NSGA WITH ELITISM APPLIED TO SOLVE MULTIOBJECTIVE OPTIMIZATION PROBLEMS

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Abstract

In this paper the effects of elitism in the Nondominated Sorting Genetic Algorithm (NSGA) are analyzed. Three different kinds of elitism: standard, clustering and Parks & Miller techniques are investigated using two test problems. For the studied problems, the Parks & Miller mechanism generated the best results. Finally, the NSGA with Parks & Miller elitism was applied to determine the nondominated front for a storage magnetic energy system and the IEEE 30-node system. Simulation results obtained suggest the effectiveness of this proposed approach to solve real world problems.

I. INTRODUCTION

Many real world optimization problems the aim is to find solutions that are best with regard to various objectives. These optimization problems are named multiobjective and they are usually hard computing. Evolutionary based algorithms have been successfully used to determine the Pareto-Optimal (PO) front for these types of problems because they simultaneously work with a population of points that is crucial to find the non-dominated solution set [1].

In this paper, the Nondominated Sorting Genetic Algorithm - NSGA [2], [3] is examined with respect to different kinds of elitist techniques, that is, standard, clustering [4], and Parks & Miller [5]. Elitism is nowadays a recognized approach to improve the performance of evolutionary based algorithms. This type of approach in multiobjective optimization is not so simple as in single-objective problems. However, in multiobjective evolutionary algorithms - MOEAs, elitist techniques are used to save at generation t non-dominated individuals from the risk of disruption during genetic operations like mutation and crossover and to restore them to the population on subsequent generations.

The performance of NSGA with the aforementioned elitist techniques was investigated taking into account evaluations of three metrics: generational distance, spacing and timing analysis [6]. As a set of solutions is expected in solving multiobjective optimization problems, just one metric is not enough to compare results obtained due to modifications performed on the used algorithm. So, the aim of using these metrics is to provide an understandable sight spotting the differences amongst the results from the NSGA implemented with one of the elitist approaches. Two analytical test functions were used in this analysis: the Schaffer's test functions F_3 and F_5 [7] .

The NSGA with the best elitist approach found in the previous investigation is subsequently

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applied to determine a nondominated front for two problems: *i*) a storage magnetic energy system – SMES proposed as a benchmark problem TEAM22 [8] and *ii*) the IEEE 30-node system. The numerical analysis of SMES problem was performed by using a finite element code with a mesh of triangular elements of first order.

II. DESCRIPTION OF ELITIST TECHNIQUES

Through elitism implementation, one intends to improve the multiobjective genetic based algorithms performance by preventing loss of efficient individuals and reinserting them in the internal population to guide the search. Different elitist approaches were described in the last years with distinct features and some of them are discussed here. Before doing that, it is important to point out the following adopted terminology: internal population (P_{in}) is the on-line or current population in which the genetic operations are performed; external population (P_{ext}) is an auxiliary population that is used to preserve the nondominated points found during the optimization process; *Npop* is the number of sample points or individuals in the internal population.

A. Standard Elitism

The simplest elitist technique in multiobjective optimization is refereed here as Standard Elitism. Basically, at each generation *t*, nondominated individuals of P_{int} are copied to P_{ext}. At the next generation *t+1* or *t+n*, where *n* is an integer number, *Npop* individuals are chosen from P_{ext} and reinserted in P_{int}. Individuals in P_{ext} see your number augmenting at each generation. As any criterion of discarding individuals is applied in P_{ext}, it is possible to have many similar or identical individuals in the internal and external populations.

B. Clustering

The clustering technique begins like standard elitist strategy. At each generation, efficient individuals of P_{int} are incorporated to P_{ext}. When the external population size exceeds a maximum admissible number, say N_{pext}, the dominance criterion is applied on the external individuals, keeping in P_{ext} only the nondominated set. If P_{ext} is reduced under N_{pext} by the dominance criterion, the program backs to increase the external set P_{ext} until N_{pext}. Else, if P_{ext} is not so reduced, the clustering technique is used. This technique consists in dividing the external population in clusters. It is possible to have some clusters represented just by one individual. To separate these elements, all distances between individuals are evaluated and the closest ones are grouped in clusters. The number of clusters is made equal to N_{pext}. The individual representing a cluster will be the one which mean distance to all others individuals in its cluster is the smallest. It is named centroid. Once N_{pext} centroids are found, they constitute an elite set. From this set, some individuals are randomly picked and reinserted in P_{int}, and the process continues. These clusters are formed in the objective space.

C. Parks and Miller

The P&M technique consists in incorporating to P_{ext}, at each generation, the efficient individuals of P_{int}. When P_{ext} size exceeds a threshold, say N_{pext}, the dominance criterion is applied,

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eliminating all dominated solutions. If P_{ext} continues bigger than N_{pext} the distance criterion is applied. It consists in measuring the distance between the external individuals, taking two per turn, and if they are within some distance just one of them is kept in P_{ext}, chosen randomly. This distance is measured in the objective space. After that some individuals of P_{ext} are reinserted in P_{int}.

III. MEASUREMENT OF ALGORITHM PERFORMANCE

To evaluate the multiobjective optimization performance, three practical metrics: generation distance, spacing and timing analysis are used to permit spotting the differences between each analyzed technique [6].

For simplicity, the nondominated solutions found are termed Pn, on the other hand, the Pareto optimal solutions are designated Po. Similarly, the associated fronts for each of these solution sets are named as PFn and PFo.

D. Generational Distance

This metric is a value representing how "far" PFn is from PFo and is defined as:

$$G = \frac{\left(\sum_{i=1}^{n} d_i^{\,p}\right)^{1/p}}{n} \tag{1}$$

where n is the number of nondominated solutions, $p=2$, and d_i is the Euclidean distance on the objective space between each point in PFn and the nearest of PFo.

E. Spacing

This metric is the spread (distribution) measuring of objectives throughout PFn, and it is defined as:

$$S = \sqrt{\frac{1}{n-1}\sum_{i+1}^{n}(\bar{d} - d_i)^2} \tag{2}$$

$$d_i = \min_{j \neq i}\left(\left| f_1^{\,i}(\vec{x}) - f_1^{\,j}(\vec{x}) \right| + \ \ldots\ + \left| f_M^{\,i}(\vec{x}) - f_M^{\,j}(\vec{x}) \right|\right)$$
$$i, j \in [1, \ldots, n]$$

Above, $\bar{d}$ is the mean value of all d_i, n is as before and M is the size of the vector of objectives. A value of zero for this metric indicates all members of PFn are equidistantly spaced.

F. Timing Analysis:

A third metric is used to make the comparison of execution time. This parameter can only be used

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to compare procedures implemented within the same language, and ran under the same CPU. Besides, the internal parameters of one GA must be equivalent to other GA under analysis.

IV. COMPARISON OF ELITIST TECHNIQUES

The three elitist techniques described were implemented in the Nondominated Sorting Genetic Algorithm - NSGA. For convenience of notation, the NSGA with standard elitism will be refered to as NSGA-S, clustering as NSGA-C and NSGA-PM to the Parks and Miller technique. They were tested with Schaffer test functions F_3 and F_5 [7]. The obtained results are compared with the NSGA without elitism (NSGA-N).

The genetic parameters used to Schaffer F3 function were: maximum number of generations = 50; crossover probability = 0.9; mutation rate = 0.01. The population size is a specific entry for each technique. The internal population is denoted as Npint and the external one as Npext. The sum Npint + Npext was kept constant. These parameters received the following values: Npint = 80 for the NSGA-N, Npint = 30 and Npext = 50 for all others, NSGA-S, NSGA–C and NSGA–PM. It should be observed that the number of individuals in the internal population for NSGA with elitism is small if compared when it works without elitism. Also, the size of Npext in NSGA-S and NSGA-PM is dynamically augmented when is necessary to capture more efficient points.

The Schaffer F5 function has two variables in the decision space. The same genetic parameters were used, except Npint and Npext. In this problem, Npint was made equal to 200 for NSGA-S and 40 in the other cases. Npext was taken equal to 160 for NSGA with elitist techniques. The total population (Npint + Npext) here was bigger than that used for Schaffer F3.

V. RESULTS

Tables I and II give the number of noninferior solutions found and the results of the three metrics aforementioned to evaluate the GA performance in the four simulated cases for each function F_3 and F_5, respectively. Figs. 1 and 2 show the nondominated fronts obtained for these two test functions.

Although NSGA-C and NSGA-PM have less individuals than NSGA-S, they could represent better the Pareto-optimal front (PFo), once the NSGA-S has many individuals equal or very similar. These results show that elitist techniques can enhance the NSGA performance.

Table I - Function Schaffer F_3

Algorithm	Efficient Individuals	Generational Distance (G)	Spacing (S)	CPU Time (p.u)
NSGA-N	68	265.3×10^{-5}	4.129	2.85
NSGA-S	577	1.349×10^{-5}	10.840	1.58
NSGA-C	50	1.766×10^{-5}	3.066	1.70
NSGA-PM	92	0.2364×10^{-5}	3.770	1.00

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Table II - Function Schaffer F5

Algorithm	Efficient Individuals	Generational Distance (G)	Spacing (S)	CPU Time (p.u)
NSGA-N	165	7.803×10^{-3}	1.627	5.53
NSGA-S	1141	2.116×10^{-3}	1.684	3.05
NSGA-C	160	11.57×10^{-3}	0.625	2.12
NSGA-PM	214	6.445×10^{-3}	0.573	1.00

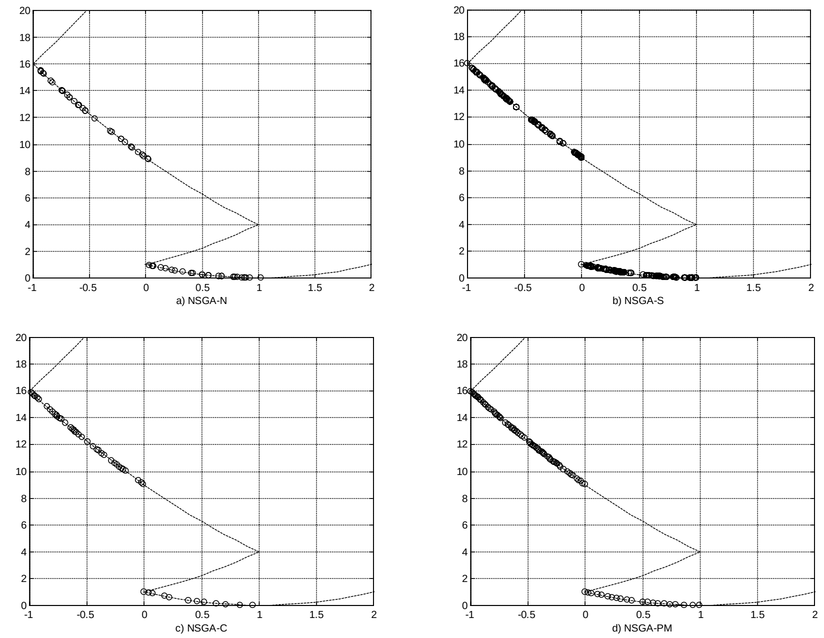

Fig. 1. Nondominated fronts (F_3)

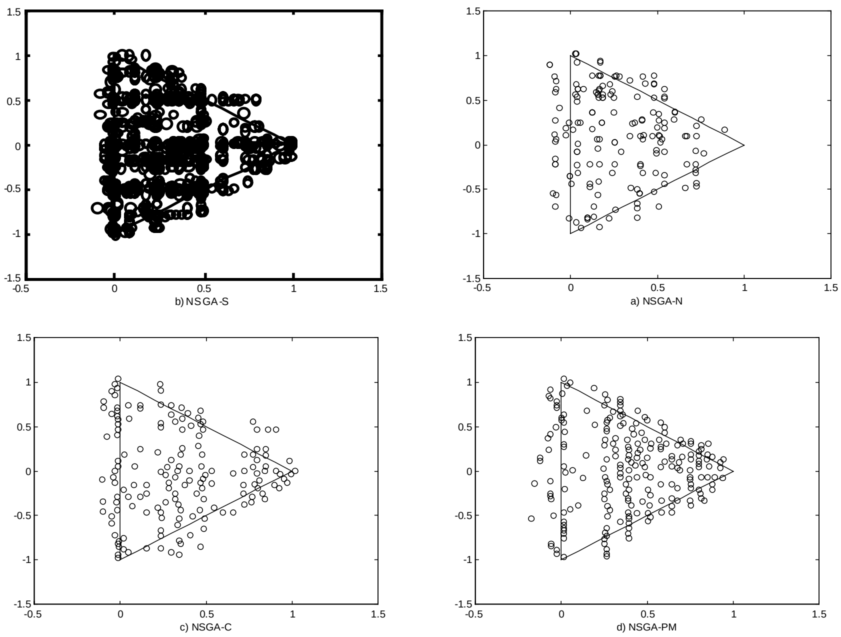

Fig. 2. Nondominated fronts (F_5)

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As can be seen, the best results for the test functions were found using NSGA-PM. It spent less time and gave a nondominated set larger than NSGA-N and NSGA-C. Although the NSGA-S problem got many individuals, they are equal or very similar. Besides, spacing and generational distance parameters show that the PFn found by NSGA-PM is very close to PFo and the spread of the nondominated solutions in objective space throughout the PFn is very small. These results show that Parks & Miller elitism approach could give good representation of the Pareto-optimal front without premature convergence problem. These results are possible due to the characteristic of this approach that does not impose any limitation to the number of dissimilar efficient individuals added to the external set. Furthermore, Parks & Miller spent less computation effort, about 60%, than clustering elitism in both cases. This is possible due to the simplicity to eliminate closer individuals in NSGA-PM, whereas NSGA-C needs more time to group, to choose representative individuals and to eliminate the others into the clusters.

VI. MULTIOBJECTIVE OPTIMIZATION OF BENCHMARK PROBLEM 22

The "TEAM benchmark problem 22" was chosen to demonstrate the performance of the NSGA with P&M elitism in multiobjective optimization problem in electromagnetics. The aim of this problem is to optimize the Super-Conducting Energy Storage configuration [**8**] as shown in Fig. 3, with respect to two objectives and one constraint, to ensure the minimal stray field (f_1), 180MJ of stored energy (f_2) and that physical quench condition (g) is met.

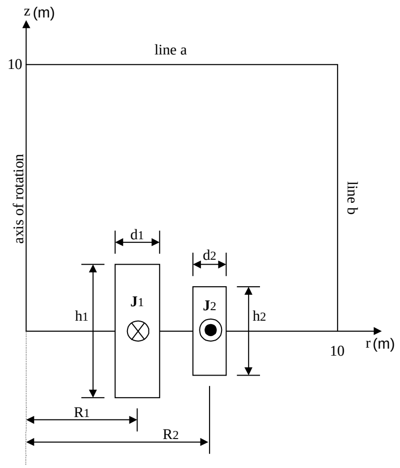

Fig.3 – SMES basic configuration (not to scale)

Table III – Three Variables Problem Ranges

	R_1 m	H_1 m	D_1 m	R_2 m	H_2 m	D_2 m	J_1 MA/m^2	J_2
Min	-	-	-	2.6	0.408	0.1	-	-
Max	-	-	-	3.4	2.2	0.4	-	-
Fixed	2.0	1.6	0.27	-	-	-	22.5	-22.5

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The problem was specified with three continuous variables keeping the others fixed. The details are shown in Table III. Mathematically, the multiobjective optimisation problem was stated as:

$$\min \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{Bmatrix} \left(\dfrac{B_{stray}}{B_{normal}}\right)^2 \\ \dfrac{|\,Energy - E_{ref}\,|}{E_{ref}} \end{Bmatrix} \tag{3}$$

where Bnormal = 3*10-3 (T) and Eref = 1.8*108 (J). The quench physical condition was neglected in this simulation.

The genetic parameters: population size, maximum number of generations, crossover and mutation probabilities used were taken equal to 40, 80, 0.9 and 0.01, respectively.

Figure 4 shows the nondominated front found, traced in the objective space. It can be seen that there is a uniform distribution of the points in the graph except between 0.15 and 0.33 on the F1 axis. In this range, it is hard and not so important to get many individuals because there are just a few variations on the value of F2.

Using the same results, the noninferior front can be seen using different axes. Figure 5 shows the nondominated front in a graph where y-axis represents the storage energy and x-axis represents the stray field.

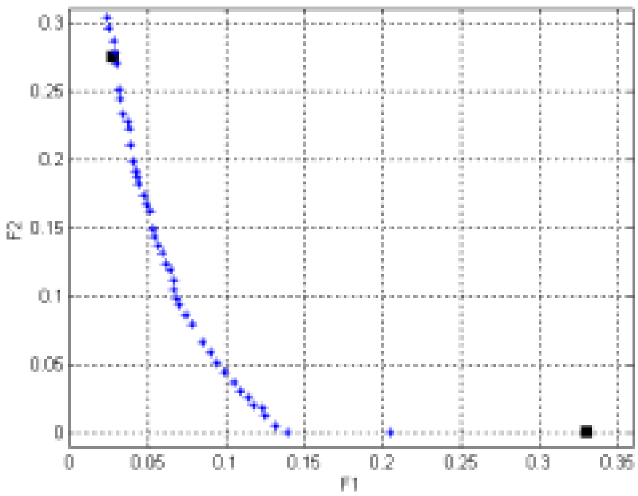

Fig. 4. Nondominated front. F1 x F2

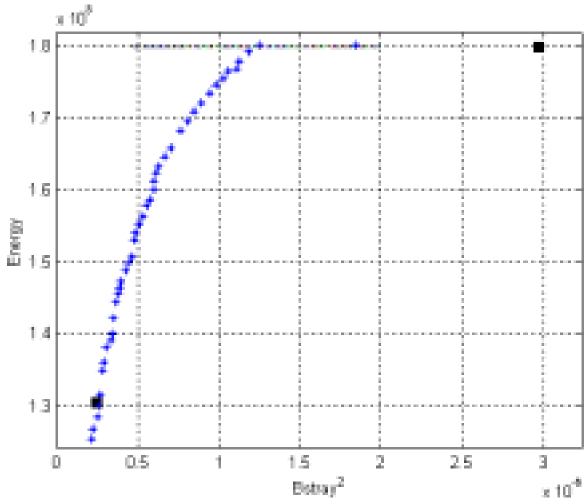

Fig. 5. Nondominated front. Energy x Bstray2

VII. Multiobjective Optimal Load Flow

Electrical Power Systems require an operational state that attends the system load, saves the available resources and respects the operational limits of the equipments. The problem consists of finding conditions of operation that satisfies these requirements simultaneously, through minimization of objectives with non-linear constraints. It is a nonlinear and a non-convex problem.

Mathematically, the multiobjective problem of minimizing the system transmission loss and improving the voltage profile can be stated as:

minimize:

$$\bar{f}(\bar{x}) = \begin{Bmatrix} f_1(\bar{x}) \\ f_2(\bar{x}) \end{Bmatrix} \tag{4}$$

subjected to:

$$\bar{g}(\bar{x}) = \begin{Bmatrix} \bar{g}_1(\bar{x}) \\ \bar{g}_2(\bar{x}) \end{Bmatrix} \le \begin{Bmatrix} \bar{0} \\ \bar{0} \end{Bmatrix} \tag{5}$$

$$\bar{h}(\bar{x}) = \begin{Bmatrix} \bar{h}_1(\bar{x}) \\ \bar{h}_2(\bar{x}) \end{Bmatrix} = \begin{Bmatrix} \bar{0} \\ \bar{0} \end{Bmatrix} \tag{6}$$

where:

$f_1(\bar{x}) = P_{LOSS}$ is the objective that denotes the system transmission loss;

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$f_2(\bar{x}) = \sum_{i=1}^{NB} \left(V_i - V_i^{esp}\right)^2$ is the objective function that represents the voltage profile;

$\bar{g}_1(\bar{x}) = \left\{ \begin{matrix} \left[(V_1 - V_1^{min})(V_1 - V_2^{max})\right] \\ \cdots \\ \left[(V_{NBPQ} - V_{NBPQ}^{min})(V_{NBPQ} - V_{NBPQ}^{max})\right] \end{matrix} \right\}$ is the vector of inequality constraint functions that limits the voltage on P,Q-nodes;

$\bar{g}_2(\bar{x}) = \left\{ \begin{matrix} \left[(Q_1 - Q_1^{min})(Q_1 - Q_2^{max})\right] \\ \cdots \\ \left[(Q_{NBPV} - Q_{NBPV}^{min})(Q_{NBPV} - Q_{NBPV}^{max})\right] \end{matrix} \right\}$ is a vector of inequality constraint functions that limits the reactive power on P,V-nodes;

$\bar{h}_1(\bar{x})$ and $\bar{h}_2(\bar{x})$ are the load-flow equations.

Moreover, $\bar{x}$ represents the control variables, *NB* is the total number of nodes in the system, *NBPQ* the number of P,Q-nodes, *NBPV* the number of P,V-nodes, V_i is the calculated voltage to the i-th node, V_i^{esp} is the specified (rated) voltage to the i-th node, V_j^{min} and V_j^{max} are the inferior and superior voltage limits to the j-th P,Q-node, Q_j, Q_j^{min} and Q_j^{max} are, respectively, the calculated reactive power, the inferior and superior limits to the j-th P,V-node.

The IEEE 30-node system, see Fig. 6, was chosen to demonstrate the performance of NSGA with P&M elitism in solving the multiple load flow problem. The control variables are the voltages on the nodes 1, 2, 5, 8, 11 and 13. The inferior and superior limits for these variables were defined as 0.90pu and 1.10pu respectively. The limits of reactive power in the controlled voltage busbars are shown in Table IV.

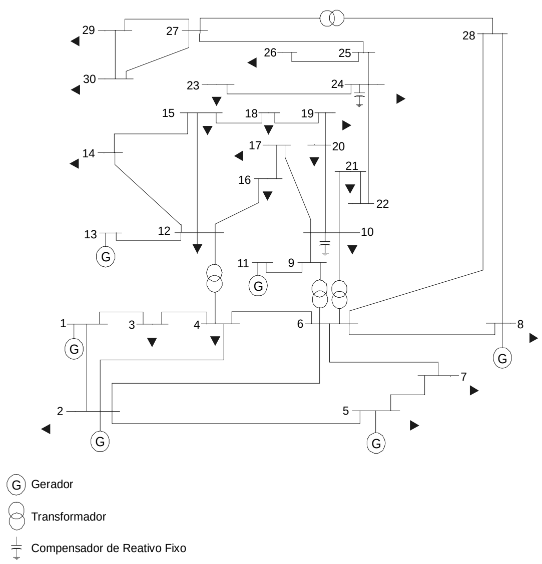

Fig. 6. IEEE 30-node system.

Table IV - Reactive Bounds

Busbar number	Q_{min} (MVar)	Q_{max} (MVar)
2	-20.0000	100.0000
5	-15.0000	80.0000
8	-15.0000	60.0000
11	-10.0000	50.0000
13	-15.0000	60.0000

This problem was solved using NSGA and the inequalities constraints were handled as objectives [9], [10], together with the P&M elitism that was modified as described in [9]. The first front, Fig. 7.a, was obtained using population size and generation number equal to 40 and 150, respectively. The load flow problem was solved using the fast decoupled method [11]. The maximum error accepted to consider convergence was 0.001pu for both active and reactive power. The maximum iteration number was 10. The results presented in Fig 7.b were obtained using the same parameters except the generation number that was taken equal to 500. Both results show the effectiveness of the multiobjective approach used.

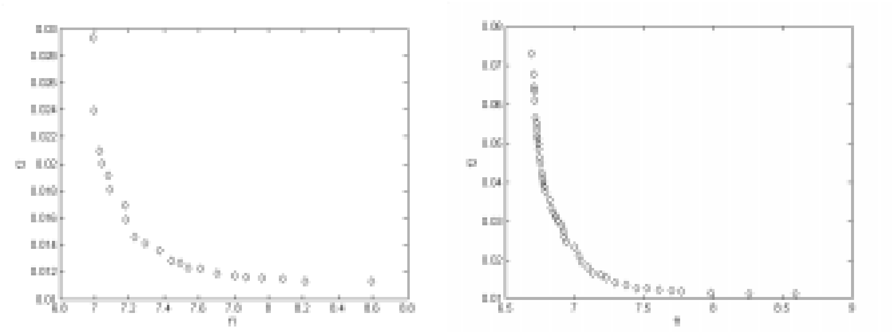

Fig. 7. Non-dominated front found for the IEEE 30-node system:
a) Nbpop = 40, Nbgen = 150; b) Nbpop = 40, Nbgen = 500. [f1 (MW) X f2 (V^2)]

VIII. CONCLUSION

The results obtained on analyzing the three elitist techniques suggest the effectiveness in mapping nondominated fronts in multiobjective optimization. The results with NSGA together with the Parks & Miller elitism approach pointed out the potential of this approach in all cases investigated. The results of both SMES problem and the IEEE 30-node system show that the proposed approach can be used to solve complex multiobjective optimization problems.

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