point of view. This has been argued by Casti (1986) to be paramount since the complexity of a system only has meaning through the interaction with its observer, particularly in more subjective areas such as behavioral complexity. As he puts it,

“...system complexity is a contingent property arising out of the interaction I between a system S and an observer/decision-maker O . Thus, any perception and measure of complexity is necessarily a function of S , O , and I .” (Casti 1986, p.149)

More importantly, he highlights the fact that

“Conditioned by the physical sciences, we typically regard S as the active system, with O being a passive observer or disengaged controller. Such a picture misses the crucial point that generally the system S can also be regarded as an observer of O and that the interaction I is a two-way path.” (Casti 1986, p.149)

Since a Pareto set is the result of optimization across two (or more) objectives, the solutions can be viewed as the result of a two-way interaction that occurs between the different objectives during the optimization process. Hence, a Pareto approach provides a distinct advantage when used to capture complexity by generating a set

of solutions that inherently exhibits properties of a two-way interaction and which can be reversibly used simply by looking at the results from the other optimization objective's view.

Secondly, we contend that the Pareto approach achieves a certain level of pragmatism when used as a complexity measure as opposed to simply providing a syntactic or semantic measure of complexity. In other words, it does not simply measure the complexity at the level of the language or symbols used to construct the system as in a typical syntactic measure nor does it measure the system's complexity within some predefined context or environment as would a semantic measure. Cariani (1992) explains that the syntactic axis represents operations conducted at the symbolic level, the semantic axis represents operations where symbolic information is extracted from the environment through measurement and control while the pragmatic axis represents the selection of appropriate measurements and controls that are advantageous to the operation of the system. In this sense, the proposed EMO methodology towards capturing complexity goes one step further in that it captures complexity through an evolutionary optimization process that continually generates new solutions from modification of previous solutions arising from testing and measurement of the system's performance within a given context or environment, which in turn is guided by the Pareto approach that imposes evolutionary pressures from multiple dimensions. In other words, it provides a view of complexity from a practical standpoint since a Pareto set comprises of solutions from a selection and adaptation process thereby constituting a pragmatic approach when such a Pareto set is used as a measure of complexity.

7.3.2 The Complexity Measure

We now present the formulation of our proposed complexity measure and demonstrate how it can be applied to characterize as well as compare the behavioral and morphological complexities of embodied artificial creatures. First, we define an embodied organism as the interaction between five components: morphology, behavior, controller, environment, and the learning algorithm. We will then show

how complexity can be defined as a partial order relation over this five-dimensional hyperspace. Accordingly, the complexity of two embodied organisms can be compared using this partial order relation. Finally, we support our argument with some experimental results which is presented in Section 7.4.

What follows is our proposal of a generic definition for complexity using the multi-objective paradigm. However, before we proceed with our definition, we need first to explain the concept of partial order.

Definition 1: Partial and Lexicographic Order. Assume the two sets A and

B . Assume the l -subsets over A and B such that $A = \{a_1 < \dots < a_l\}$ and $B = \{b_1 < \dots < b_l\}$.

A partial order is defined as $A \leq_j B$ if $a_j \leq b_j$, $\forall j \in \{1, \dots, l\}$

A Lexicographic order is defined as $A <_j B$ if $\exists a_k < b_k$ and $a_j = b_j$, $j < k$, $\forall j, k \in \{1, \dots, l\}$

In other words, a lexicographic order is a total order. In multi-objective optimization, the concept of Pareto optimality is normally used. A solution x belongs to the Pareto set if there is not a solution y in the feasible solution set such that y dominates x (that is x has to be at least as good as y when measured on all objectives and better than y on at least one objective). The Pareto concept thus forms partial orders in the objective space.

Let us recall the embodied cognition problem. The problem is to study the relationship between the behavior, controller, environment, learning algorithm and morphology. A typical question that one may ask is “What is the optimal behavior for a given morphology, controller, learning algorithm and environment?”. We can formally represent the problem of embodied cognition as the five sets B , C , E , L , and M for the five-dimensional hyperspace of behavior, controller, environment, learning algorithm, and morphology respectively. We also need to differentiate between the robot behavior B and the desired behavior \hat{B} . The former can be seen as the actual value of the fitness function and the latter can be seen as the real maximum of the fitness function. For example, if the desired behavior (task) is to maximize

the locomotion distance, then the global maximum of this function is the desired behavior, whereas the distance achieved by the robot (what the robot is actually doing) is the actual behavior. In traditional robotics, the problem can be seen as Given the desired behavior \hat{B} , find L which optimizes C subject to $E \cup M$. In psychology, the problem can be formulated as Given C , E , L and M , study the characteristics of the set B . In co-evolving morphology and mind, the problem is Given the desired behavior \hat{B} and L , optimize C and M subject to E . A general observation is that the learning algorithm is usually fixed during the experiments.

In asking a question such as “Is a human more complex than a Monkey?”, a natural question that follows would be “In what sense?”. Complexity is not a unique concept. It is usually defined or measured within some context. For example, a human can be seen as more complex than a Monkey if we are looking at the complexity of intelligence, whereas a Monkey can be seen as more complex than the human if we are looking at the number of different gaits the monkey has for locomotion. Therefore, what is important from an artificial life perspective is to establish the complexity hierarchy on different scales. Consequently, we introduce the following definition for complexity.

Definition 2: Complexity is a strict partial order relation.

According to this definition, we can establish an order of complexity between the system’s components/species. We can then compare the complexities of two species $S_1 = (B_1, C_1, E_1, L_1, M_1)$ and $S_2 = (B_2, C_2, E_2, L_2, M_2)$ as:

S_1 is at least as complex as S_2 **with respect to concept Ψ iff**

$$S_2^\Psi = (B_2, C_2, E_2, L_2, M_2) \leq_j S_1^\Psi = (B_1, C_1, E_1, L_1, M_1), \forall j \in \{1, \dots, l\}, \text{ **Given**}$$

$$B_i = \{B_{i1} < \dots < B_{il}\}, C_i = \{C_{i1} < \dots < C_{il}\}, E_i = \{E_{i1} < \dots < E_{il}\},$$

$$L_i = \{L_{i1} < \dots < L_{il}\}, M_i = \{M_{i1} < \dots < M_{il}\}, i \in \{1, 2\} \quad (7.4)$$

where Ψ partitions the sets into l non-overlapping subsets.

We can even establish a complete order of complexity by using the lexicographic order as:

S_1 is more complex than S_2 **with respect to concept Ψ iff**

$$S_2^\Psi = (B_2, C_2, E_2, L_2, M_2) <_j S_1^\Psi = (B_1, C_1, E_1, L_1, M_1), \forall j \in \{1, \dots, l\}, \text{ **Given**}$$

$$B_i = \{B_{i1} < \dots < B_{il}\}, C_i = \{C_{i1} < \dots < C_{il}\}, E_i = \{E_{i1} < \dots < E_{il}\},$$

$$L_i = \{L_{i1} < \dots < L_{il}\}, M_i = \{M_{i1} < \dots < M_{il}\}, i \in \{1, 2\} \quad (7.5)$$

The lexicographic order is not as flexible as partial order since the former requires a monotonic increase in complexity. The latter however, allows individuals to have similar levels of complexity. Therefore, it is more suitable for defining hierarchies of complexity. Hence, our definition of complexity based on the Pareto approach conforms to the set of axioms desirable in a binary operator for measuring complexity as discussed earlier in Section 7.3.1.

The concept of Pareto optimality is a special case of the partial order concept in that Pareto optimality is a strict partial order. In other words, Pareto optimality does not satisfy reflexivity; that is, a solution cannot dominate itself. Therefore two copies of the same solution cannot co-exist as Pareto solutions. Usually, when we have copies of one solution, we discard one of them. Therefore this problem does not arise when the Pareto set is generated. As a result, we can assume here that Pareto optimality imposes a complexity hierarchy on the solution set.

The previous definition will simply order the sets based on their complexities according to some concept Ψ . However, they do not provide an exact quantitative measure for complexity. In the simple case, given the five sets B , C , E , L , and M : assume the function f , which maps each element in each set to some value called the fitness, and assuming that C , E and L do not change, a simple measure of morphological change of complexity can be

$$\frac{\partial f(b)}{\partial m}, b \in B, m \in M \quad (7.6)$$

In other words, assuming that the environment, controller, and the learning algorithm are fixed, the change in morphological complexity can be measured in the

eyes of the change in the fitness of the robot (actual behavior). The fitness will be defined later in Section 7.4.2. Therefore, we introduce the following definition

Definition 3: Change of Complexity Value for the morphology is the rate of change in behavioral fitness when the morphology changes, given that both the environment, learning algorithm and controller are fixed.

The previous definition can be generalized to cover the controller and environment quite easily by simply replacing “morphology” by either “environment”, “learning algorithm”, or “controller”. Based on this definition, if we can come up with a good measure for behavioral complexity, we can use this measure to quantify the change in complexity for morphology, controller, learning algorithm, or environment. In the same manner, if we have a complexity measure for the controller, we can use it to quantify the change of complexity in the other four parameters. Therefore, we propose the notion of defining the complexity of one object as viewed from the perspective of another object. This is not unlike Emmeche’s idea of complexity as put in the eyes of the beholder (Emmeche 1994). However, we formalize and solidify this idea by putting it into practical and quantitative usage through the multi-objective approach. We will demonstrate that results from an EMO run of two conflicting objectives results in a Pareto-front that allows a comparison of the different aspects of an artificial creature’s complexity.

In the literature, there are a number of related topics which can help here. For example, the Vapnik-Chervonenkis (VC) dimension (Vapnik and Chervonenkis 1971) can be used as a complexity measure for the controller. A feed-forward neural network using a threshold activation function has a VC dimension of $O(W \log W)$ while a similar network with a sigmoid activation has a VC dimension of $O(W^2)$, where W is the number of free parameters in the network (Haykin 1999). It is apparent from here that one can control the complexity of a network by minimizing the number of free parameters which can be done in a number of ways, the most obvious being the minimization of the number of synapses and/or the number of hidden units. It is important to separate between the learning algorithm and the model itself. For example, two identical neural networks with fixed architectures

may perform differently if one of them is trained using back-propagation while the other is trained using an evolutionary algorithm. In this case, the separation between the model and the algorithm helps us to isolate their individual effects and gain an understanding of their individual roles.

In this set of experiments, we are essentially posing two questions, what is the change of (1) behavioral complexity, and (2) morphological complexity of the artificial creature in the eyes of its controller. In other words, how complex is the behavior and morphology in terms of evolving a successful controller?

7.3.3 Complexity Measures Revisited

Before we proceed with an empirical experiment of how this complexity measure based on the Pareto concept can be applied to capturing the morphological and behavioral complexities of artificially evolved creatures, we first provide some examples of how this methodology can be applied in a more general manner to the biological, social and physical sciences.

First, we provide a Pareto view to complexity in the biological sciences. More specifically, we will use two existing measures for biological complexity, namely genome length and number of genes which were discussed previously in Section 7.2.2. The data used in this example are actual genomic information extracted from the **EnsEMBL** on-line database (EnsEMBL.Org 2002)². We will compare the complexities of five different organisms (the version of the organism's genomic database is given in following parentheses): human (v.8.30a.1), mouse (v.8.3b.1), zebrafish (v.8.08.1), fugu or pufferfish (v.8.1.1) and mosquito (v.8.1b.1). In terms of genome length, the order of complexity from least to greatest number of DNA base-pairs, we obtain

1. zebrafish (0.04×10^9 BP)

²**EnsEMBL** is a joint project between European Molecular Biology Laboratory, European Bioinformatics Institute and the Sanger Institute to develop a software system that produces and maintains automatic annotation on eukaryotic genomes. It is one of the three main repositories for genomic information (Gibas and Jambeck 2001).

2. mosquito (0.28×10^9 BP)
3. fugu (0.33×10^9 BP)
4. mouse (2.73×10^9 BP)
5. human (3.34×10^9 BP)

However, if we take the number of genes instead as the measure for biological complexity, then we obtain the following ordering

1. zebrafish (1511 genes)
2. mosquito (15088 genes)
3. mouse (22444 genes)
4. human (22980 genes)
5. fugu (31059 genes)

As such, by simply changing the complexity measure (scale) from genome length to number of genes, we have dramatically changed the ordering of complexity for the mouse, human and fugu, as depicted in Figure 7.2.

Now let us take a multi-objective approach to characterizing the complexities of these different organisms by combining the two biological complexity measures into a 2D graph.

Let us assume that a real biological organism can be made analogous to an artificial embodied creature. Now we can compare between the organisms' complexities by making the following representations: assume that all the organisms share a common environment E being the Earth, acquire knowledge through some common learning mechanism L such as reinforcement learning, that the organisms have different morphologies M , that the genome is acting as a master controller C and that the primal behavior in the organism B is a reflection of its genes.

Using the Pareto approach, we can now characterize the complexity of these five organisms at four different levels. Firstly, we can make some observations

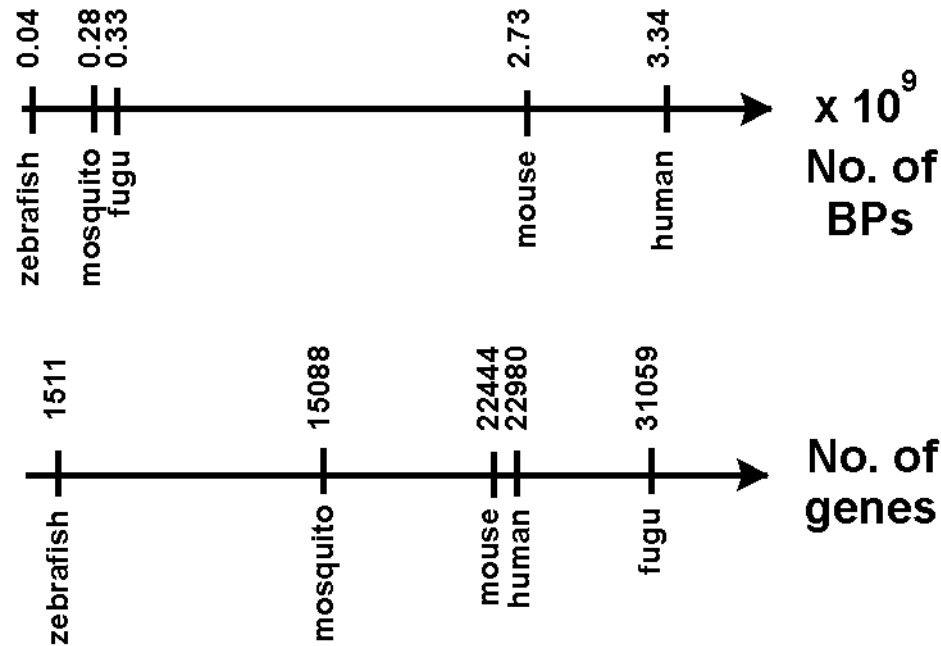


Figure 7.2: Conflicting ordering of biological complexity when taking a single-objective view to complexity measures (scales).

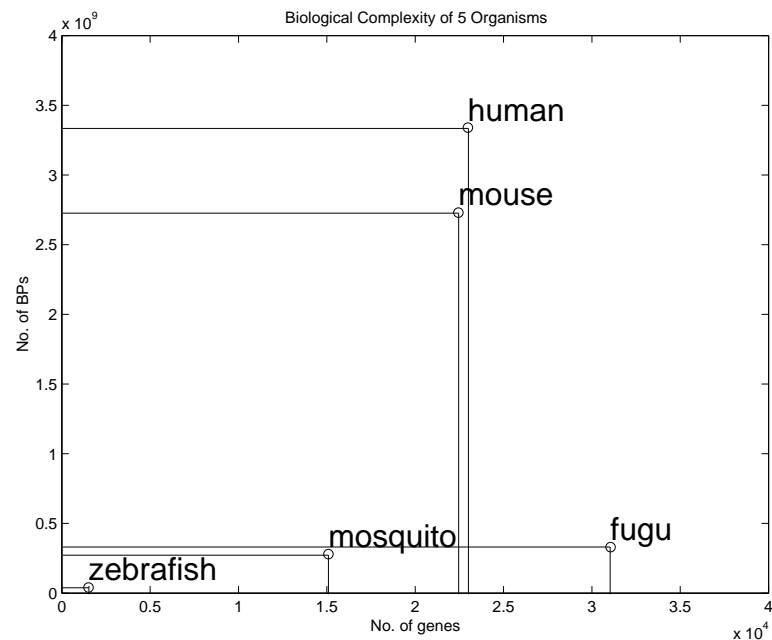


Figure 7.3: Comparing the biological complexity of 5 organisms by combining two different complexity measures (scales). X-axis: No. of genes, Y-axis: No. of BPs.

about which organisms are more complex than others by considering the dominance ordering present in one set of organisms defined within the two complexity dimensions B and C . Since the mosquito dominates the zebrafish along both complexity dimensions, we can say that the mosquito is more complex than the zebrafish. In a similar manner, we can also say that the mouse is more complex than the mosquito and that both the human and fugu is in turn more complex than the mouse. However, if we compare the human against the fugu, we cannot say that either is more complex than the other because they both each dominate the other along one of the complexity dimensions. In other words, the human and fugu are at the Pareto-front of this particular set of organisms when compared using these two measures of biological complexity. In this case, the hierarchy of complexity can be illustrated as in Figure 7.4.

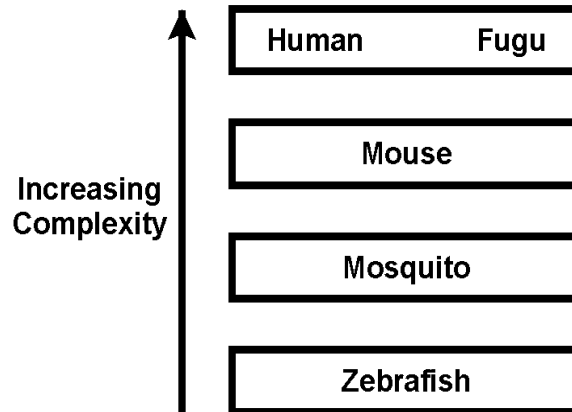


Figure 7.4: Diagram illustrating the complexity hierarchy of 5 organisms constructed using a Pareto approach.

Secondly, we can make some quantitative comparisons by looking at the Euclidean distance between these organisms. We can see that the increase in complexity from the zebrafish to the mosquito was smaller than the increase in complexity from the mosquito to the mouse. We can also look at the change in complexity along one dimension relative to the other. For example, a relatively small change in the zebrafish's number of DNA base-pairs resulted in a surprisingly large increase in number of genes as compared to the required increase in number of DNA base-pairs

going from the mosquito to the mouse for a roughly similar increase in the number of genes.

Thirdly, if we had another dimension of complexity, say in terms of the environment E and that E now represents the planet Mars, then this second set of organisms from Mars can be compared with the first set of organisms from Earth by taking the two Pareto-fronts present in these two sets of organisms and finding some common ground for complexity along one dimension and comparing the relative change in the other. In this case, let us presume that Organism M-human and Organism M-fugu represent the two organisms placed at the Pareto-front of organisms from Mars, then these two organisms can be compared against the human and fugu from Earth, since these form the Pareto-front of organisms from Earth. For example, say if we find that Organism M-human has a comparable number of DNA base-pairs as the fugu but has double the number of genes, then we might conclude that the change in the environment from Earth to Mars shows a more complex environment in Mars in terms of the required number of genes to survive as compared to on Earth since for the same number of DNA base-pairs, the organism in Mars required more number of genes to allow for the survival of Organism M-fugu.

In terms of the social sciences, an example can be taken from the political governance of two different countries, which can be regarded as two distinct complex systems. Let us assume that Country A has less citizens than Country B and define the complexity of the country based on the number of citizens that needs to be governed. Using this measure, Country A would be considered less complex than Country B. Let us assume that another measure of complexity can be formulated in terms of the governance structure of the different countries. Assume Country A is democratic with a government that is lead by a group of elected representatives while Country B is autocratic with a government that is lead by a single dictator. It is reasonable to assume that Country A requires many interactions between its politicians before any decision can be made compared with no interaction required whatsoever in the case of a decision made by the sole dictator in Country B. In this sense, Country A may now be considered to be more complex than Country B on

this new scale. A Pareto view will again help in comparing between the political complexities of these countries by considering one complexity measure in the eyes of the other complexity measure rather than taking these conflicting viewpoints as stand-alone indicators, which would then reflect a failure in providing a reasonable characterization of political complexity between the two countries by virtue of the contrasting quantizations.

Let us now see how this can be done in terms of our formulated measure of complexity. First, let us consider that the actual political complexity of these countries are the result of interactions between both its citizens and governance structure, rather than just either of these singular components. Then let us assume that the political complexities of the countries represent the morphology M of the complex system, that the environment E is unchanged if we consider them to be geographically located in the same region of the world, that learning occurs through some common medium L such as the mass media, that the populations represent the behavior B and that the governance structures represent the controller C . Now we can compare the change in political complexity between these two different countries ∂M by measuring some quantitative change in the behavior of the population ∂B through some commonality that can be found in the hyperspace of the controlling governance structure C . Conversely, we can compare the governance complexity between the two countries ∂M from the reverse viewpoint by measuring some quantitative change in terms of the controlling governance structure ∂C through establishing some commonality in the hyperspace of the population's behavior B . Casti (1986) provides an elegant example of how such a complex system emerges from the interactions between its governance structure and its citizens. Here, he states that the citizens views the governance structure as complex if the actions taken by its political leaders seem to be incomprehensible:

“... they [*the citizens*] see a byzantine and unwieldy government bureaucracy and a large number of independent decision-makers (government agencies) affecting their day-to-day life.” (p.150)

Similarly, the government typically also views its citizens as being very complex:

“They [*the political leaders*] would see a seemingly fickle, capricious public, composed of a large number of independent self-interest groups clamoring for more and more public goods and services.” (p.150)

Hence, we can see that a multi-objective view to the study of social sciences such as political complexity can again be very insightful and valuable.

In revisiting complexity measures for the physical sciences, let us turn to an example from computer science itself. Wuensche (1999) has recently devised a method for automatic classification of 1D cellular automata (CA) rules into one of three dynamical groups, that is ordered, complex and chaotic systems based on the frequency of particular updating rules being looked-up over time called the input-entropy. Based on Shannon’s entropy measure, Wuensche (1999) formulated input-entropy S at time-step t as

$$S^t = - \sum_{i=1}^{2^k} \left(\frac{Q_i^t}{n} \times \log\left(\frac{Q_i^t}{n}\right) \right) \quad (7.7)$$

where k and n are the neighborhood and system size of the CA, and Q_i^t is the lookup frequency of neighborhood i at time t . One of the proposed classification methods was based on indications given by two measures: (1) the input-entropy itself, and (2) the variability (standard deviation) of the input-entropy. An example from Wuensche (1999) classified three rules from a Boolean CA system with $k = 5$ and $n = 150$, described as typical examples of CA behaviors, as given in Table 7.2:

Rule No.	Classification	Input-Entropy	Variability of Input-Entropy
01 dc 96 10	Ordered	low	low
6c 1e 53 a8	Complex	medium	high
99 4a 6a 65	Chaotic	high	low

Table 7.2: Classification of 3 cellular automata rules according to Wuensche (1999).

Let us now consider two rules arising from the same CA setup that after experimentation and analysis gave the following indications:

- Rule X: moderately high entropy, moderately high variability

- Rule Y: very high entropy, very high variability

It would be difficult to classify these two rules since they are placed mid-way between the existing classes. However, if we take a multi-objective view to this problem, we would be able to provide the following perspective:

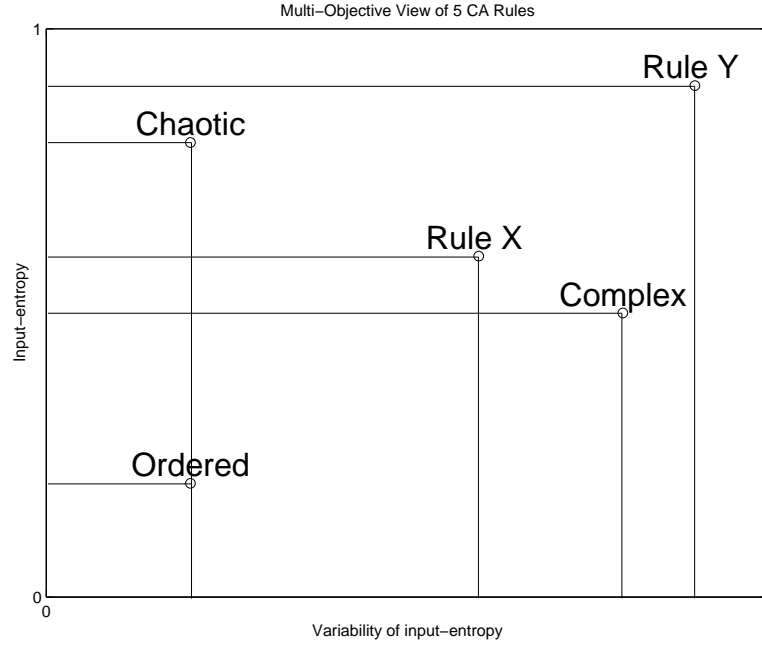


Figure 7.5: Multi-objective view of 5 cellular automata rules by combining input-entropy and variability of input-entropy. X-axis: Variability of input-entropy, Y-axis: Input-entropy.

If we now cast the cellular automata system into our proposed measure of complexity, we can formulate the following representations. The environment E is represented by the CA's overall system setup as defined by the neighborhood size, periodic boundary conditions and dimensionality. As such, E remains constant since both rules arise from the same CA setup. L would be null in all cases since no learning occurs in a CA system. Let us now consider that the difference between Rule X and Rule Y represents a change in the controller C of the CA system since the dynamics of a CA is dependent on the rule being used in the CA. Next, we shall consider that the morphology M of the CA is represented by the input-entropy and that the behavior B of the CA is represented by the variability in the input-entropy.

Firstly, we can see that Rule Y dominates all other rules in this particular system since it has higher values for both complexity dimensions of B and M . As such, we can say that Rule Y is the most complex rule from a multi-objective perspective. Also, we can see that a Pareto-front is formed by the Chaotic and Complex rules, and that both rules are as complex as each other since they both dominate each other in one dimension. Furthermore, since Rule X is not dominated by either the Chaotic nor the Complex rule in both dimensions, it too belongs to the Pareto set for this particular CA system and that its complexity can be characterized as being similar to that of the Chaotic and Complex rules in terms of B and M . As for the Ordered rule, it has the same value for B (variability of input-entropy) as the Chaotic rule and has a lower value for M (input-entropy) than the Chaotic rule. As such, the Ordered rule is dominated by the Pareto-front of which the Chaotic rule is a member. Hence, it can be characterized as being the least complex among all the rules in this particular CA system since it is dominated by all other rules. As with the biological example visited earlier, a change in the environment E , for example increasing the neighborhood size, will produce a second set of observations which can then be compared with this first set of observations by comparing the two Pareto-fronts obtained from these two different CA systems.

In the next section, we will describe the setup of the experiments which demonstrate empirically how our proposed measure for capturing complexity can be applied to the comparison of the morphological and behavioral complexities of artificially evolved creatures.

7.4 Experimental Setup

7.4.1 Two Artificial Creatures

Two artificial creatures were used in this study (Figure 7.6): (1) a quadruped creature with four legs, (2) and a hexapod creature with six legs. The first artificial creature (Figure 7.6.1) is the same quadruped used in Chapters 4, 5, and

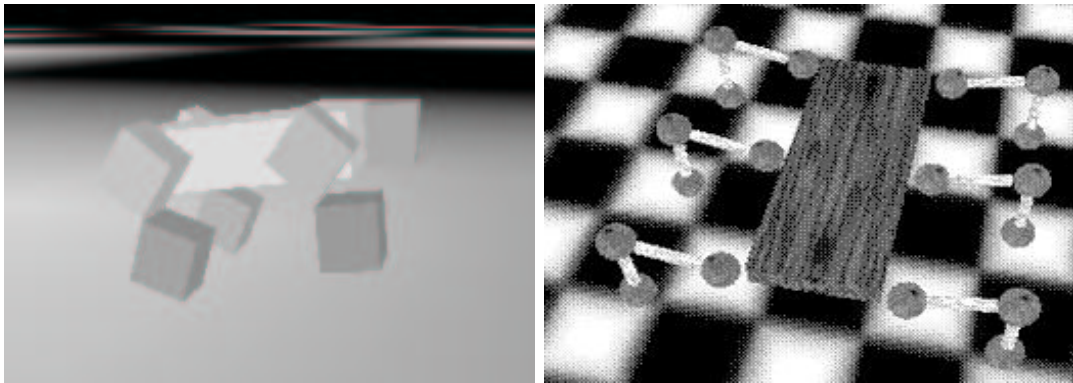


Figure 7.6: Screen dump of the 1. quadruped (left), 2. hexapod (right) artificial creatures.

6. The second artificial creature (Figure 7.6.2) is a hexapod³ with 6 long legs that are connected to the torso by insect hip joints. Each insect hip joint consists of two hinges, making it a joint with two degrees of freedom: one to control the back-and-forth swinging and another for the lifting of the leg. Each leg has an upper limb connected to a lower limb by a hinge (one degree of freedom) joint. The hinges are actuated by motors in the same fashion as in the first artificial creature.

Morphological Characteristic	Simulated Quadruped	Simulated Hexapod
No. of legs	4	6
Degrees of freedom	8	24
No. of sensors	12	24
No. of motors	8	18

Table 7.3: A comparison of the simulated quadruped and hexapod creatures' morphological characteristics.

Table 7.3 presents a comparison of the main features of the two artificial creatures. It would appear that the quadruped has a much simpler design compared to the hexapod creature. However, this is only a subjective observation from a human designer's perspective. It remains to be seen whether this view will hold when

³The design and experimentation of the hexapod creature was carried out jointly with another graduate student Ms. Minh Ha Nguyen.

we compare the complexities of these two artificial creatures from the controller's and behavior's perspectives.

7.4.2 Controller Architecture

The Pareto-frontier of our evolutionary runs are obtained from optimizing two conflicting objectives as in Chapter 5: (1) minimizing the number of hidden units used in the ANN that act as the creature's controller, and (2) maximizing horizontal locomotion distance of the artificial creature. What we obtain at the end of the runs are again Pareto sets of ANNs that trade-off between number of hidden units and locomotion distance. The locomotion distances achieved by the different Pareto solutions will provide a common ground where locomotion competency can be used to compare different behaviors and morphologies. It will provide a set of ANNs with the smallest hidden layer capable of achieving a variety of locomotion competencies. The structural definition of the evolved ANNs can now be used as a measure of complexity for the different creature behaviors and morphologies.

The type of ANN architecture used for the experiments in this chapter is NNType3 as presented in Section 3.3.3, which has fully-connected feed-forward network with recurrent connections on the hidden units as well as direct input-output connections. Only one type of architecture was used since the results from Chapter 5 showed no significant differences between the four architectures. Of the four architectures, NNType3 was chosen since the best overall locomotion distance was achieved using this particular architecture. A diagrammatic representation of part of the ANN architecture is illustrated in Figure 3.4.4. The genotype representation used for specifying the ANN controller remains unchanged as explained in Section 3.4 and the SPANN algorithm as presented in Section 5.4.1 was again used to drive the artificial evolutionary process.

7.4.3 Assumptions

Two assumptions need to be made. First, the Pareto set obtained from evolution is considered to be the actual Pareto set. This means that for the creature

on the Pareto set, the maximum amount of locomotion is achieved with the minimum number of hidden units in the ANN. We do note however that the evolved Pareto set in the experiments may not have converged to the optimal set. Nevertheless, it is not the objective of this paper to provide a method which guarantees convergence of EMO but rather to introduce and demonstrate the application of measuring complexity in the eyes of the beholder. It is important to mention that although this assumption may not hold, the results can still be valid. This will be the case when creatures are not on the actual Pareto-front but the distances between them on the intermediate Pareto-front are similar to that of creatures on the actual Pareto-front.

The second assumption that we are making is that there are no redundancies present in the ANN architectures of the evolved Pareto set. This simply means that all the input and output units as well as the synaptic connections between layers of the network are actually involved in and required for achieving the observed locomotion competency. We have investigated the amount of redundancy present in evolved ANN controllers in Section 6.4.5 and found that the self-adaptive Pareto EMO approach produces networks with virtually no redundancy.

Before excluding these assumptions, it is important to emphasize that none of these assumptions will dramatically change our findings. Since we are interested in the partial order and the rate of change, getting to the exact Pareto-front or obtaining the neural network with zero redundancy may not affect the results. Take for example the solutions on the Pareto set found during a hypothetical EMO run $X_1 = (0, 5)$, $X_2 = (1, 4)$. Let us assume that the two solutions on the actual Pareto-front are $Y_1 = (0, 5.5)$, $Y_2 = (1, 4.5)$. What we are interested in is the difference between the two solutions and the partial order between them, which as can be seen in this case, are not affected by the evolved Pareto-front not being the actual Pareto-front. Obviously this is a hypothetical example but at least it demonstrates that the assumptions may hold in a large number of actual cases.

7.4.4 Evolutionary Runs

Two series of experiments were conducted. Behavioral complexity was investigated in the first series of experiments and morphological complexity was investigated in the second. For both series of experiments, each evolutionary run was allowed to evolve over 1000 generations with a randomly initialized population size of 30. The maximum number of hidden units was again fixed at 15 as in previous experiments carried out in Chapters 4, 5, and 6. The number of hidden units used and maximum locomotion achieved for each genotype evaluated as well as the non-dominated set of solutions obtained in every generation were recorded. The Pareto solutions obtained at the completion of the evolutionary process were compared to obtain a characterization of the behavioral and morphological complexity.

To investigate behavioral complexity in the eyes of the controller, the morphology was fixed by using only the quadruped creature but the desired behavior was varied by having two different fitness functions. The first fitness function measured only the maximum horizontal locomotion achieved but the second fitness function measured both maximum horizontal locomotion and static stability achieved. By static stability, we mean that the creature achieves a statically stable locomotion gait with at least three of its supporting legs touching the ground during each step of its movement. The two problems we have are:

(P1)

$$f_1 = \uparrow d(g) \quad (7.8)$$

$$f_2 = \Downarrow \sum_{h=0}^H \rho_h \quad (7.9)$$

(P2)

$$f_1 = \uparrow d(g)/20 + s(g)/500 \quad (7.10)$$

$$f_2 = \Downarrow \sum_{h=0}^H \rho_h \quad (7.11)$$

where $P1$ and $P2$ are the two sets of objectives used. f_1 and f_2 represent the respective fitness functions used to evaluate the genotypes g . d refers to the locomotion distance achieved and s is the number of times the creature is statically stable as

controlled by the ANN at the end of the evaluation period of 500 timesteps. $P1$ is using the locomotion distance as the first objective while $P2$ is using a linear combination of the locomotion distance and static stability. Minimizing the number of hidden units is the second objective in both problems.

To investigate morphological complexity, another set of 10 independent runs was carried out but this time using the hexapod creature. This is to enable a comparison with the quadruped creature which has a significantly different morphology in terms of its basic design. The $P1$ set of objectives was used to keep the behavior fixed. The results obtained in this second series of experiments were then compared against the results obtained from the first series of experiments where the quadruped creature was used with the $P1$ set of objective functions. Where the $P1$ experiments involving the quadruped creature was required, the results from Section 5.6 for NNType3 were used since the setup of the experiments were identical.

7.5 Results and Discussion

7.5.1 Morphological Complexity

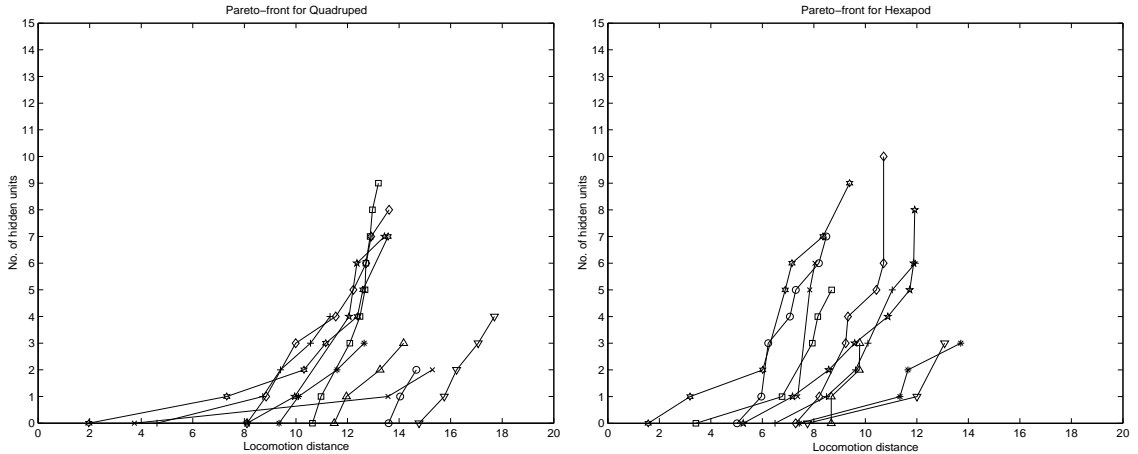


Figure 7.7: Pareto-frontiers of controllers obtained from 10 runs using the $P1$ set of objectives for the 1. quadruped (left), 2. hexapod (right). X-axis: Locomotion distance, Y-axis: No. of hidden units.

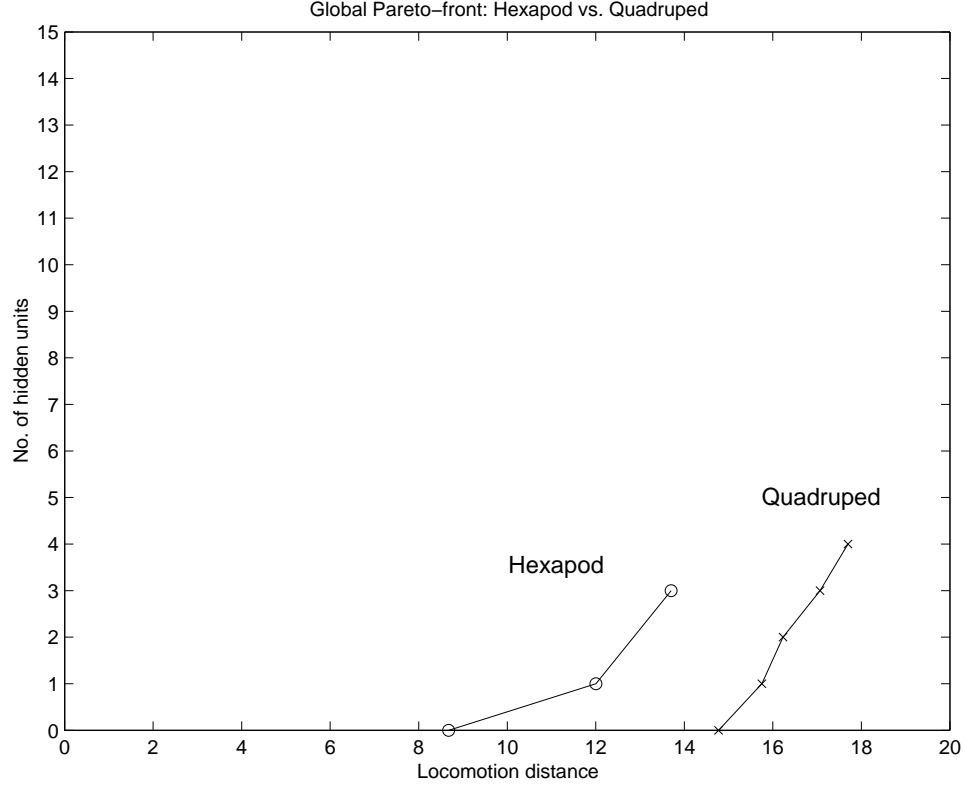


Figure 7.8: Global Pareto-front of controllers obtained from 10 runs using the $P1$ set of objectives for the quadruped and hexapod. X-axis: Locomotion distance, Y-axis: No. of hidden units.

We first present the results for the quadruped and hexapod evolved under $P1$. Figure 7.7 compares the Pareto optimal solutions obtained for the two different morphologies over 10 runs. Figure 7.8 plots the global Pareto-front for both the hexapod and quadruped. As such, we are comparing two Pareto-fronts that characterize the complexities of two different systems. Here we are fixing E and L . Therefore, we can either measure the change of morphological complexity in the eyes of the behavior or the controller: that is, $\frac{\partial f(B)}{\partial M}$ or $\frac{\partial f(C)}{\partial M}$ respectively. If we fix the actual behavior B as the locomotion competency of achieving a movement of $13 < d < 15$, then the change in the controller $\partial f(C)$ is measured according to the number of hidden units used in the ANN. At this point of comparison, we find that the quadruped is able to achieve the desired behavior with 0 hidden units whereas the hexapod required 3 hidden units (Figure 7.8). Therefore, this is an indication

that from the controller's point of view, given the change in morphology ∂M from the quadruped to the hexapod, there was an increase in complexity for the controller ∂C from 0 hidden units to 3 hidden units. Hence, the hexapod morphology can be seen as being more complex than the quadruped morphology in the eyes of the controller.

Conversely, if we would like to measure the complexity of the morphology from the eyes of the locomotion behavior, we can choose a common point of comparison in terms of the network size. If we fix the controller C to having a hidden layer size of 3 hidden units, then the change in the locomotion behavior $\partial f(B)$ is measured according to the maximum distance achieved by artificial creatures. At this point of comparison, we find that the quadruped achieves just over 17 units distance while the hexapod is only able to achieve just under 14 units distance (Figure 7.8). Thus, this is an indication that from the locomotion behavior's point of view, given the change in morphology ∂M from the quadruped to the hexapod, there was an increase in complexity for the locomotion behavior ∂B of approximately 3 units distance. In this case, the quadruped morphology can be seen as being more complex than the hexapod morphology.

As such, by taking different viewpoints, we find different interpretations of which morphology is more complex than the other. This is not unlike what we have seen in the biological sciences (see Section 7.2.2) where different complexity measures result in different orderings of organismic complexity. Therefore, a Pareto approach to capturing complexity is advantageous in the sense that it gives multiple views of complexity through a single comparison exercise.

7.5.2 Behavioral Complexity

A comparison of the results obtained using the two different sets of fitness functions $P1$ and $P2$ is presented in Table 7.4. Here we are fixing M , L and E and looking for the change in behavioral complexity. The morphology M is fixed by using the quadruped creature only. For $P1$, we can see that the Pareto-frontier offers a number of different behaviors. In this case, we are comparing complexities within a

Type of Behavior	Pareto Controller	No. of Hidden Units	Locomotion Distance	Static Stability
<i>P1</i>	1	0	14.7730	19
	2	1	15.7506	24
	3	2	16.2295	30
	4	3	17.0663	26
	5	4	17.6994	14
<i>P2</i>	1	0	5.2065	304
	2	1	3.3355	408
	3	2	3.5935	420
	4	3	3.6829	419

Table 7.4: Comparison of number of hidden units, locomotion distance and static stability for global Pareto optimal controllers obtained using the quadruped for the *P1* and *P2* sets of objective functions.

system by using the evolved Pareto-front to represent the complexity characteristics of a single system. For example, a network with no hidden units can achieve up to 14.7 units of distance while the creature driven by a network with 5 hidden units can achieve 17.7 units of distance within the 500 timesteps. This is an indication that to achieve a higher speed gait entails a more complex behavior than a lower speed gait.

We can also see the effect of static stability, which requires a walking behavior, by comparing the two Pareto-fronts that characterize the *P1* and *P2* systems respectively. By comparing a running behavior using a dynamic gait in *P1* with no hidden units against a walking behavior using a static gait in *P2* with no hidden units, we can see that using the same number of hidden units, the creature achieves both a dynamic as well as a quasi-static gait. If more static stability is required, this will necessitate an increase in controller complexity.

At this point of comparison, we find that the behavior achieved with the *P1* fitness functions consistently produced a higher locomotion distance than the behavior achieved with the *P2* fitness functions. This meant that it was much harder for the *P2* behavior to achieve the same level of locomotion competency in terms of distance moved as the *P1* behavior due to the added sub-objective of having

to achieve static stability during locomotion. Thus, the complexity of achieving the $P2$ behavior can be seen as being at a higher level of the complexity hierarchy than the $P1$ fitness function in the eyes of the controller.

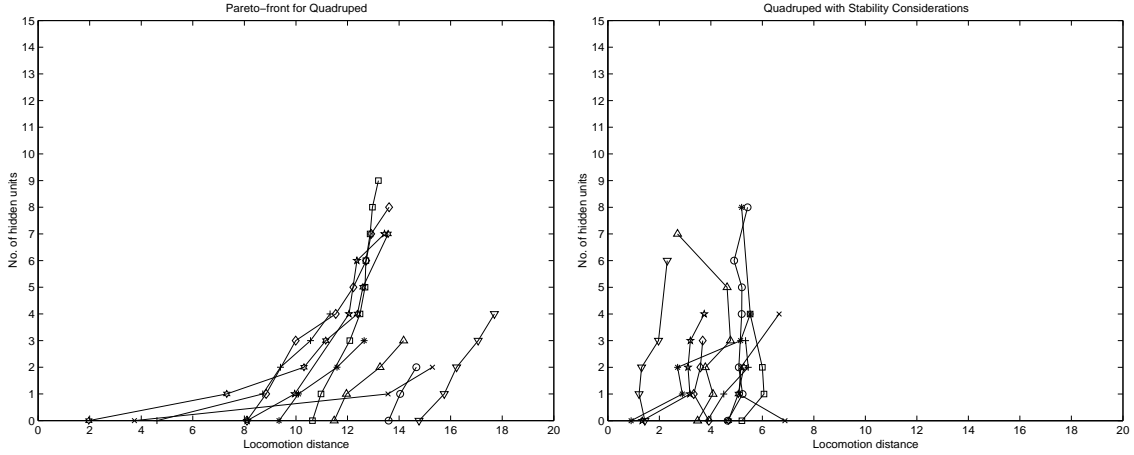


Figure 7.9: Locomotion distance of Pareto solutions obtained from 10 runs using the quadrupted with the $P1$ (left) and $P2$ (right) sets of objective functions. X-axis: Locomotion distance, Y-axis: No. of hidden units.

Figure 7.9 depicts the locomotion distance achieved using the quadrupted with the $P2$ set of objectives, which measure for both locomotion distance and static stability, along with the Pareto-fronts obtained from using the $P1$ set of objectives. Note that the graph for $P2$ does not depict Pareto-fronts since we are only interested in the locomotion distance, which is only part of the objective function. Here we see that the locomotion distance achieved was much lower due to the added sub-objective of attempting to maximize static stability. This is expected since the creature will be discouraged from jumping behaviors, which may allow for greater locomotion capabilities. No controllers could be found that achieved a locomotion distance of $d > 7$. As such, it was not possible to compare the behavioral complexities of standard locomotion versus locomotion with stability since the quadrupted evolved for standard locomotion achieved a distance of $13 < d < 15$ with the least complex network of 0 hidden units. In other words, no commonality could be found that would have enabled a comparison of the change in behavioral complexity from

the eyes of the controller.

7.5.3 Limitations

From our experiments, we note the following limitations with the proposed complexity measure based on the Pareto approach:

- To compare across Pareto-fronts from different systems, some common ground needs to be established to enable a fair comparison of complexity. The first disadvantage arising from this requirement is that the inherent complexity of the particular component being investigated may be so different between the different systems that no common ground exists. For example, we found that no common ground existed when we tried to compare between the two fitness functions $P1$ and $P2$ when viewed from the controllers perspective (see Section 7.5.2). Secondly, the actual determination of the range of values considered to be within this area of commonality is dependent on the results of the components being compared. For example, in our comparison between the quadruped and hexapod creatures, we determined that the locomotion competency range used as the common ground when viewing from the controller's perspective was $13 < d < 15$ by virtue of the results that were obtained (see Section 7.5.1).
- In empirical studies involving artificial evolutionary systems, some form of conflicting objectives needs to be present before this Pareto approach will provide any additional benefits in taking a multi-objective view to complexity. In the case where the objectives of a particular problem are not in conflict, the Pareto approach will simply reflect an hierarchical ordering similar to those obtained when the individual single objectives are used to characterize the objects in question. As such, this requirement of having conflicting objectives is not so much of a disadvantage but rather a desirable characteristic to have for the Pareto approach to be able to provide a different view towards capturing complexity. For example, if the evolutionary runs conducted in our experiments

had maximization of locomotion distance and straight-line walking behavior, then there exists no conflict since the maximum distance will be achieved when the walking behavior results in a path that is maximally straight. In other words, this complexity measure will not provide any additional value towards characterizing complexity unless the system in question can be formulated in some form of conflicting components.

7.6 Chapter Summary

In the introductory sections of this chapter, we have reviewed the concept of complexity as well as a number of existing measures of complexity as applied in the social, biological and physical sciences. We then proposed a Pareto approach towards complexity and revisited each of these areas to show how conflicting measures of complexity can be re-formulated using a Pareto approach to provide an understanding of complexity from different perspectives. Subsequently, we proceeded to demonstrate how this technique can be applied empirically for studying the behavioral and morphological complexities of artificially evolved embodied creatures. In doing so, we found that the morphological complexity of a quadruped creature was lower than the morphological complexity of a hexapod creature as seen from the perspective of an evolving locomotion controller. At the same time, the quadruped was found to be more complex than the hexapod in terms of behavioral complexity. This proposed measure will allow for artificial creatures with evolvable morphologies to be compared in terms of their morphological as well as behavioral complexity in the next chapter.