MOEA Test Suite Generation, Design & Use

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1 Introduction

Many research efforts use numeric Multiobjective Optimization Problems (MOPs) as examples to show or judge Multiobjective Evolutionary Algorithm (MOEA) performance. However, there is limited comprehensive discussion of MOP landscape issues in the MOEA literature, and there is often no explanation of why (the selected) numeric MOPs may be appropriate MOEA test functions. Extensive experimentation and analyses concerning MOEA parameters, components, and approaches are also generally lacking.

Most MOEA researchers' *modus operandi* is an algorithm's comparison (usually the researcher's own new and improved variant) against some other MOEA by analyzing results for specific MOP(s) (Schaffer's VEGA and MOP-$F2$ are typical). Results are often "clearly" shown in graphical form indicating the new algorithm is more effective. However, these empirical, relativistic experiments are incomplete as regarding general MOEA comparisons. The literature's history of visually comparing MOEA performance on non-standard and unjustified numeric MOPs does little to determine a given MOEA's actual efficiency and effectiveness. A standard suite of numeric functions exhibiting *relevant* MOP domain characteristics can provide the necessary common comparative basis [2].

The MOEA community's limited *de facto* test suite contains various functions, many of whose origins and rationale for use are unknown. [1, 2]. The lack of complex mathematical MOEA performance assessment tests implies that identification of appropriate functions to objectively determine MOEA efficiency and effectiveness is required. Thus, a documented MOP test suite is an asset to MOEA research. We provide various MOPs for use in a standardized MOEA test suite. Supporting these proposals is a discussion of general test suite issues, the MOP domain, and various MOEA evaluations.

The NFL theorems imply that if problem domain knowledge is not incorporated into the algorithm domain, no formal assurances of an algorithm's general effectiveness exist. Previously proposed Single-Objective EA (SOEA) test suites examine an EA's capability to "handle" various problem domain characteristics. These suites incorporate relevant search space features to be addressed by some particular EA instantiation. For example, De Jong suggests five single-objective optimization test functions (**F1 - F5**) and Michalewicz five single-objective *constrained* optimization test functions (**G1 - G5**). Whitley et al and Goldberg et al. offer other formalized SOEA test suites. Particular SOEA instantiations are subjected to generic test suites like these and judged on their computational performance.

2 MOP Domain Features

We assert that like SOEA optimization problems, numeric MOPs may be suitable representatives of real-world problems. Any modeled real-world problem is done so mathematically in a functional form, but MOPs arguably capture more information about the modeled problem as they allow incorporation of several functions (objectives). Regardless, modeling a real-world problem may result in a numeric or combinatorial MOP, one that is perhaps simple, perhaps complex. The MOP may contain continuous or discrete (e.g., integer-constrained) functions or even a mix of the two.

It is generally accepted that EAs are useful search algorithms when the problem domain is multidimensional (many decision variables), and/or the search space is very large. Many numerical examples used by MOEA researchers do *not* explicitly meet this criteria. Of the 30 distinct numerical MOPs in the literature (both constrained and unconstrained [2], all but three use at most two decision variables and the majority use only two objective functions. This implies that unless the search space is very large (at the least), MOEA performance claims/comparisons based on these functions may not be meaningful. The MOEA may be operating in a problem domain not particularly well-suited to its capabilities or perhaps one which is not challenging.

Any proposed MOEA test suite must offer functions spanning known MOP characteristics. Particularly, it must contain "MOEA challenging" functions. In order to then identify appropriate functions for inclusion relevant MOP domain characteristics must be identified and considered. We use the 30 known examples in the literature as the basis for discussion [2]. These MOPs incorporate 2-3 functions and 0-12 side constraints. Van Veldhuizen [1] presents a complete set of figures showing representations of the Pareto optimal set (genotype) and the Pareto optimal front (phenotype) for each of these MOPs.

We have identified salient MOP domain characteristics viewed from an MOEA perspective and classified under a genotype and phenotype rubric. These high-level characteristics were identified from empirical evaluations, whose representation (and succeeding interpretation) may slightly change based upon underlying computational resolution and graphical presentation. Thus, test suite functions should encompass (combinations of) all these possible genotype and phenotype characteristics; convex vs non-convex (concave), continuous vs discontinuous, discrete, symmetric vs non-symmetric, uniformity vs non-uniformity, unimodal, multi-modal, and scalable vs non-scalable.

3 Numeric MOEA Test Suite Functions

The proposed MOPs address the issues discussed. Note that we initially restrict functions to those with no side constraints. Their mathematical formulations (which may be slightly revised from the originals or as we elsewhere proposed [3]) are reflected in [1].

Seven MOPs are initially selected. MOP1 (convex) and MOP2 (concave) are arguably MOEA "easy" MOPs. MOP2 and MOP4 (discontinuous) are scalable as regards decision variable dimensionality. MOP6 (discontinuous) is scalable as regarding the number of Pareto curves in the Pareto front. MOP5 and MOP7 are tri-objective MOPs. All are nonlinear, and several show a lack of symmetry in both solution and objective space. Taken together these MOPs begin to form a coherent basis for MOEA comparisons. Other relevant MOP characteristics can also be addressed by selecting additional MOPs for test suite inclusion [1]. Thus, we also propose side-constrained numeric and combinatorial MOEA test functions.

Real-world applications can be considered for inclusion in any comprehensive MOEA evaluation. These MOPs may be numeric, non-numeric, or both, and are probably more constrained (in terms of resources) than the problems we have presented. We note that many real-world applications employ fitness function software (i.e., computational fluid dynamics or computational electromagnetic) requiring extensive calculations, data interchange, and data mapping. Other possible characteristics include deception and isolated multimodal functions.

4 MOEA Testing

Using our validated test suite, various MOEAs have been evaluated. These currently include the MOGA, NPGA, and the NSGA along with our new explicit building block GA, the MOMGA. The MOMGA design approach is based upon Goldberg's"messy" GA. Comprehensive testing using the MOEA test suite indicates that the MOMGA is statistically as good as or better than the other approaches. Such a statistical analysis involved the development and analysis of appropriate metrics (genotype and phenotype) generally based upon distance metrics [1, 2]. Our analysis also addresses the impact of the various MOEA crossover and mutation parameter values, selection operators, and population sizes that effectively direct search towards the extended Pareto front. Using our test suite based upon an validated set of metrics, MOEA comparisons are made more precise and the results more informative for specific MOEA selection.

References

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